Exact Schedulability Test

Tarek Abdelzaher

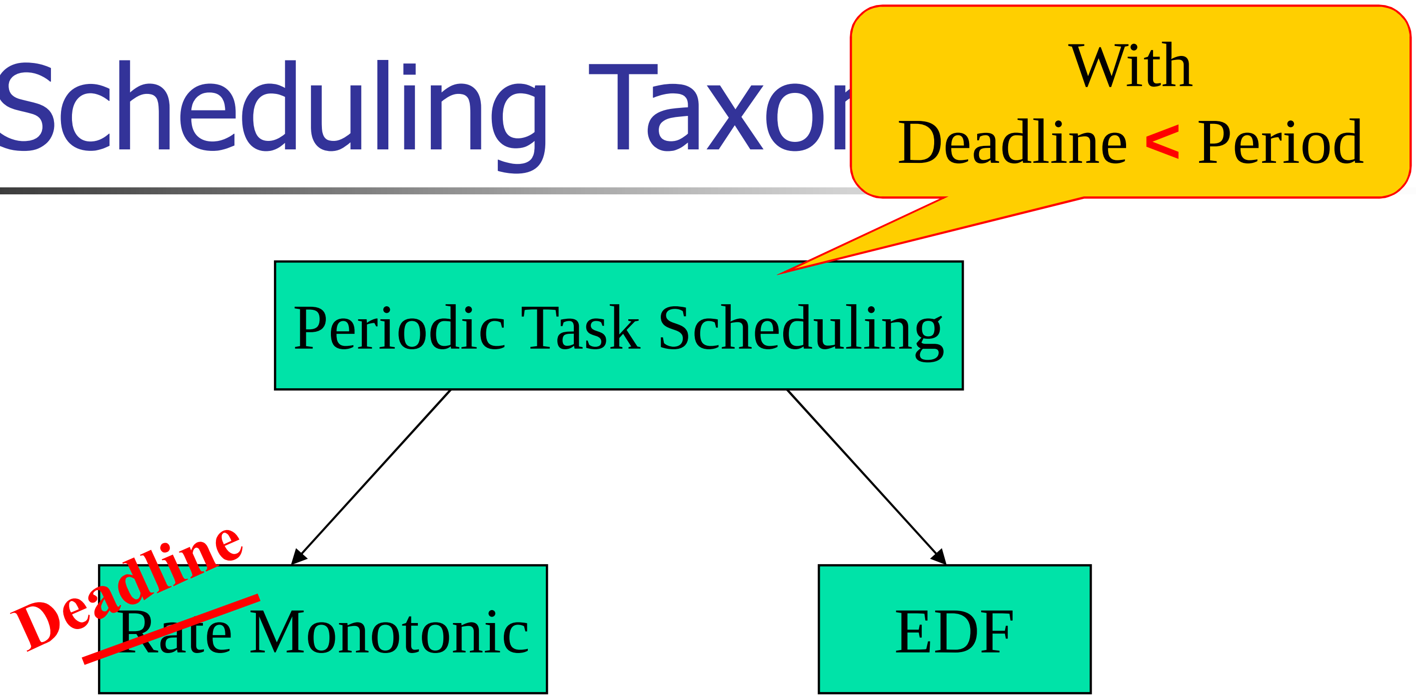
The 4th Credit Project (Suggested: 1-2 persons per project)

- Option 1: Develop a 30 min survey presentation on an advanced topic of your choice in real-time and embedded computing.
  - Topic name due 10/24.
  - Slides due 11/28.
  - Presentation the week of 12/5
- Example topics:
  - Self-driving cars: the state of the art and future challenges
  - Real-time operating systems
  - Multicore scheduling – main challenges and results
  - Space applications
  - Scheduling Map/Reduce workflows (with emphasis on time support)
  - Participatory and social sensing (crowd-sensing)
  - Software model checking (proving software correctness)
  - Energy/smart grid

The 4th Credit Project (Suggested: 1-2 persons per project)

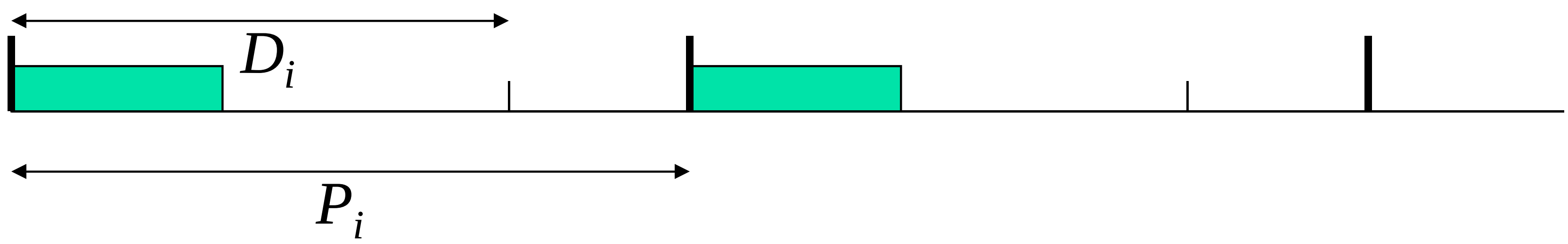
- Option 2: Implement a real-time or embedded systems service
  - Service name due 10/24.
  - Slides due 11/28.
  - Presentation + Demo the week of 11/29
- Example services:
  - A real-time scheduler for Roomba
  - Security and diagnostics
  - Real-time Hadoop
  - Social sensing services
  - Your idea here…

Scheduling Taxo



Deadline Monotonic Scheduling

- Consider a set of periodic tasks where each task, *i*, has a computation time, C_i, a period, P_i, and a relative deadline $D_i < P_i$.



