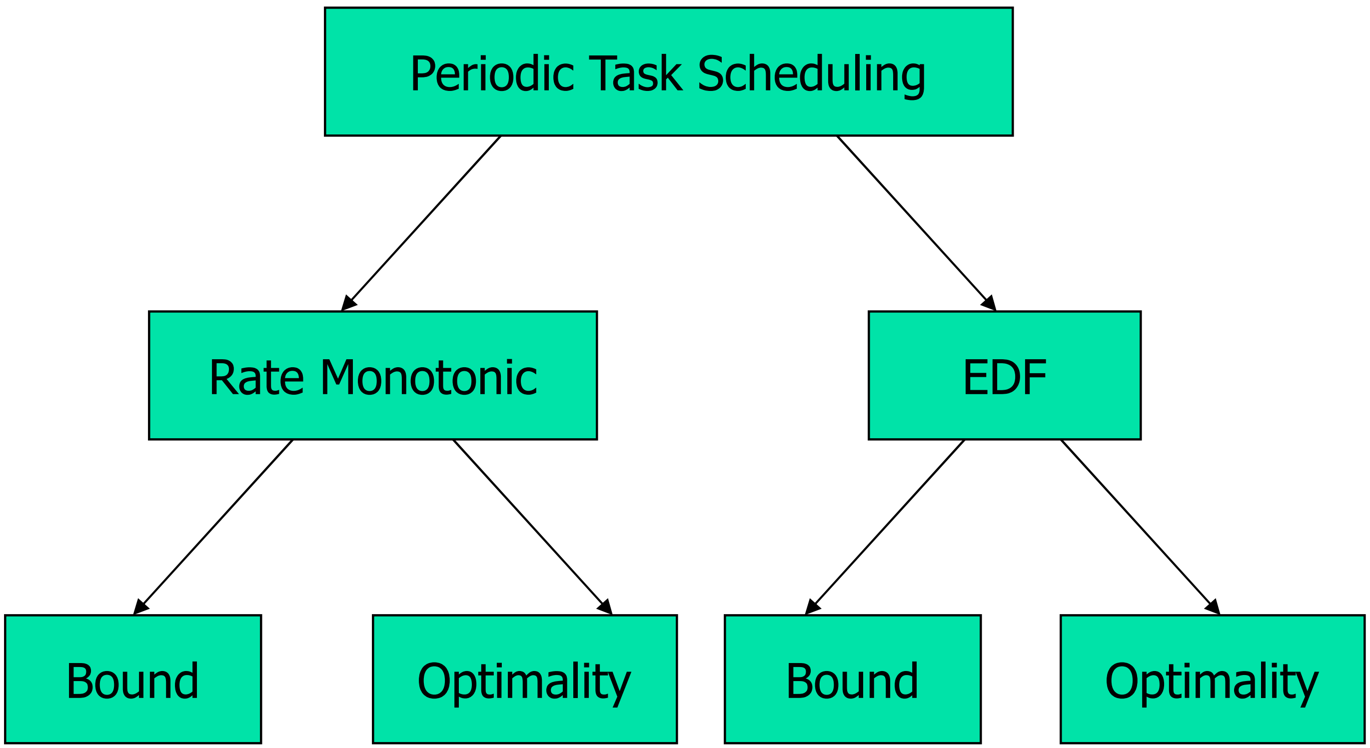
Real-time Scheduling

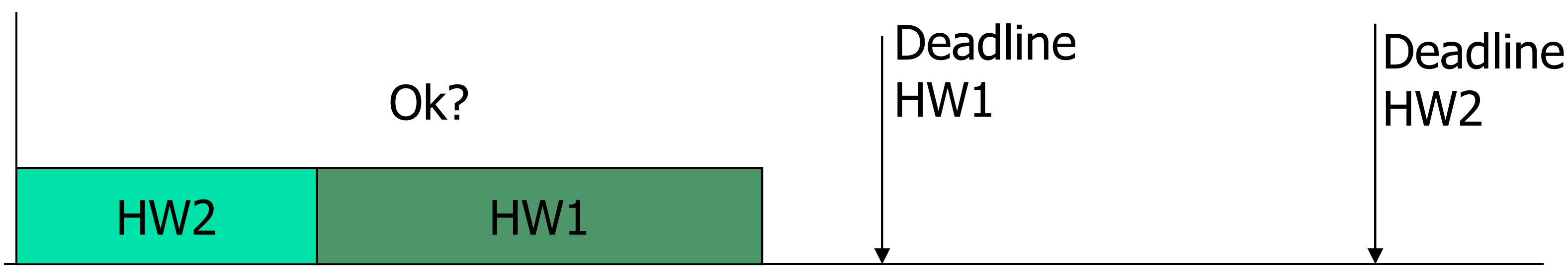
Main Results on Periodic Task Scheduling

Main Results in Real-time Scheduling of Periodic Tasks



Earliest Deadline First (EDF) Optimality Result

- EDF is the optimal dynamic priority scheduling policy
  - It can meet all deadlines whenever the processor utilization is less than 100%
  - Intuition:
    - You have HW1 due tomorrow and HW2 due the day after, which one do you do first?
    - If you started with HW2 and met both deadlines you could have started with HW1 (in EDF order) and still met both deadlines
    - EDF can meet deadlines whenever anyone else can



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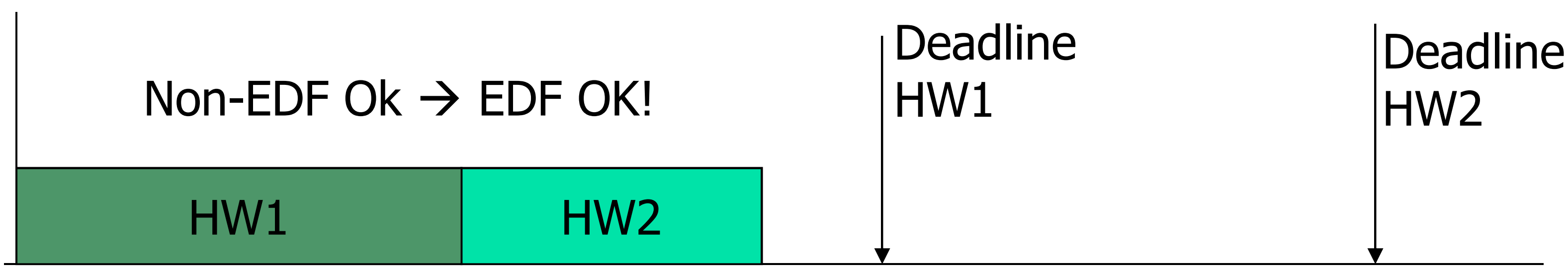
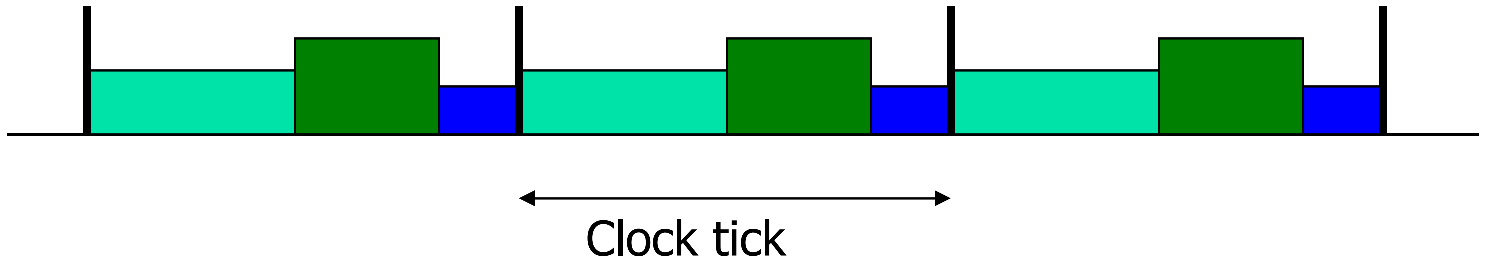


When can EDF Meet Deadlines?

- Consider a task set where:

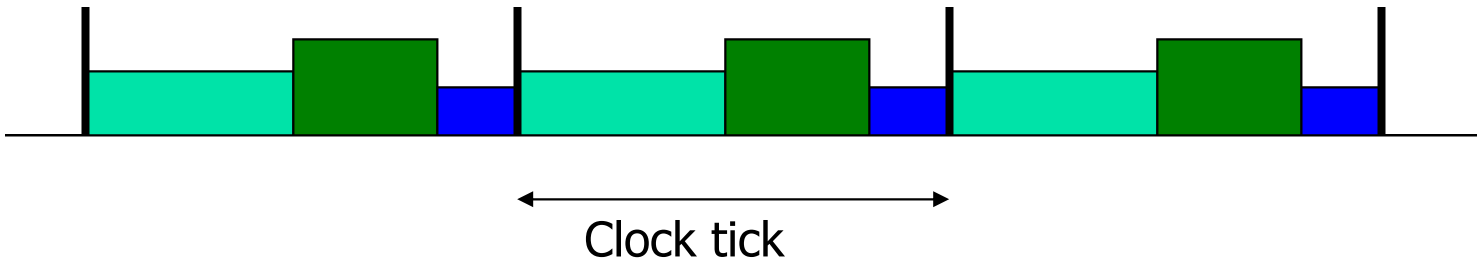
$$\sum_i \frac{C_i}{P_i} = 1$$

- Imagine a policy that reserves for each task i a fraction f_i of each clock tick, where $f_i = C_i / P_i$



Utilization Bound of EDF

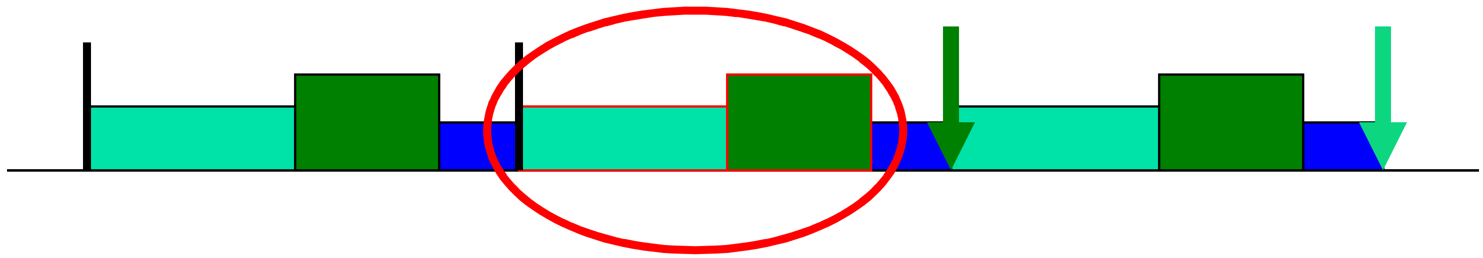
- Imagine a policy that reserves for each task i a fraction f_i of each time unit, where $f_i = C_i / P_i$



- This policy meets all deadlines, because within each period P_i it reserves for task i a total time
  - Time $= f_i\, P_i = (C_i / P_i)\, P_i = C_i$ (i.e., enough to finish)