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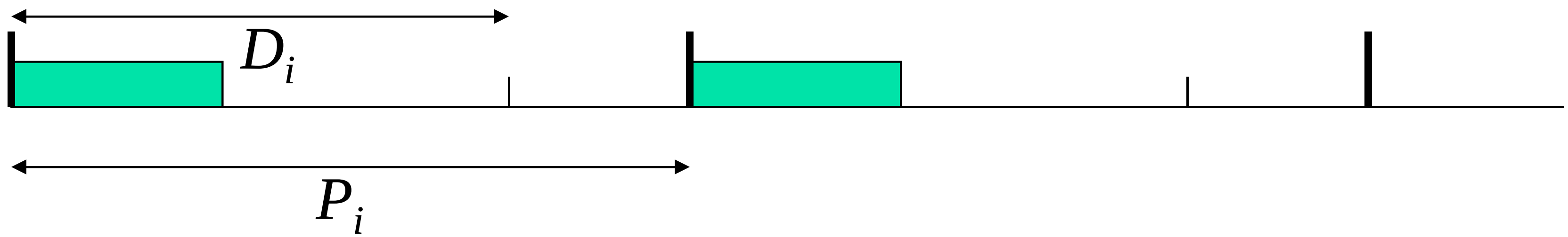

- What is the schedulability condition?

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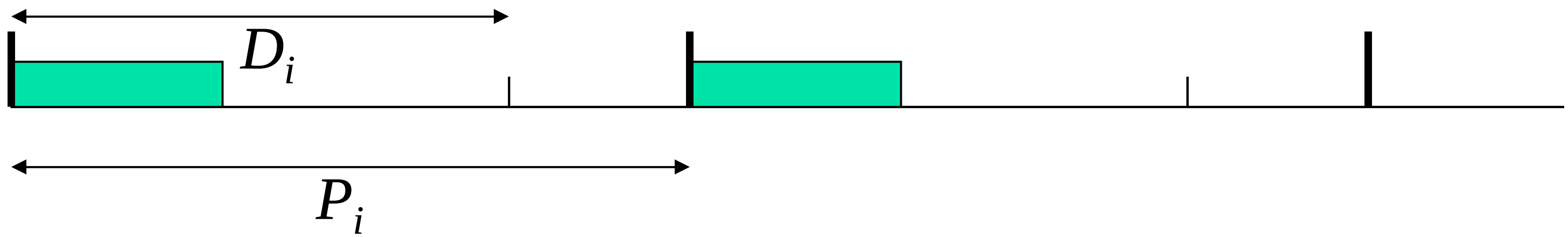

- Schedulability can't be worse than if P_i was reduced to D_i. Thus:

$$\sum_i \frac{C_i}{D_i} \leq n\left(2^{1/n} - 1\right)$$

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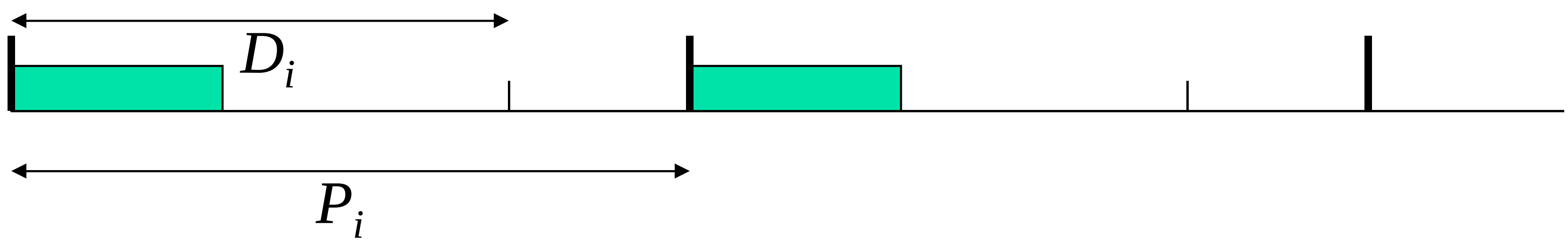

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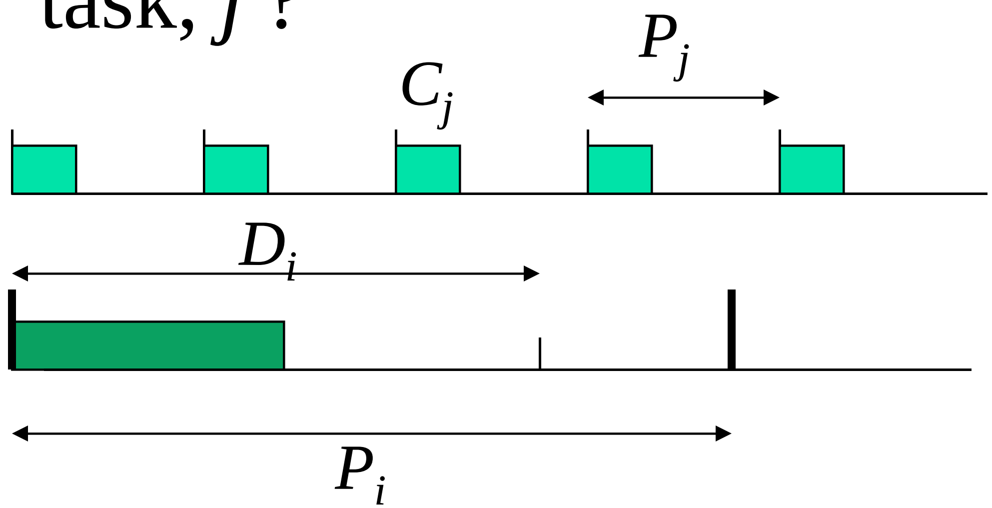
Problem?

A Better Condition

- Worst case interference from a higher priority task, j ?



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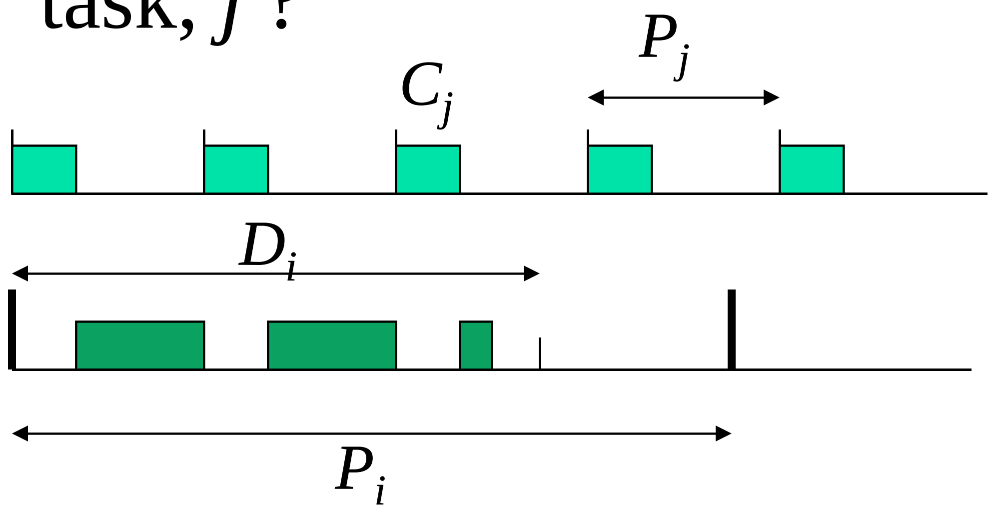

$$\left\lceil \frac{D_i}{P_j} \right\rceil C_j$$

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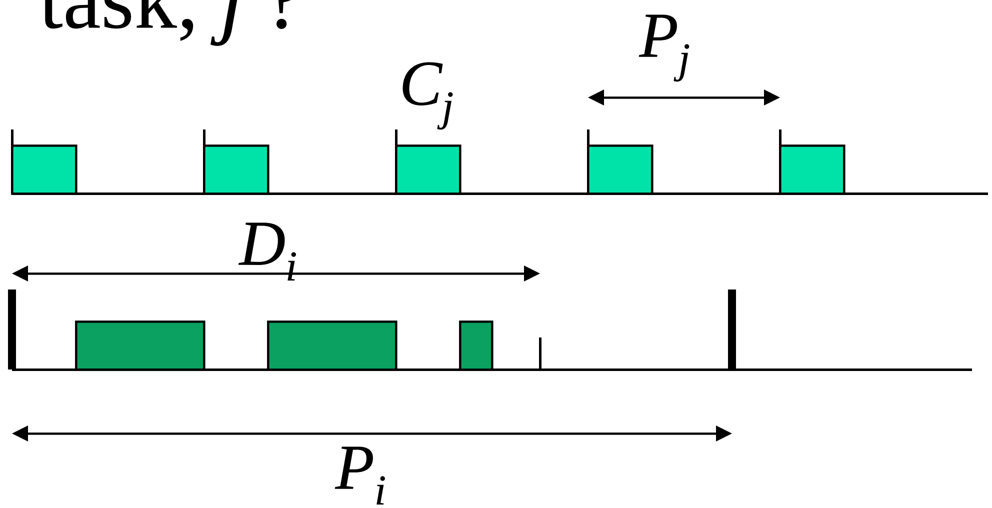

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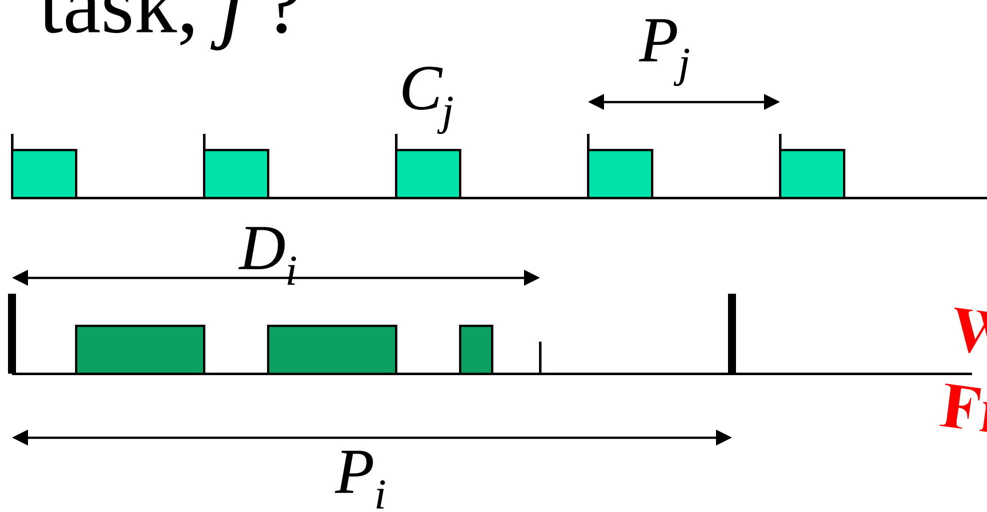

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Worst case interference, *I*, From higher priority tasks

My exec. time

My deadline

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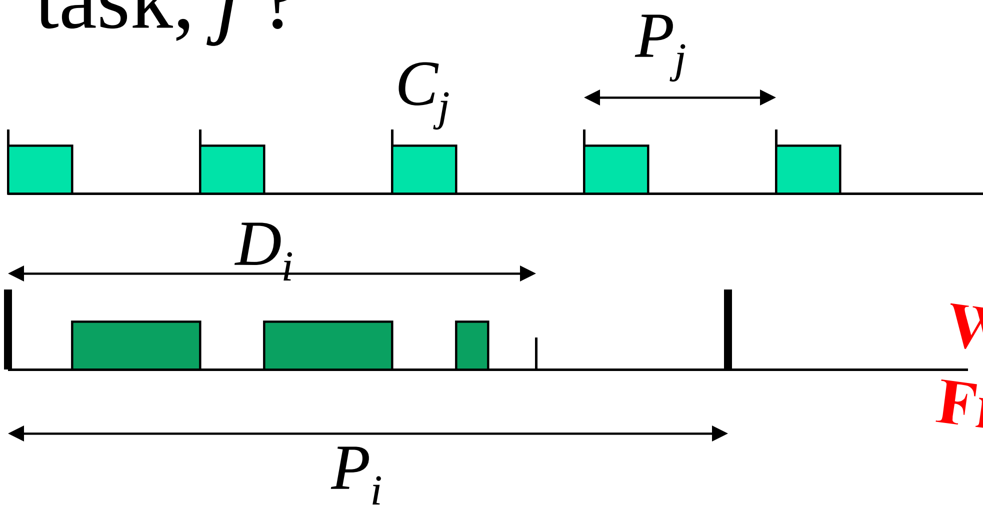