Utilization Bound of EDF

- Pick any two execution chunks that are not in EDF order and swap them



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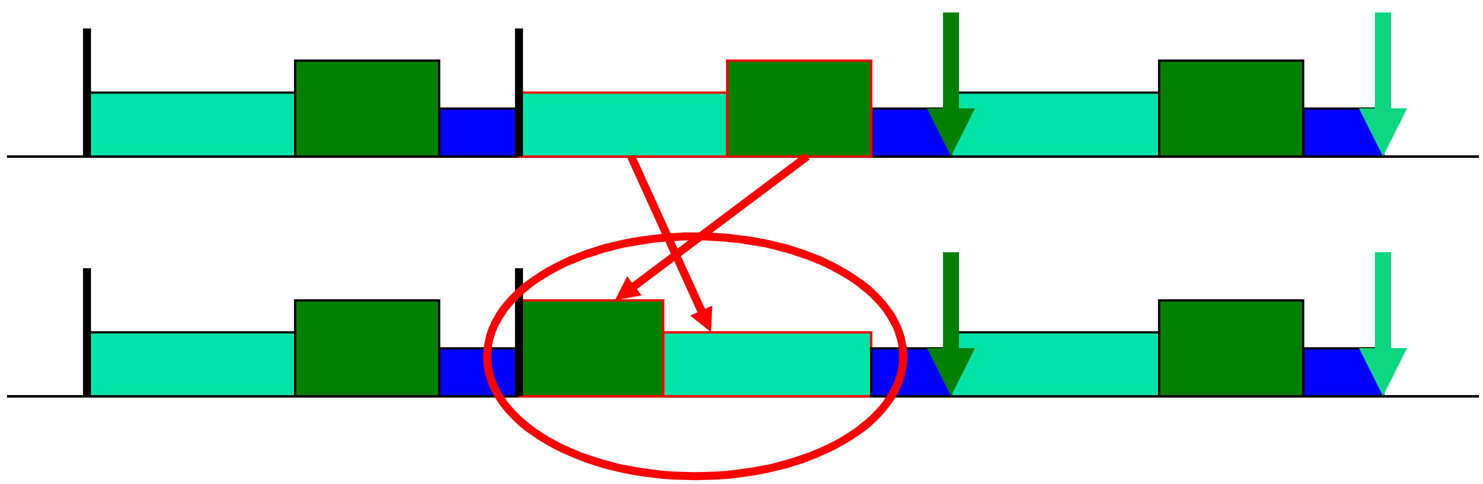
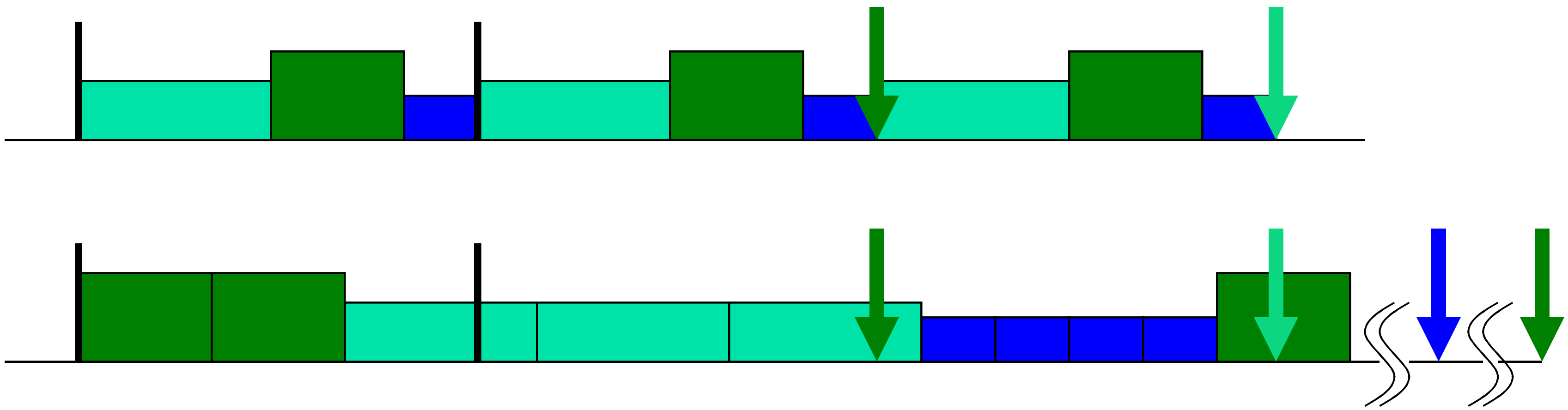


- Still meets deadlines!

Utilization Bound of EDF

- Pick any two execution chunks that are not in EDF order and swap them



- Still meets deadlines!
- Repeat swap until all in EDF order
  → EDF meets deadlines

Rate Monotonic Scheduling

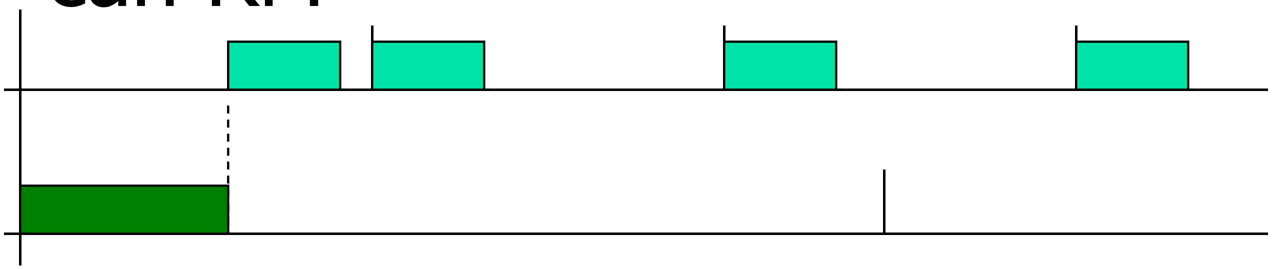
- Rate monotonic scheduling is the optimal fixed-priority scheduling policy for periodic tasks (with period = deadline).

The Worst-Case Scenario

- Consider the worst case where all tasks arrive at the same time.
- If any fixed priority scheduling policy can meet deadline, rate monotonic can!

Optimality of Rate Monotonic

- If any other policy can meet deadlines so can RM



Policy X meets deadlines?

Optimality of Rate Monotonic

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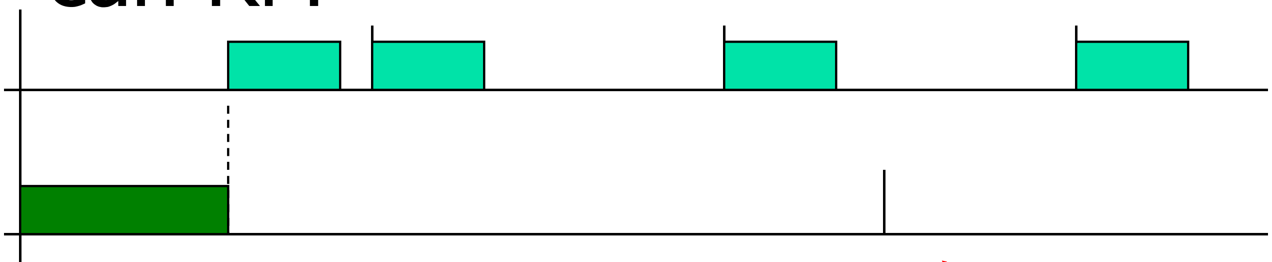
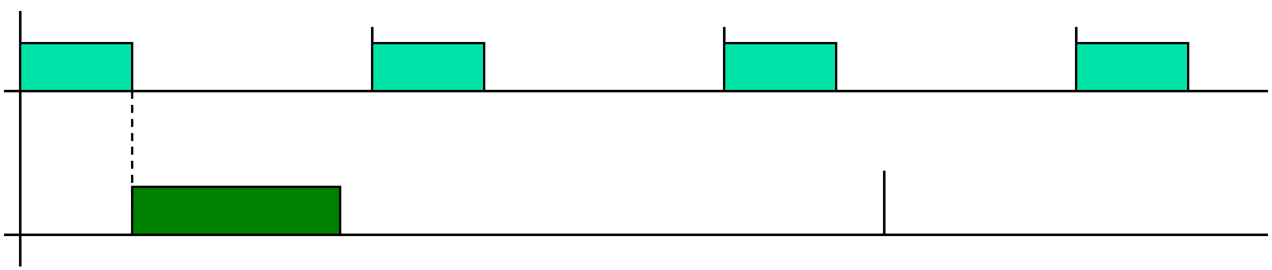


Policy X meets deadlines? YES

→ RM meets deadlines



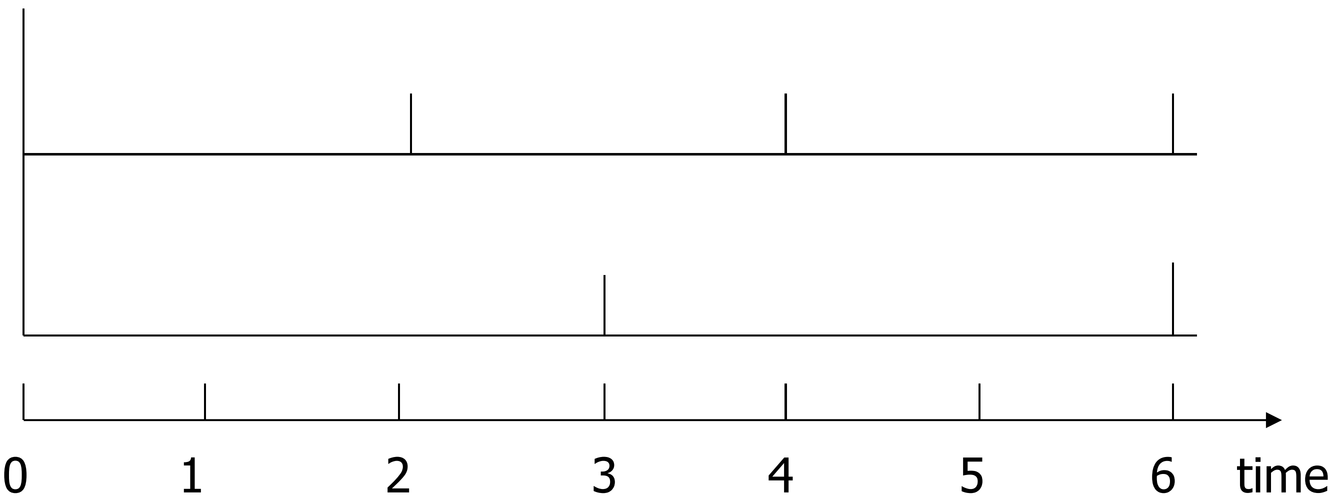
Utilization Bounds

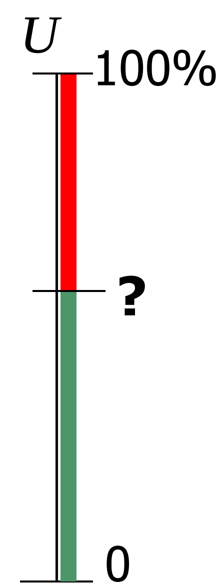
- Intuitively:
  - The lower the processor utilization, U, the easier it is to meet deadlines.
  - The higher the processor utilization, U, the more difficult it is to meet deadlines.
- Question: is there a threshold U_{bound} such that
  - When $U < U_{bound}$ deadlines are met
  - When $U > U_{bound}$ deadlines are missed