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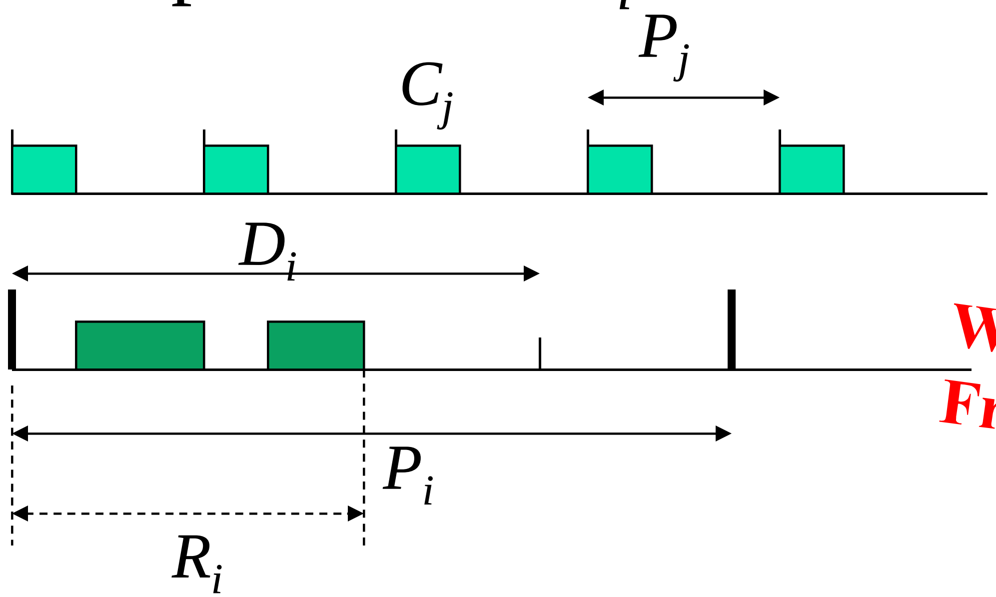
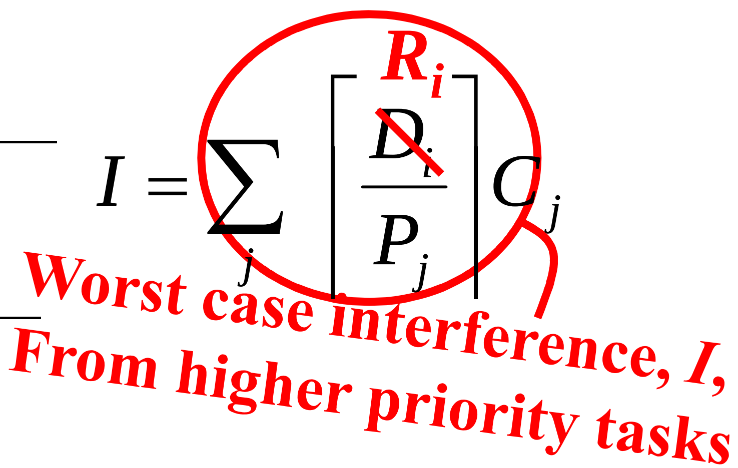
An Exact Condition

- Note: Interference exists only during the response time R_i not the entire D_i



$$I = \sum_j \left\lceil \frac{D_i}{P_j} \right\rceil C_j$$



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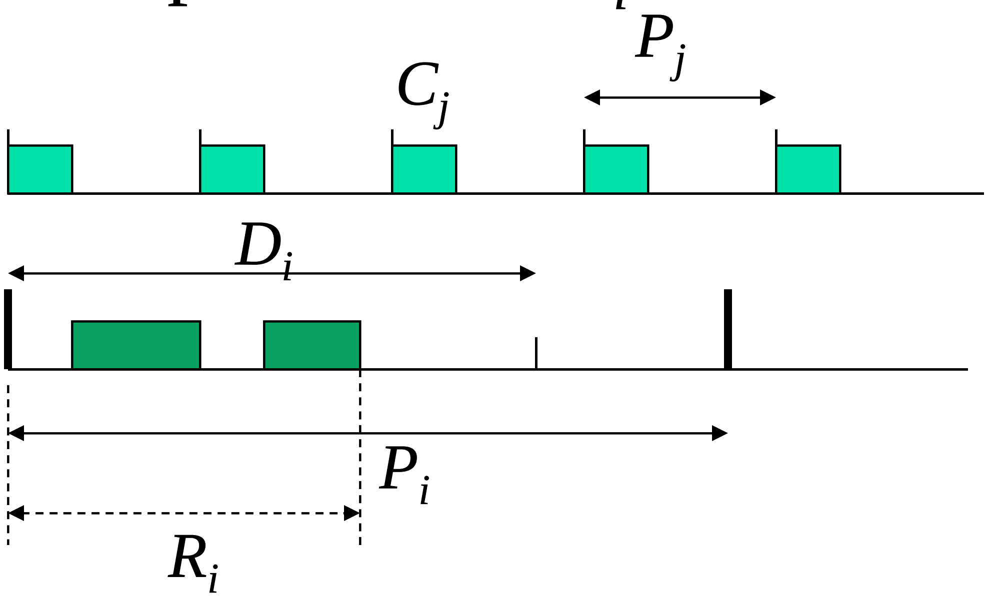

$$I = \sum_j \left\lceil \frac{\overset{R_i}{\cancel{D_i}}}{P_j} \right\rceil C_j$$