Example (Rate-Monotonic Scheduling)

Task 1
P_1=2
C_1=1

Task 2
P_2=3
C_2=1.01





$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = \frac{1}{2} + \frac{1.01}{3} \approx 83.3\%$$

- Question: is there a threshold U_{bound} such that
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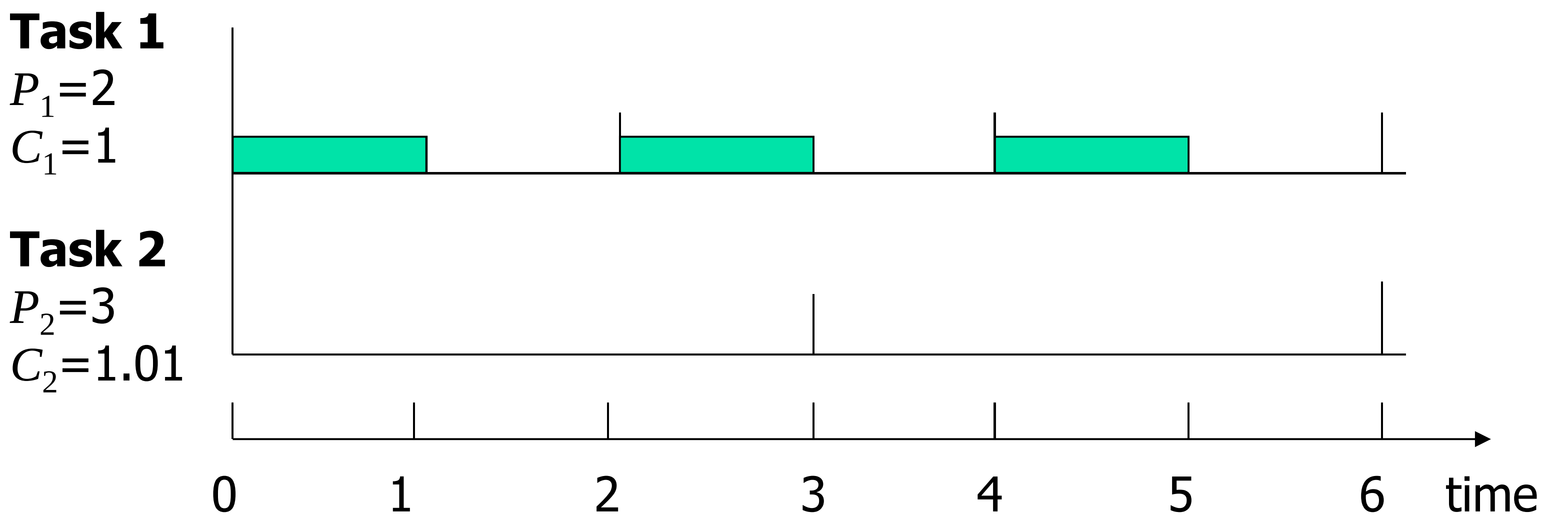

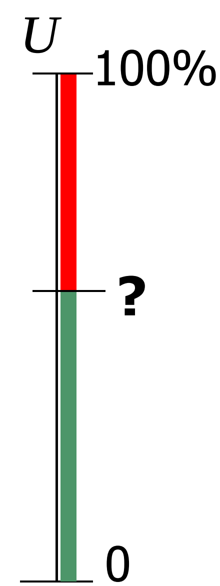

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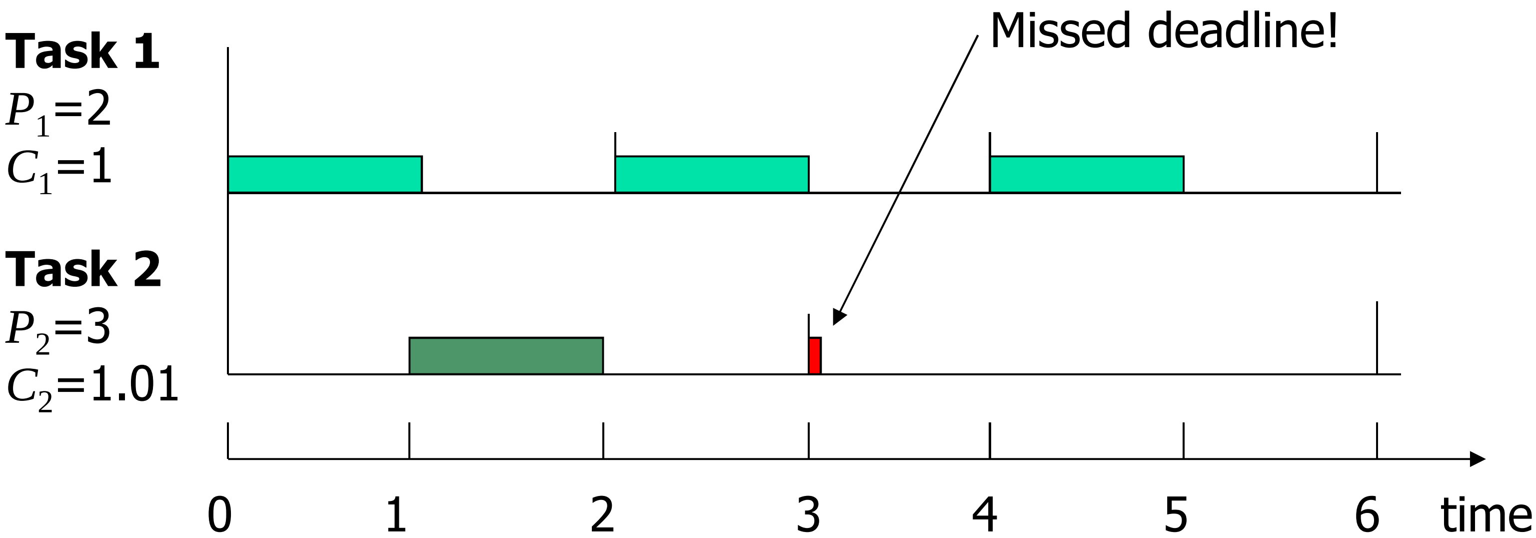
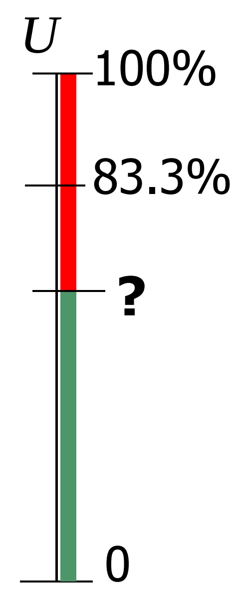
(Rate-Monotonic Scheduling)





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Unschedulable

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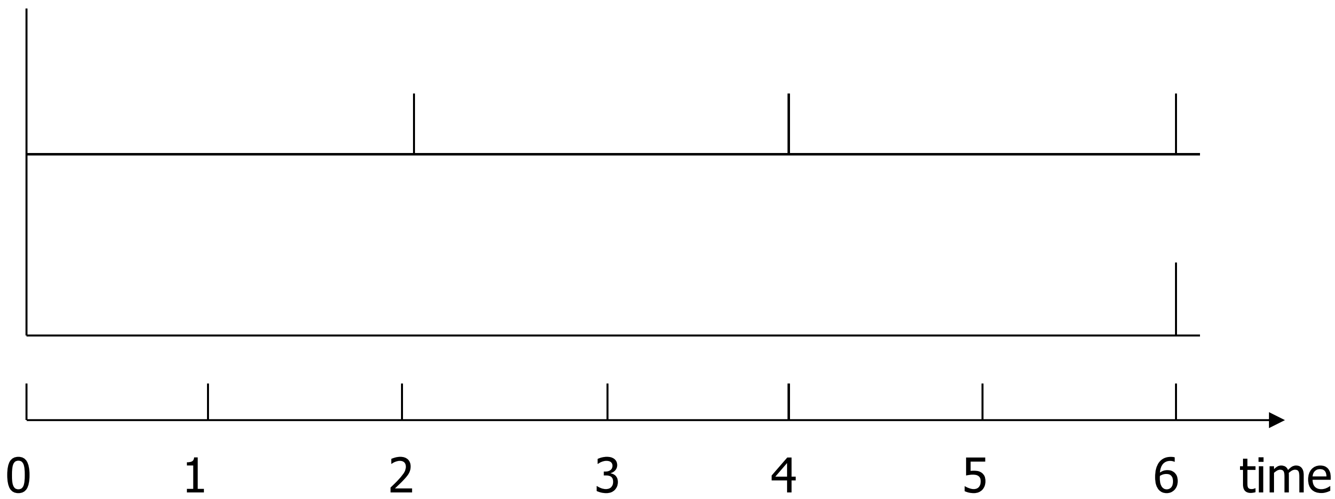
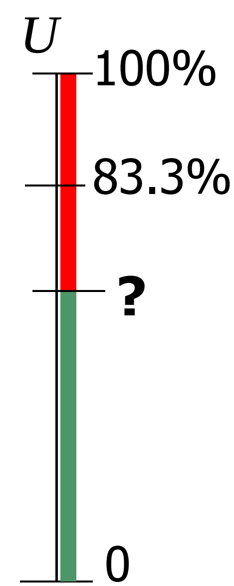
Another Example (Rate-Monotonic Scheduling)

Task 1
P_1=2
C_1=1

Task 2
P_2=6
C_2=2.4





$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = \frac{1}{2} + \frac{2.4}{6} = 90\%$$

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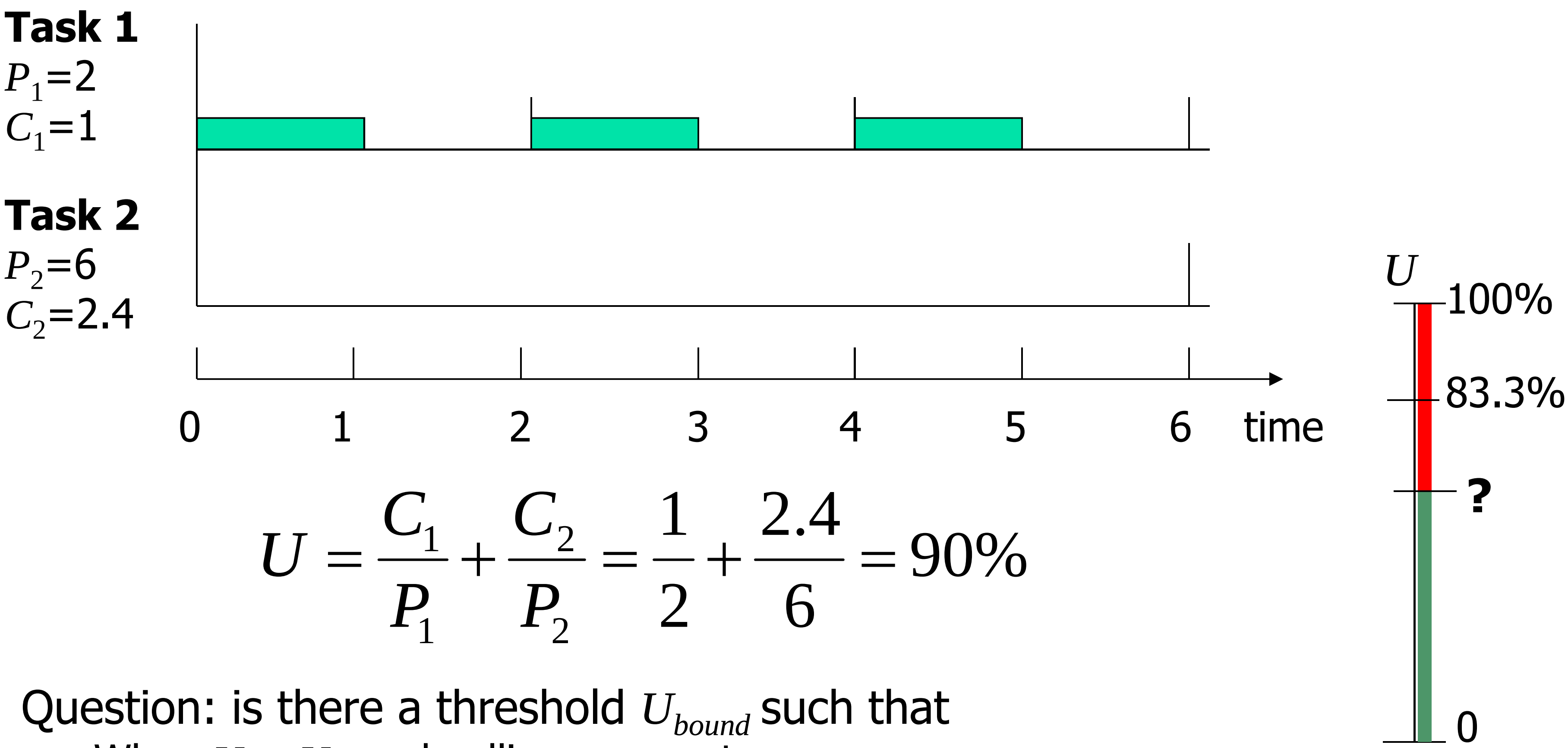

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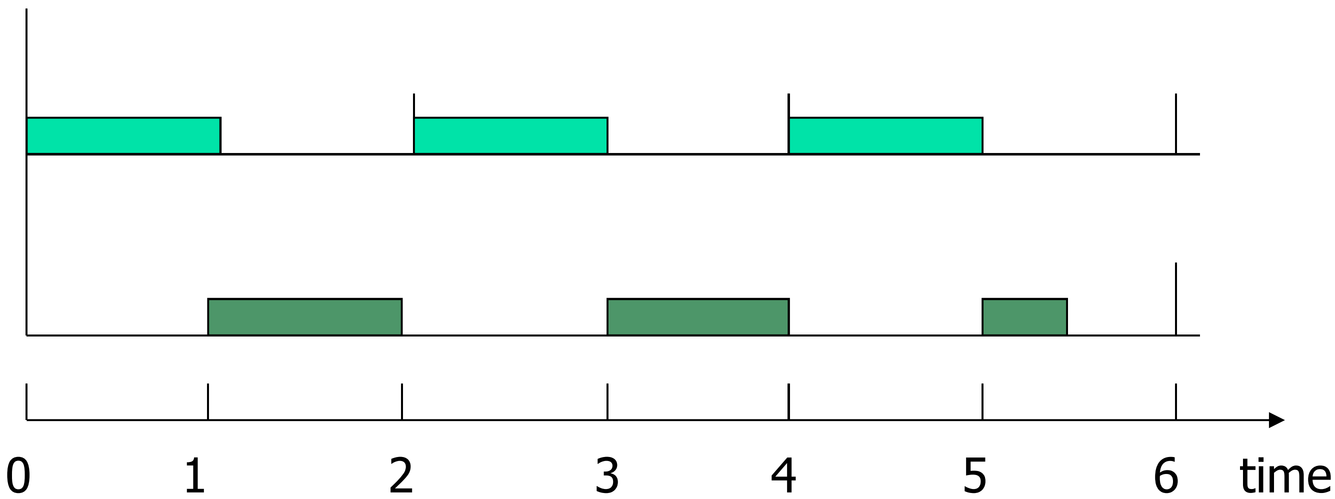

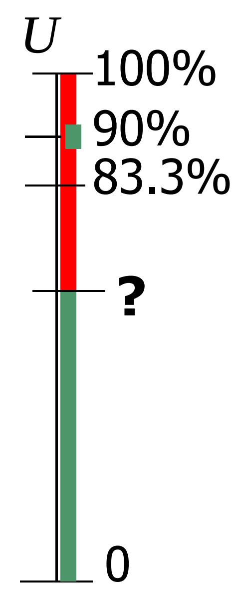

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Schedulable!

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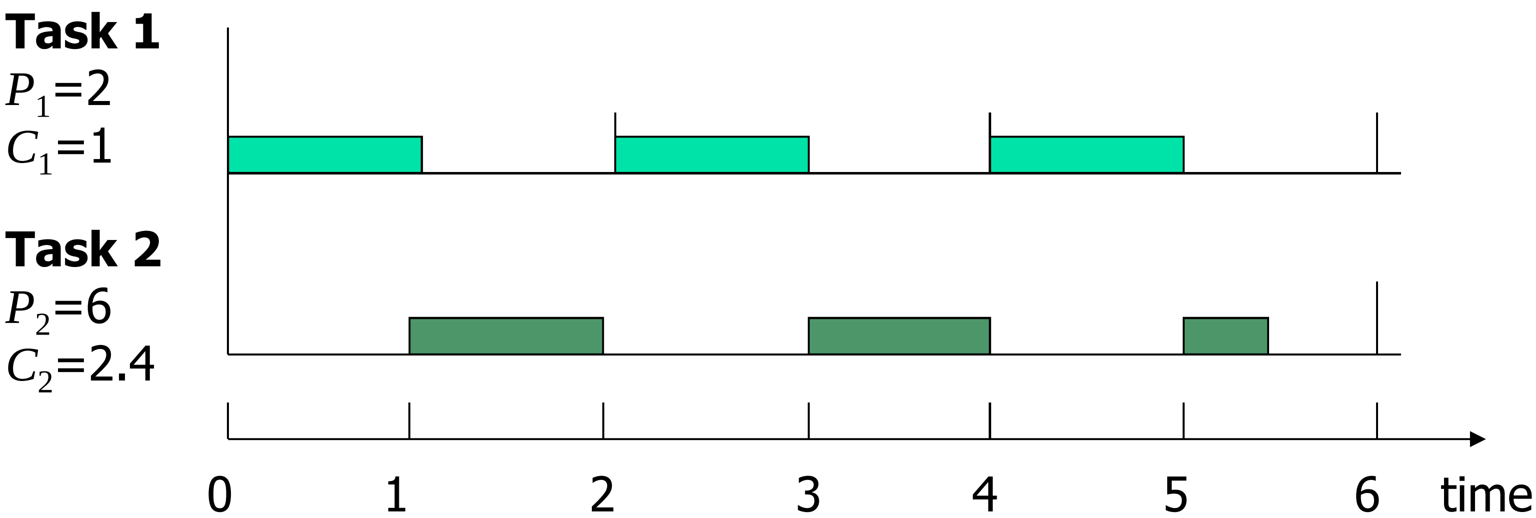

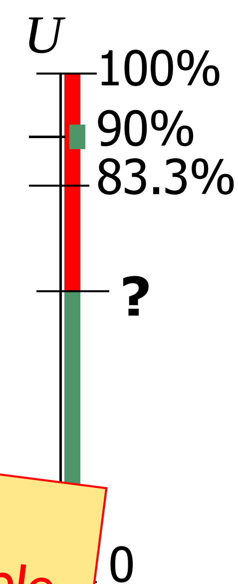

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Schedulability depends on task set!
No clean utilization threshold between schedulable and unschedulable task sets!

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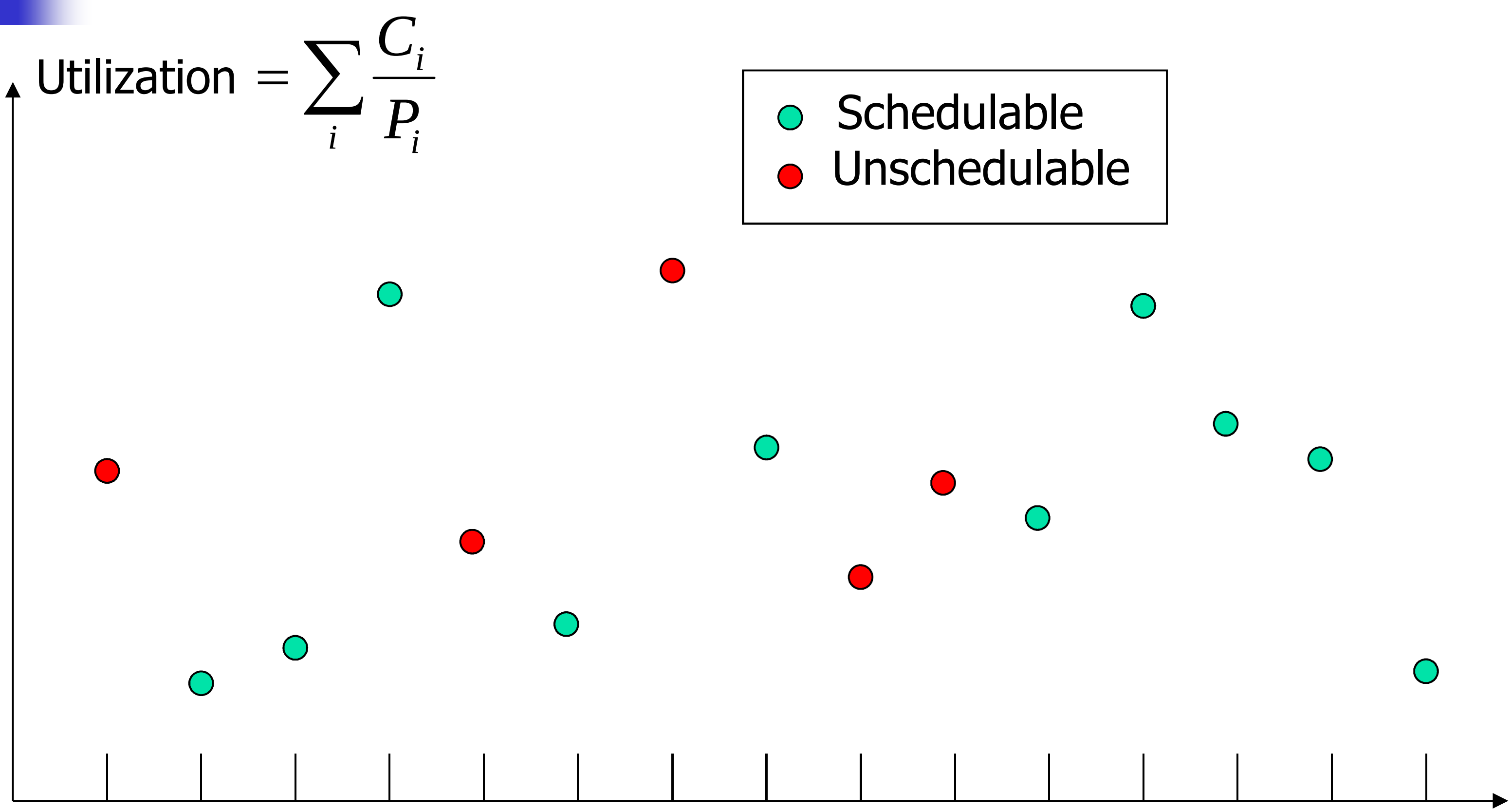
A Conceptual View of Schedulability