where

$$R_i = I + C_i$$

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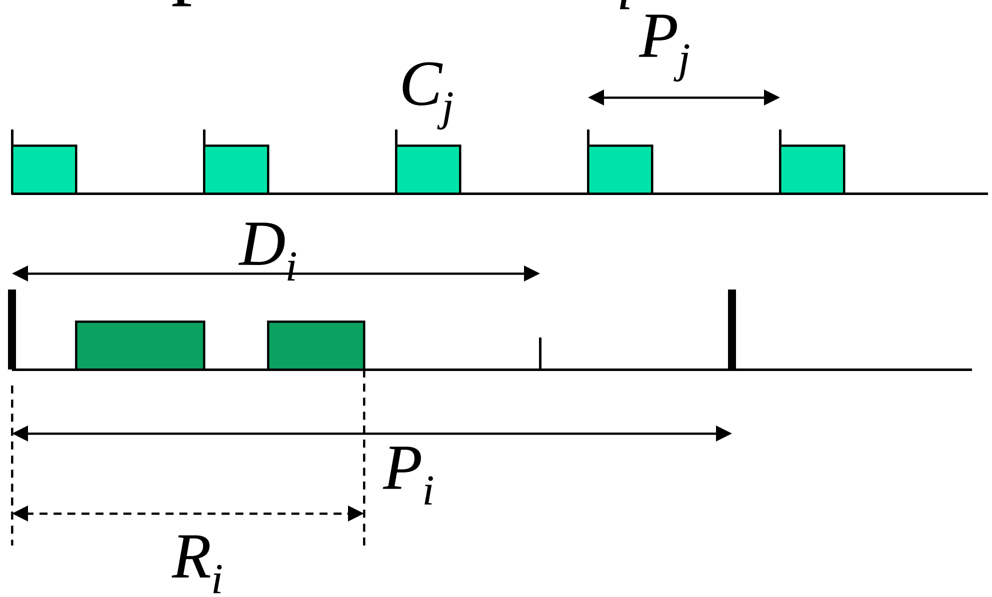

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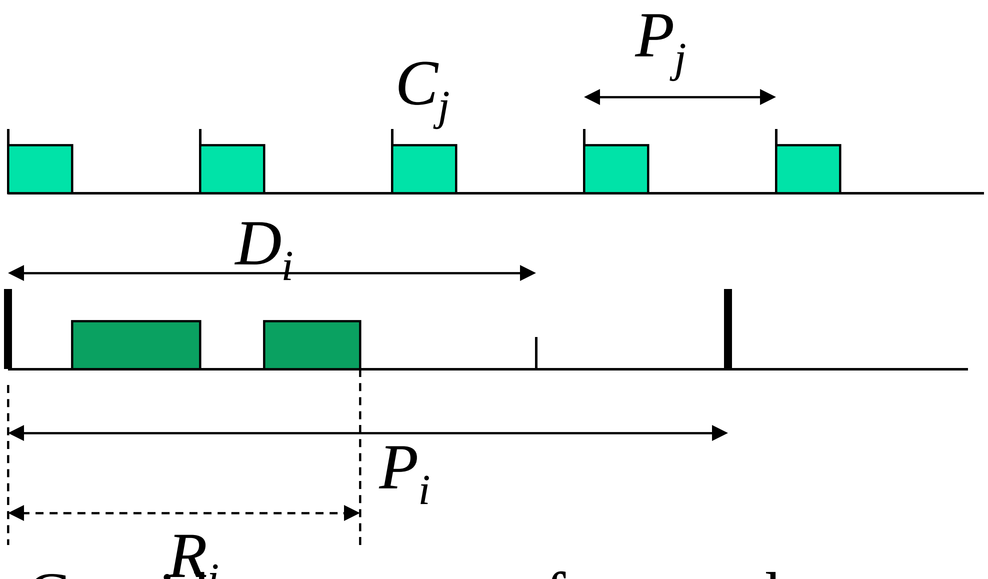
Solve iteratively for the smallest R_i that satisfies both equations

$$I = \sum_j \left\lceil \frac{R_i}{P_j} \right\rceil C_j$$

$$R_i = I + C_i$$

Example



Consider a system of two tasks:

Task 1: P_1=1.7, D_1=0.5, C_1=0.5
Task 2: P_2=8, D_2=3.2, C_2=2

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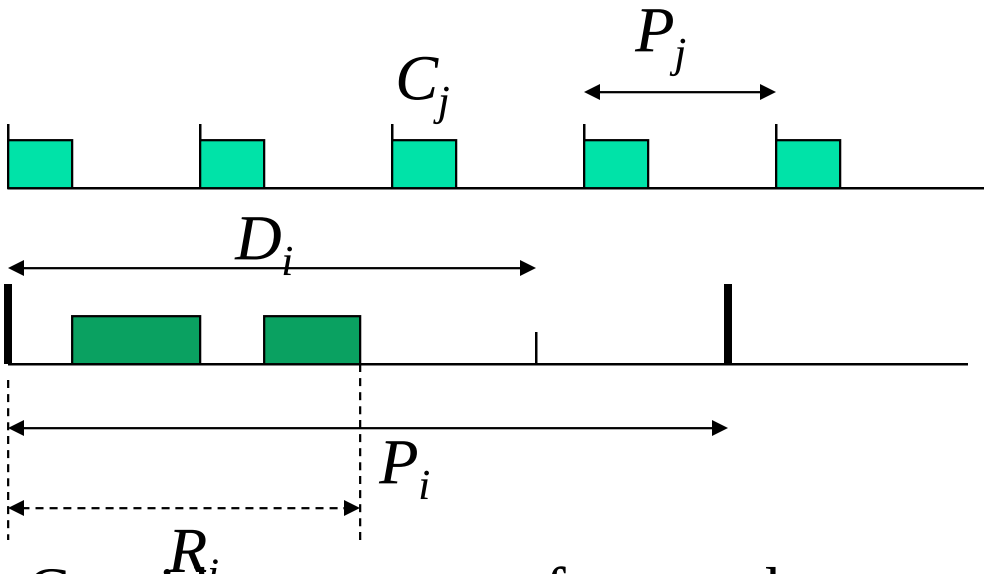

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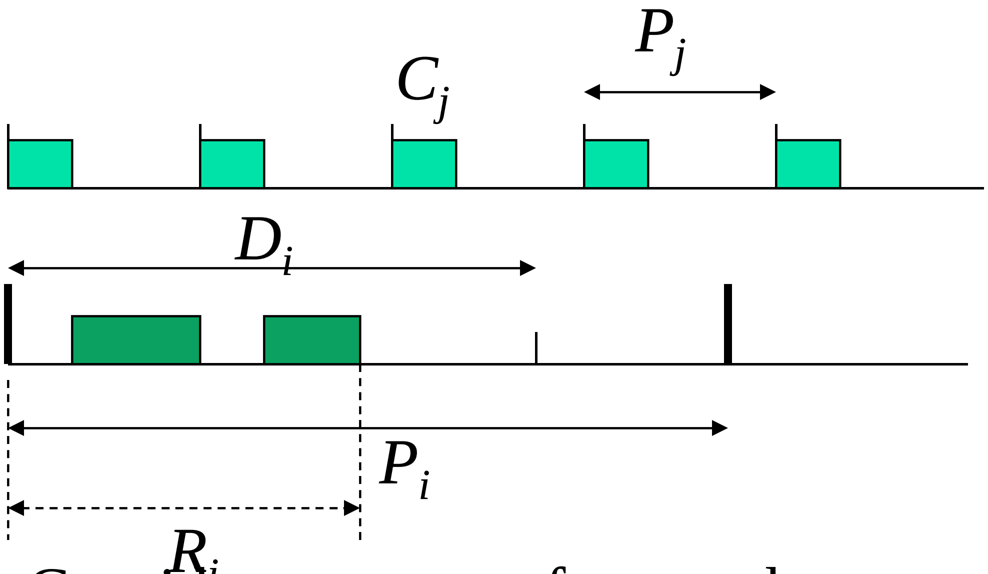

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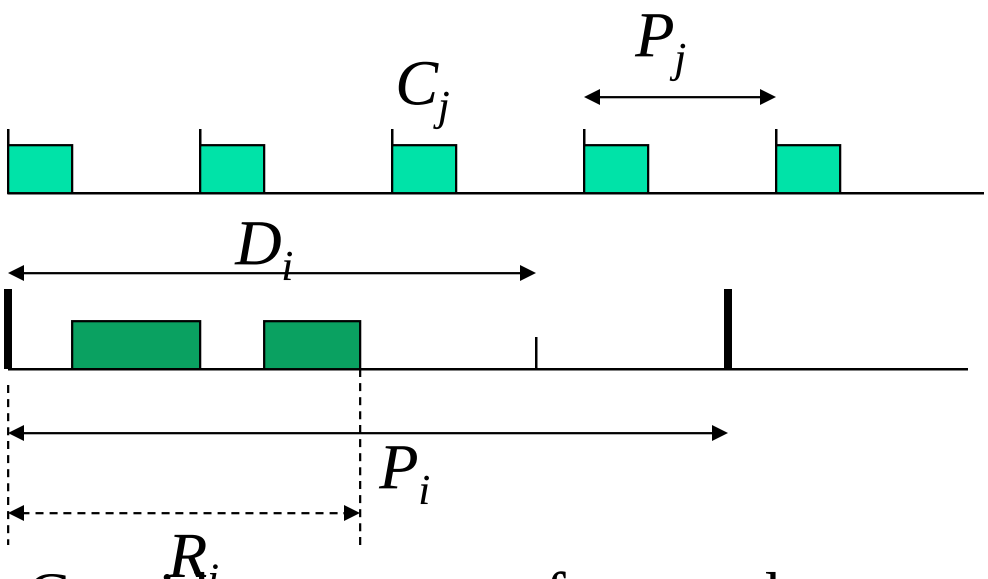

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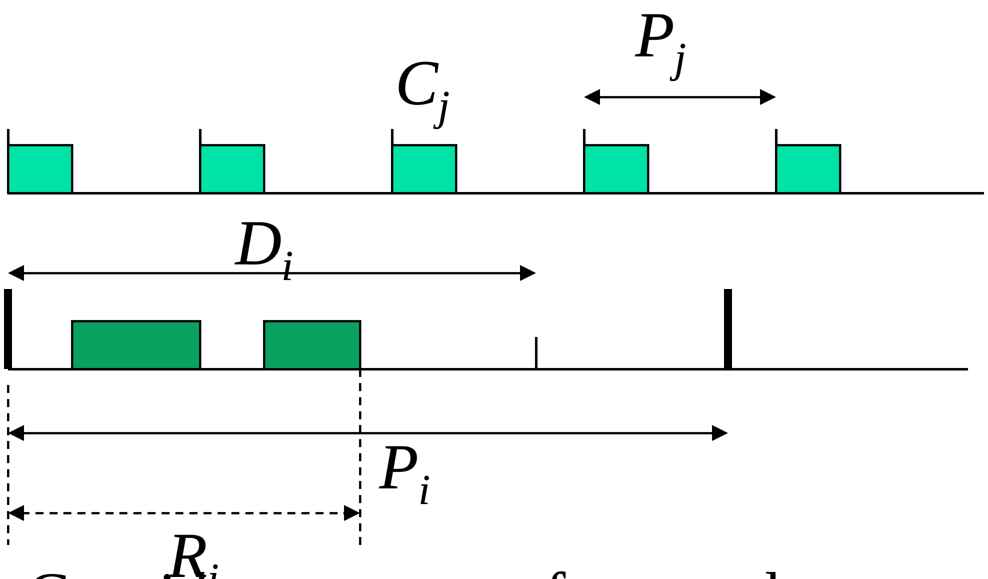

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$3 < 3.2$ → Ok!

Mixed Periodic and Aperiodic Task Systems

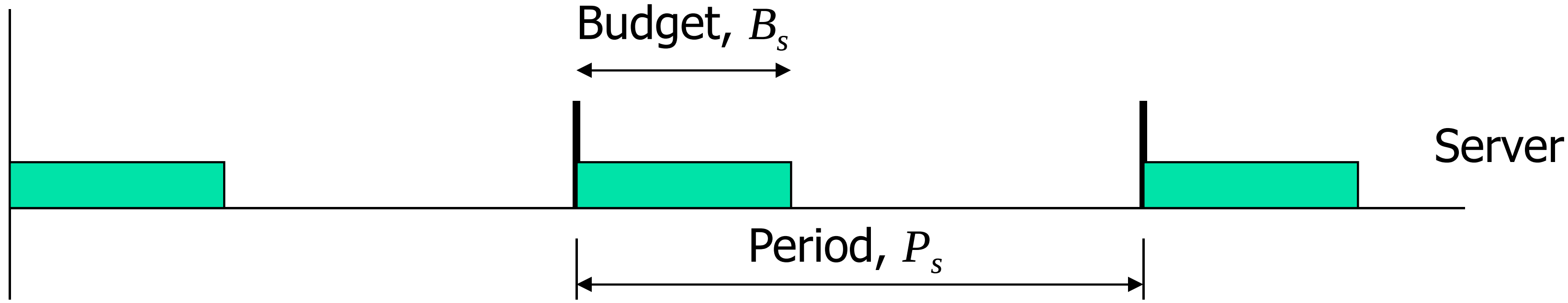
- Question: how to execute aperiodic tasks without violating schedulability guarantees given to periodic tasks?