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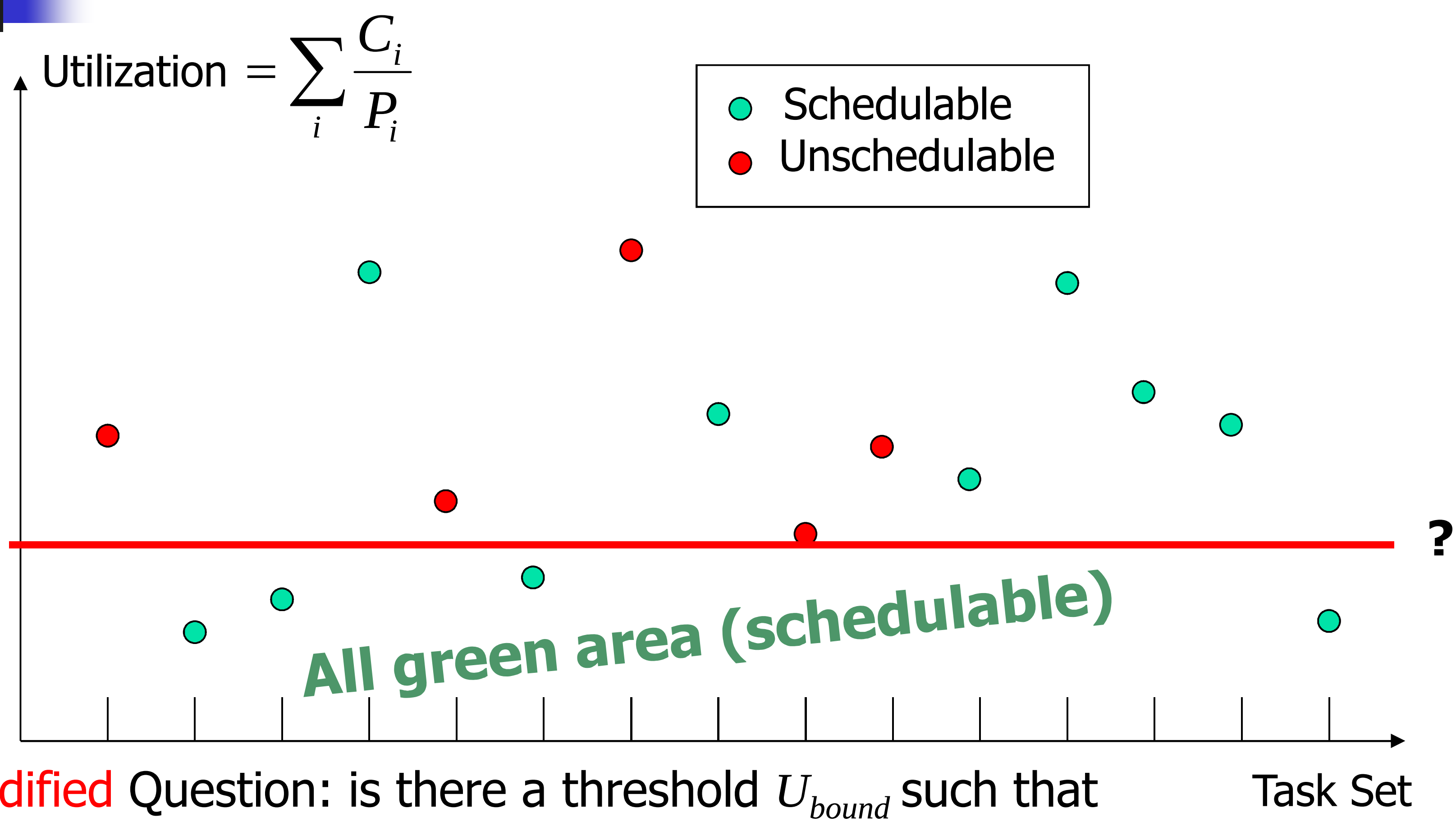

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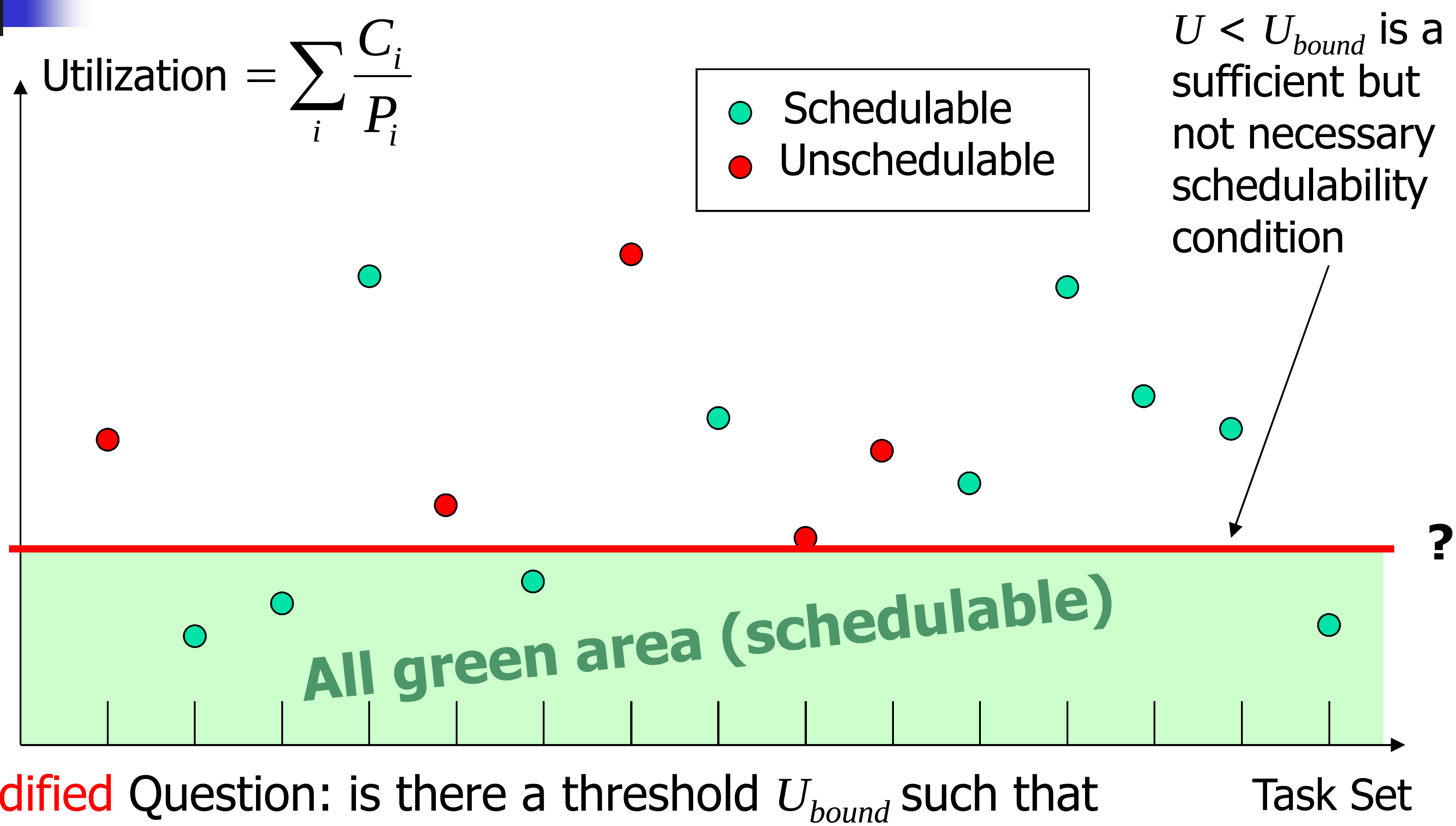

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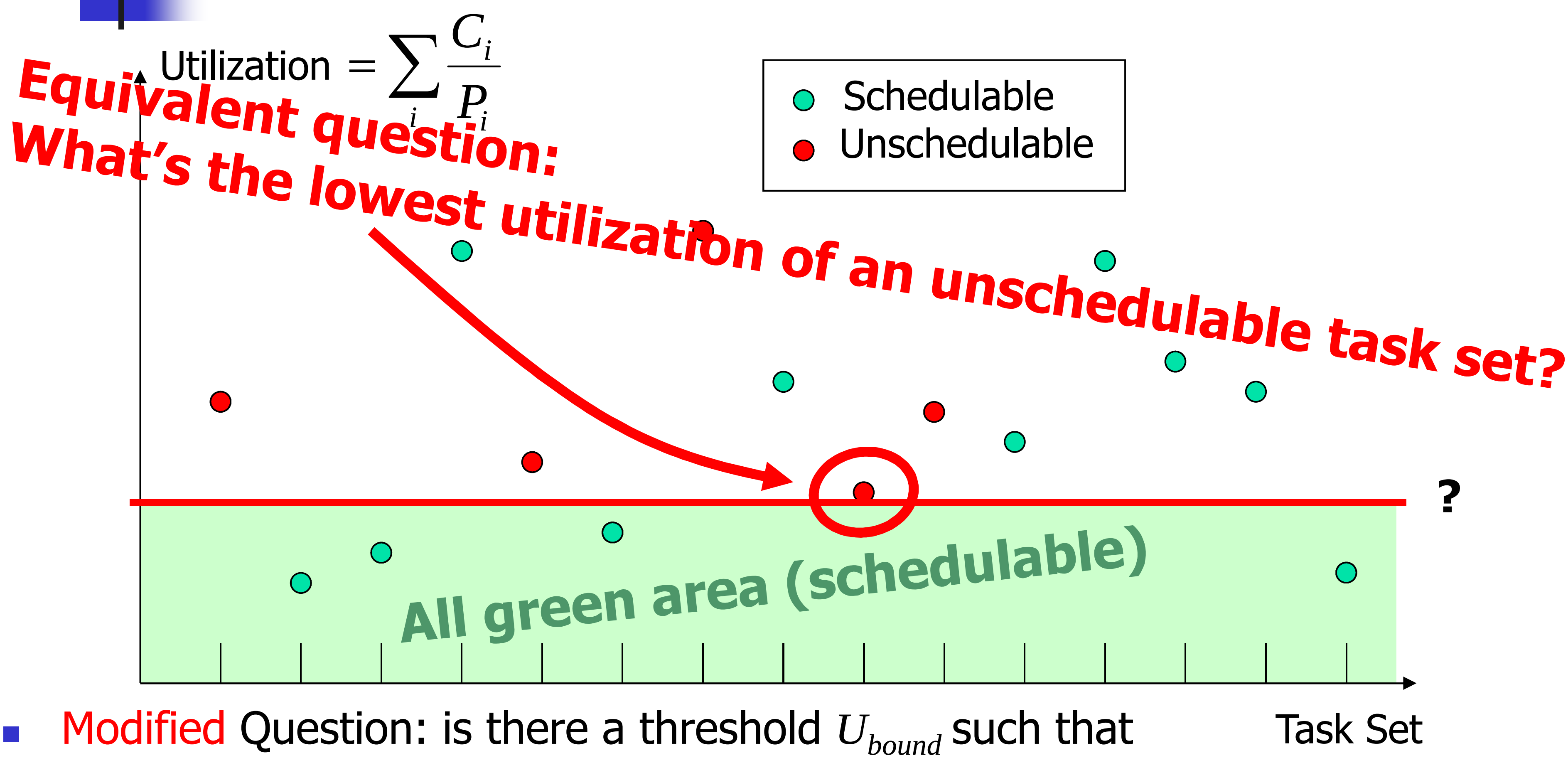