Mixed Periodic and Aperiodic Task Systems

- Question: how to execute aperiodic tasks without violating schedulability guarantees given to periodic tasks?
- One Answer: Execute aperiodic tasks at lowest priority
  - Problem: Poor performance for aperiodic tasks

Mixed Periodic and Aperiodic Task Systems

- Idea: aperiodic tasks can be served by periodically invoked servers
- The server can be accounted for in periodic task schedulability analysis
- The server has a period P_s and a budget B_s
- Server can serve aperiodic tasks until budget expires
- Servers have different flavors depending on the details of when they are invoked, what priority they have, and how budgets are replenished



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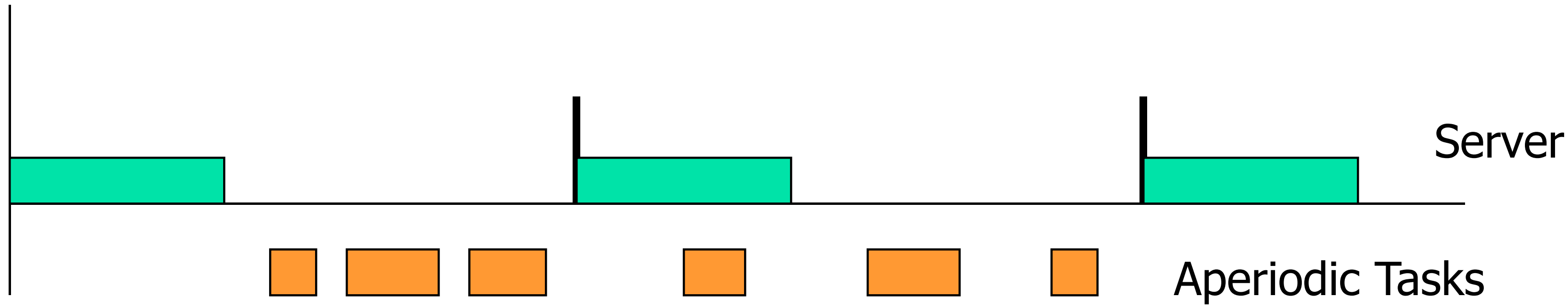

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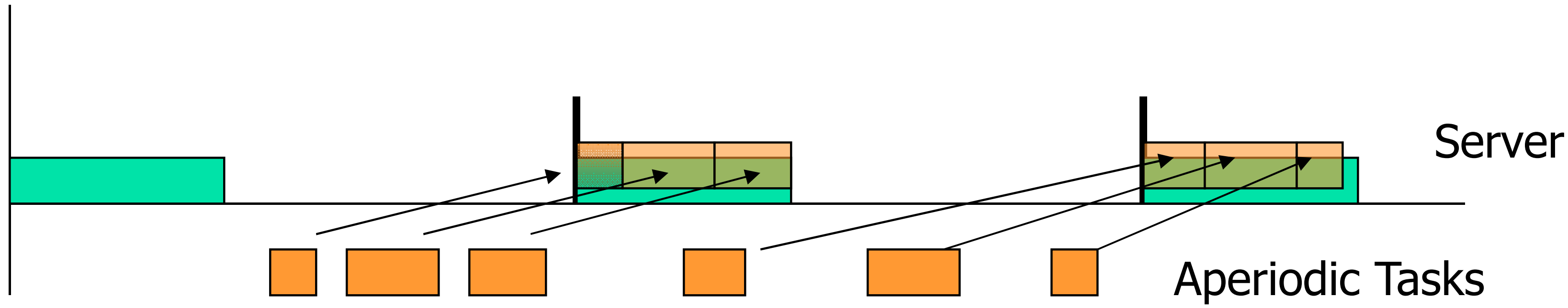
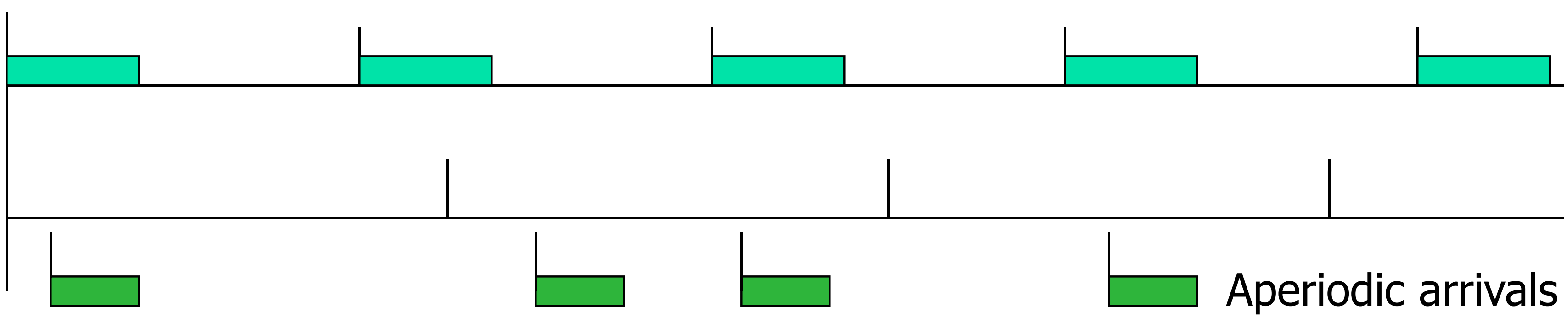


Polling Server

- Runs as a periodic task (priority set according to RM)
- Aperiodic arrivals are queued until the server task is invoked
- When the server is invoked it serves the queue until it is empty or until the budget expires then suspends itself
  - If the queue is empty when the server is invoked it suspends itself immediately.
- Server is treated as a regular periodic task in schedulability analysis

Example of a Polling Server

- Polling server:
  - Period $P_s = 5$
  - Budget $B_s = 2$
- Periodic task
  - $P = 4$
  - $C = 1.5$
- All aperiodic arrivals have C=1