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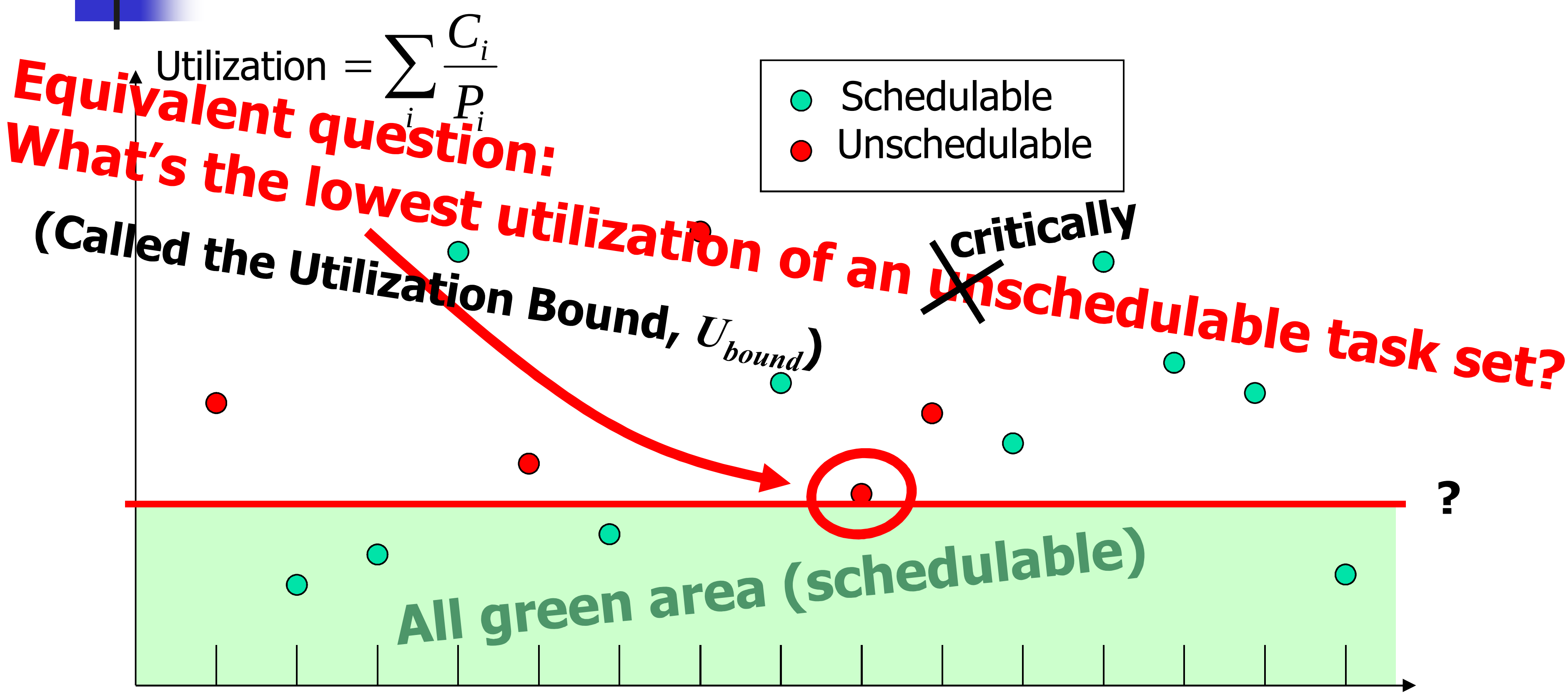

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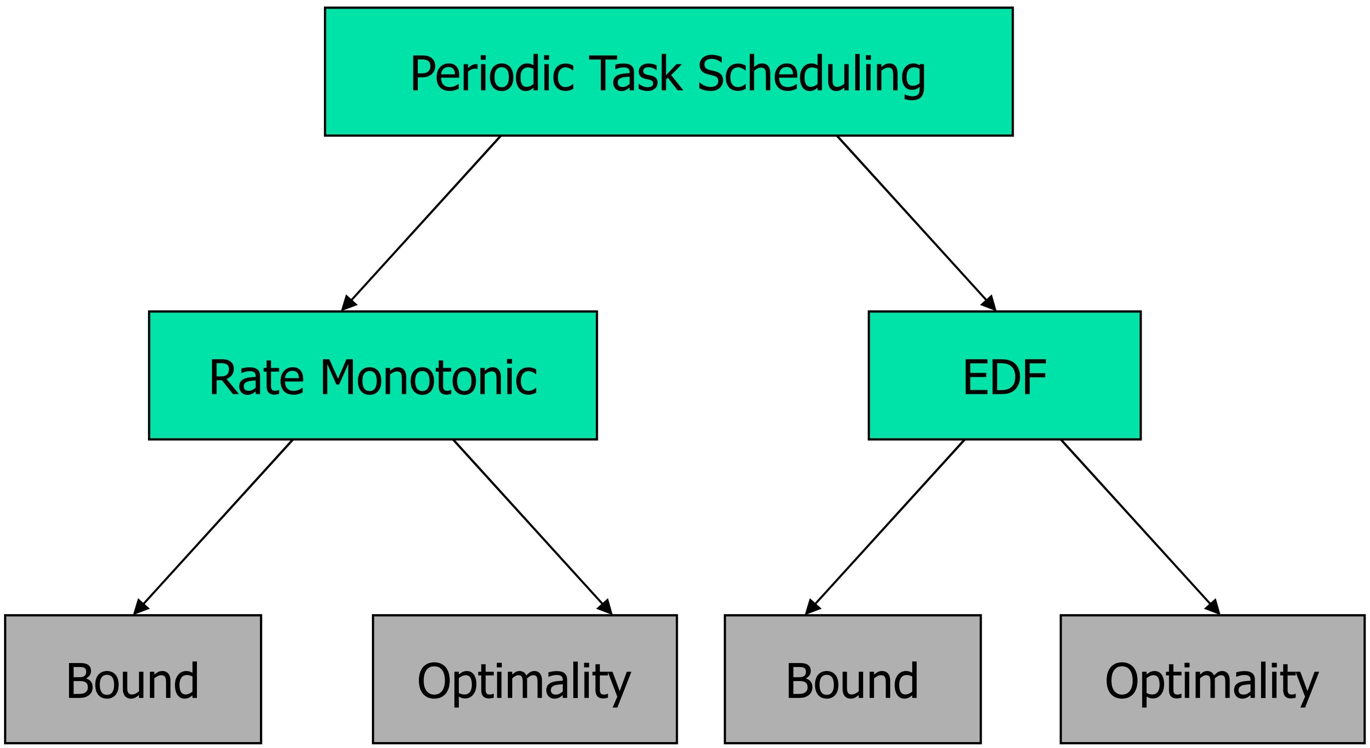
The Schedulability Condition

For n independent periodic tasks with periods equal to deadlines, the utilization bound is:

$$U = n\left(2^{1/n} - 1\right)$$

$$n \to \infty \quad U \to \ln 2$$

Done Today



Periodic Task Scheduling
Rate Monotonic
EDF
Bound
Optimality
Bound
Optimality

Deriving the Utilization Bound for Rate Monotonic Scheduling

- Consider a simple case: 2 tasks

Find some task set parameter x
such that
   Case (a): $x<x_o$ → $U(x)$ decreases with x
   Case (b): $x>x_o$ → $U(x)$ increases with x
Thus $U(x)$ is minimum when $x=x_o$
   Find $U(x_o)$

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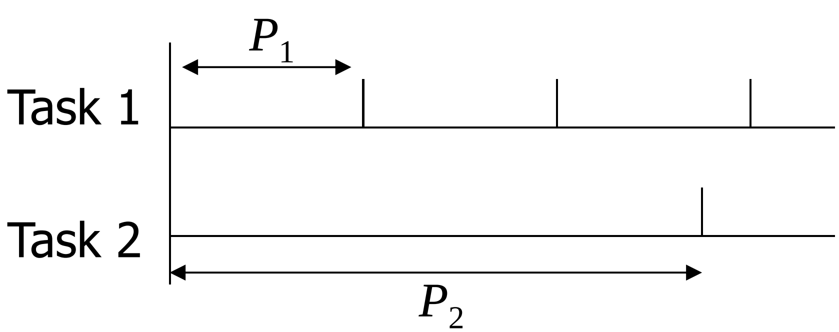

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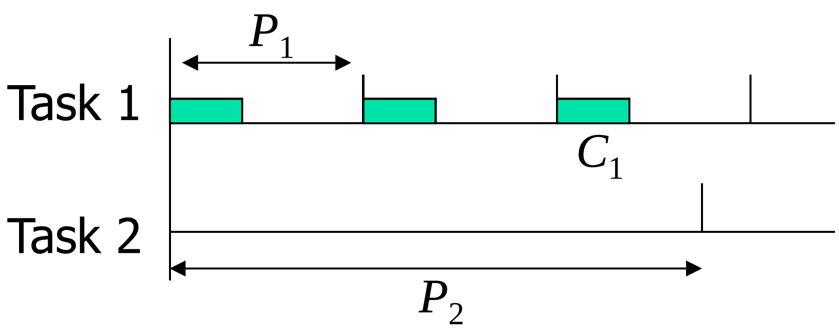

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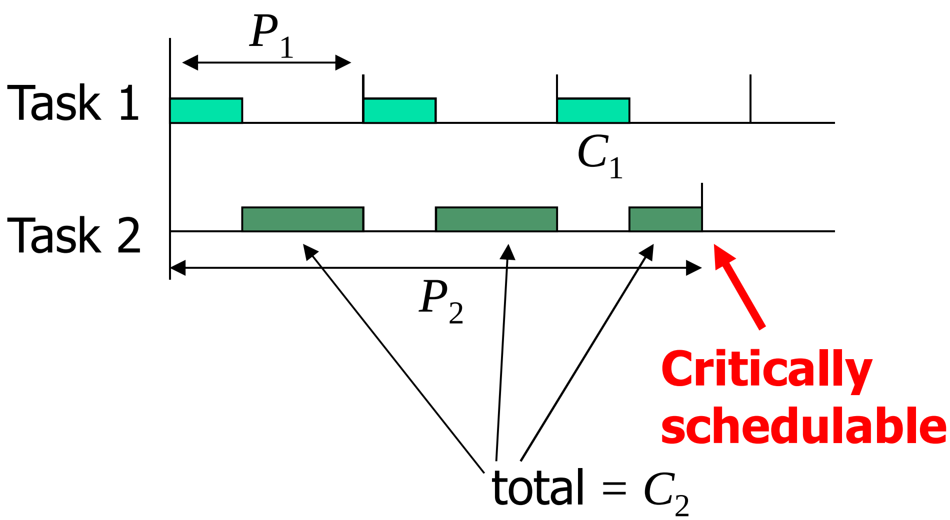

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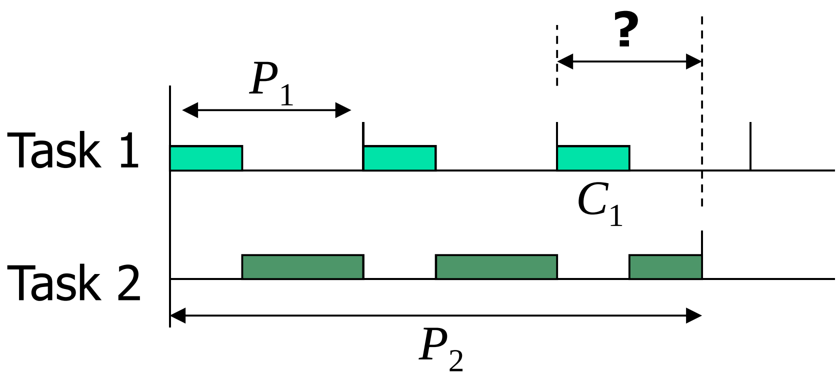

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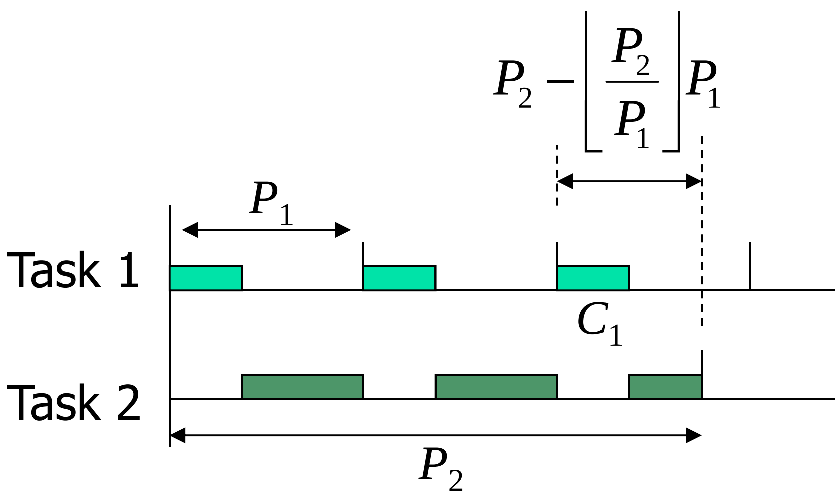

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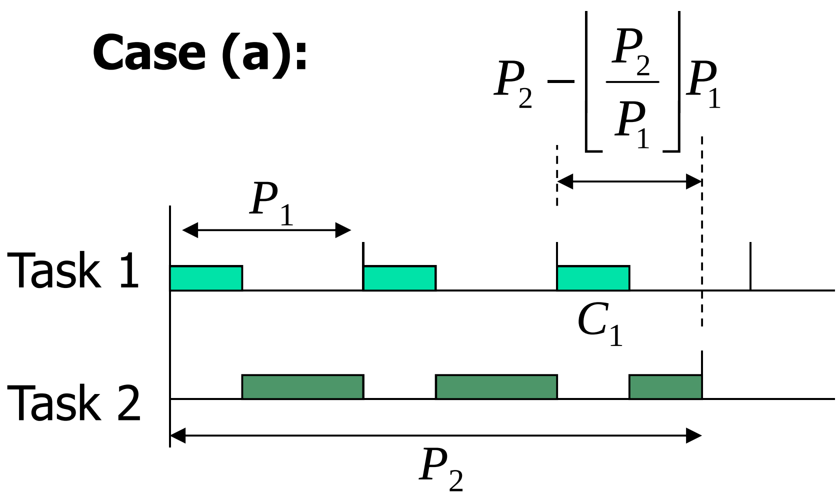

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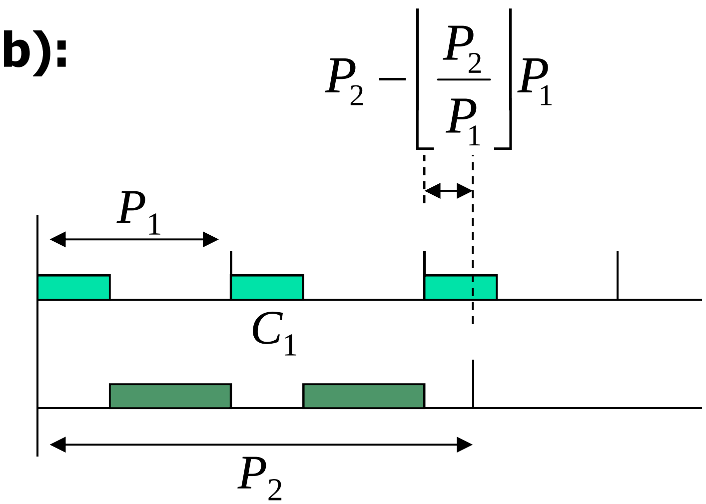

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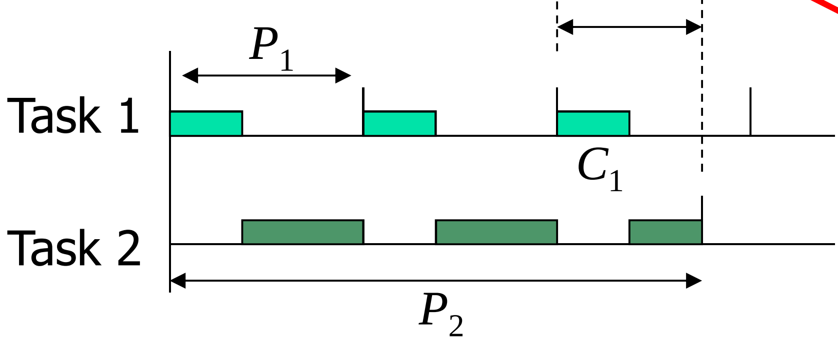
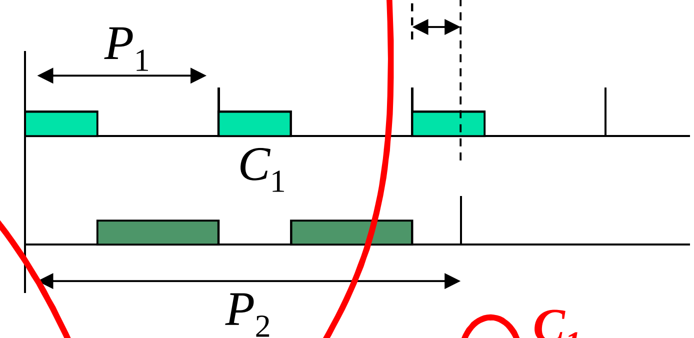


Case (b): $C_1 > P_2 - \left\lfloor \frac{P_2}{P_1} \right\rfloor P_1$



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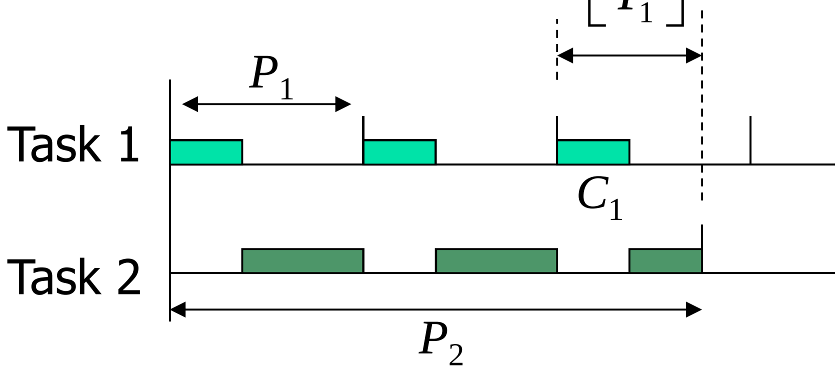

$$C_2 = P_2 - C_1\left(\left\lfloor \frac{P_2}{P_1} \right\rfloor + 1\right)$$

$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = 1 + \frac{C_1}{P_2}\left[\frac{P_2}{P_1} - \left\lfloor \frac{P_2}{P_1} \right\rfloor - 1\right]$$

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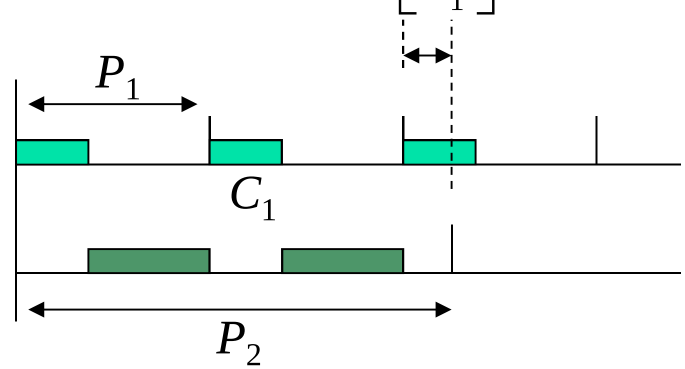

$$C_2 = (P_1 - C_1)\left\lfloor \frac{P_2}{P_1} \right\rfloor$$

$$U = \frac{P_1}{P_2}\left\lfloor \frac{P_2}{P_1} \right\rfloor + \frac{C_1}{P_2}\left[\frac{P_2}{P_1} - \left\lfloor \frac{P_2}{P_1} \right\rfloor\right]$$

Deriving the Utilization Bound for Rate Monotonic Scheduling

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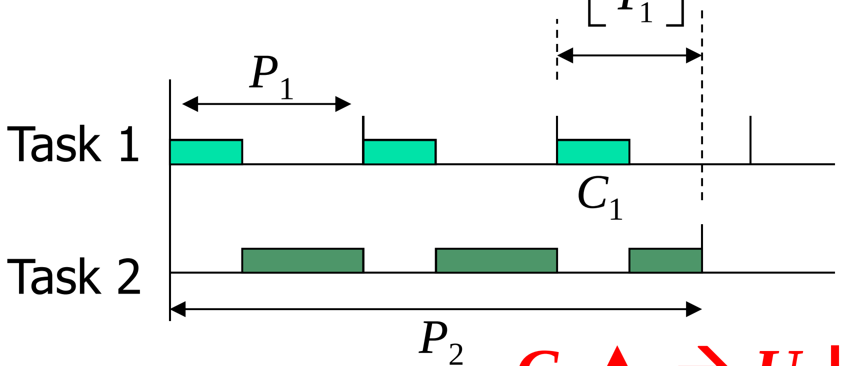

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$C_1 \uparrow \;\rightarrow\; U \downarrow$

$$U = \frac{C_1}{P_1} + \frac{C_2}{P_2} = 1 + \frac{C_1}{P_2}\left[\frac{P_2}{P_1} - \left\lfloor \frac{P_2}{P_1} \right\rfloor - 1\right]$$

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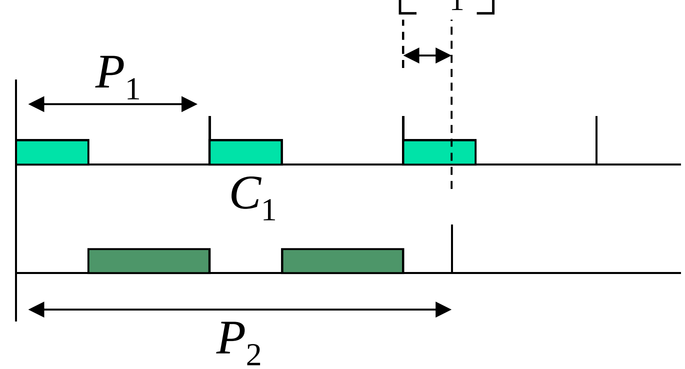


$$C_2 = (P_1 - C_1)\left\lfloor \frac{P_2}{P_1} \right\rfloor$$

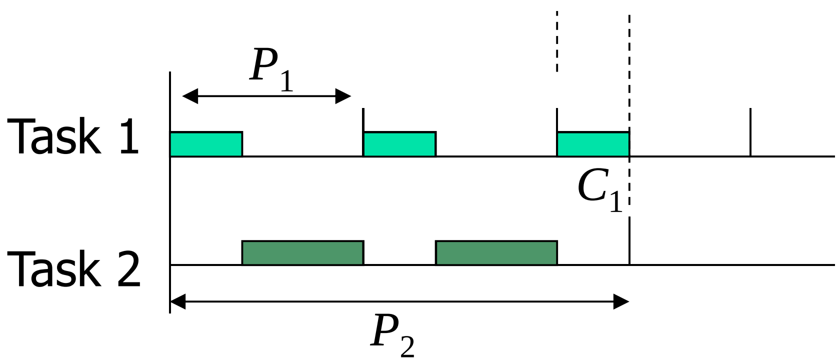
$C_1 \downarrow \;\rightarrow\; U \downarrow$

$$U = \frac{P_1}{P_2}\left\lfloor \frac{P_2}{P_1} \right\rfloor + \frac{C_1}{P_2}\left[\frac{P_2}{P_1} - \left\lfloor \frac{P_2}{P_1} \right\rfloor\right]$$

Deriving the Utilization Bound for Rate Monotonic Scheduling

- The minimum utilization case:

$$C_1 = P_2 - \left\lfloor \frac{P_2}{P_1} \right\rfloor P_1$$



$$U = 1 + \frac{C_1}{P_2}\left[\frac{P_2}{P_1} - \left\lfloor \frac{P_2}{P_1} \right\rfloor - 1\right]$$

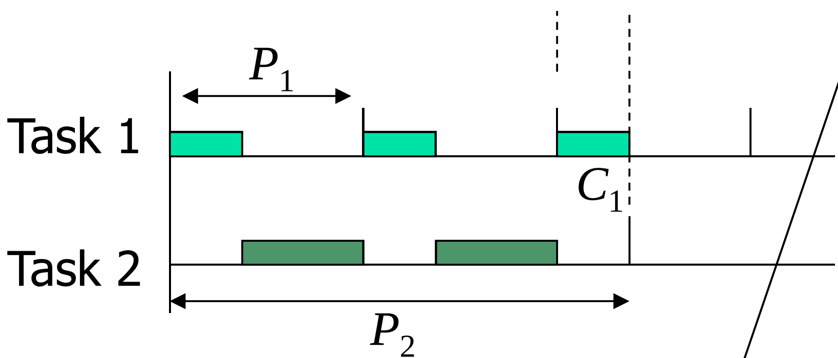
Deriving the Utilization Bound for Rate Monotonic Scheduling

- The minimum utilization case:

$$C_1 = P_1\left(\frac{P_2}{P_1} - \left\lfloor \frac{P_2}{P_1} \right\rfloor\right)$$

$$C_1 = P_2 - \left\lfloor \frac{P_2}{P_1} \right\rfloor P_1$$

$$U = 1 + \frac{P_1}{P_2}\left(\frac{P_2}{P_1} - \left\lfloor \frac{P_2}{P_1} \right\rfloor\right)\left[\frac{P_2}{P_1} - \left\lfloor \frac{P_2}{P_1} \right\rfloor - 1\right]$$



$$\Rightarrow \left\lfloor \frac{P_2}{P_1} \right\rfloor = 1$$

$$\Rightarrow U = 1 + \frac{\left(P_2/P_1 - 1\right)\left(P_2/P_1 - 2\right)}{P_2/P_1}$$

$$\frac{dU}{d\left(P_2/P_1\right)} = 0 \qquad \Rightarrow \frac{P_2}{P_1} = \sqrt{2}$$

$$\Rightarrow U \approx 0.83$$

$$U = 1 + \frac{C_1}{P_2}\left[\frac{P_2}{P_1} - \left\lfloor \frac{P_2}{P_1} \right\rfloor - 1\right]$$

Note that $C_1 = P_2 - P_1$

Generalizing to N Tasks

$$\left.\begin{aligned} C_1 &= P_2 - P_1 \\ C_2 &= P_3 - P_2 \\ C_3 &= P_4 - P_3 \\ &\ldots \end{aligned}\right\} \qquad U = \frac{C_1}{P_1} + \frac{C_2}{P_2} + \frac{C_3}{P_3} + \ldots$$

Generalizing to N Tasks

$$\left.\begin{aligned} C_1 &= P_2 - P_1 \\ C_2 &= P_3 - P_2 \\ C_3 &= P_3 - P_2 \end{aligned}\right\} \qquad U = \frac{C_1}{P_1} + \frac{C_2}{P_2} + \frac{C_3}{P_3} + \ldots$$

$$\ldots$$

$$\frac{dU}{d\left(P_2 / P_1\right)} = 0 \qquad \frac{dU}{d\left(P_3 / P_2\right)} = 0 \qquad \frac{dU}{d\left(P_4 / P_3\right)} = 0 \qquad \ldots$$

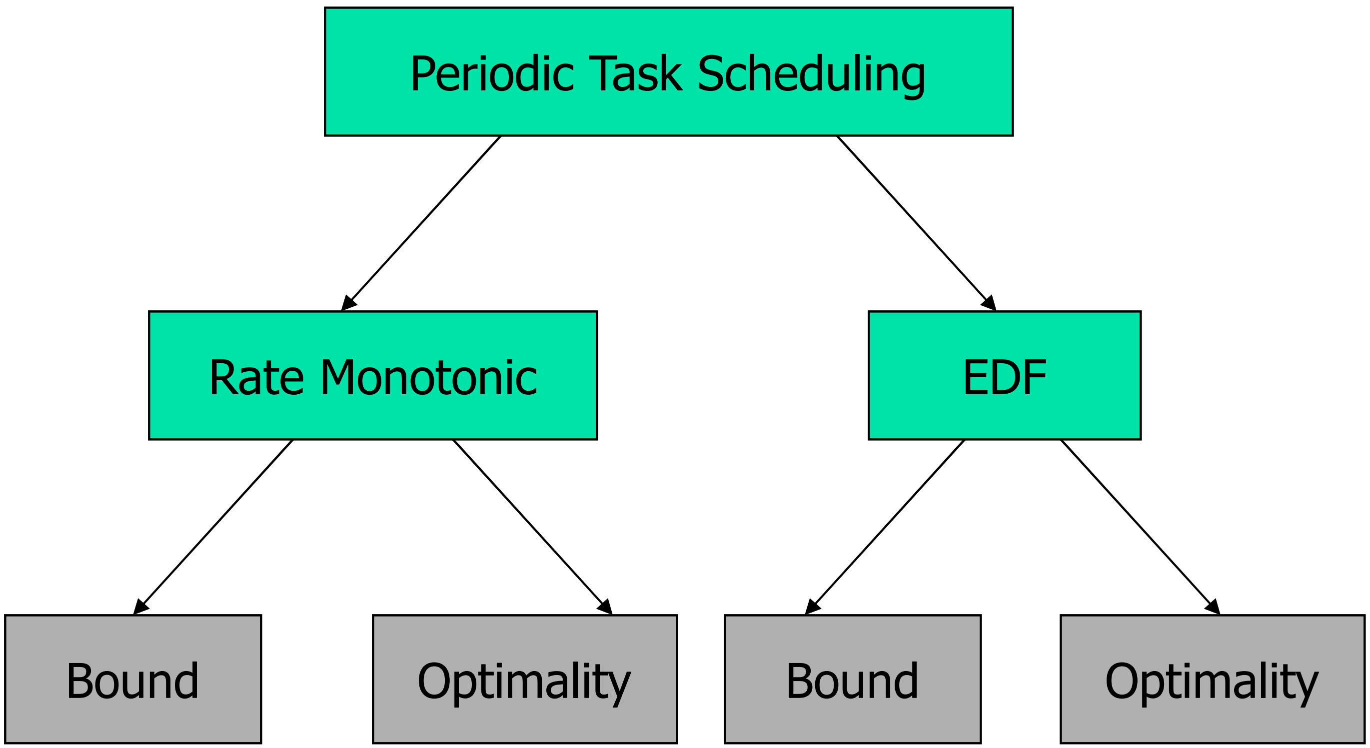
Generalizing to N Tasks

$$\left.\begin{aligned} C_1 &= P_2 - P_1 \\ C_2 &= P_3 - P_2 \\ C_3 &= P_3 - P_2 \\ &\cdots \end{aligned}\right\} \qquad U = \frac{C_1}{P_1} + \frac{C_2}{P_2} + \frac{C_3}{P_3} + \ldots$$

$$\frac{dU}{d\left(P_2/P_1\right)} = 0 \qquad \frac{dU}{d\left(P_3/P_2\right)} = 0 \qquad \frac{dU}{d\left(P_4/P_3\right)} = 0 \qquad \cdots$$

$$\Rightarrow \frac{P_{i+1}}{P_i} = 2^{1/n} \qquad \Rightarrow U = n\left(2^{1/n} - 1\right)$$

Done Today



Periodic Task Scheduling
Rate Monotonic
EDF
Bound
Optimality
Bound
Optimality