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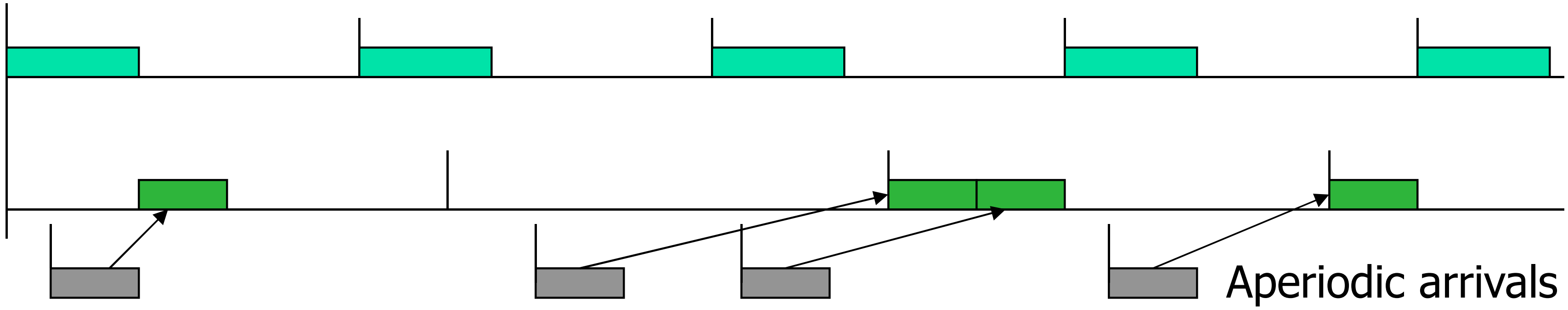

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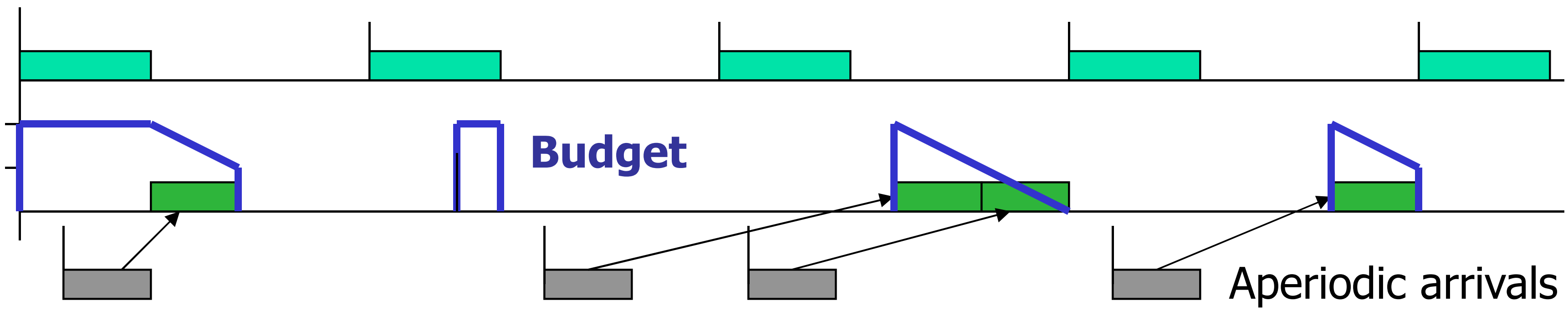

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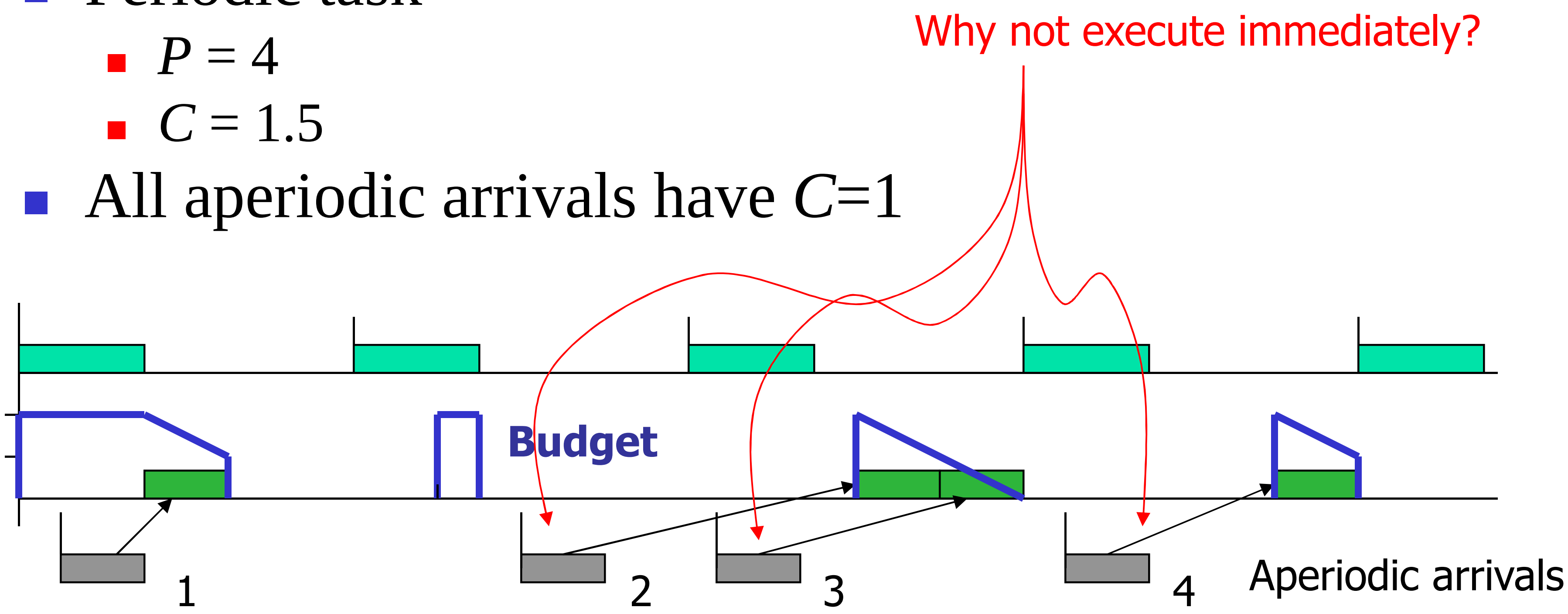
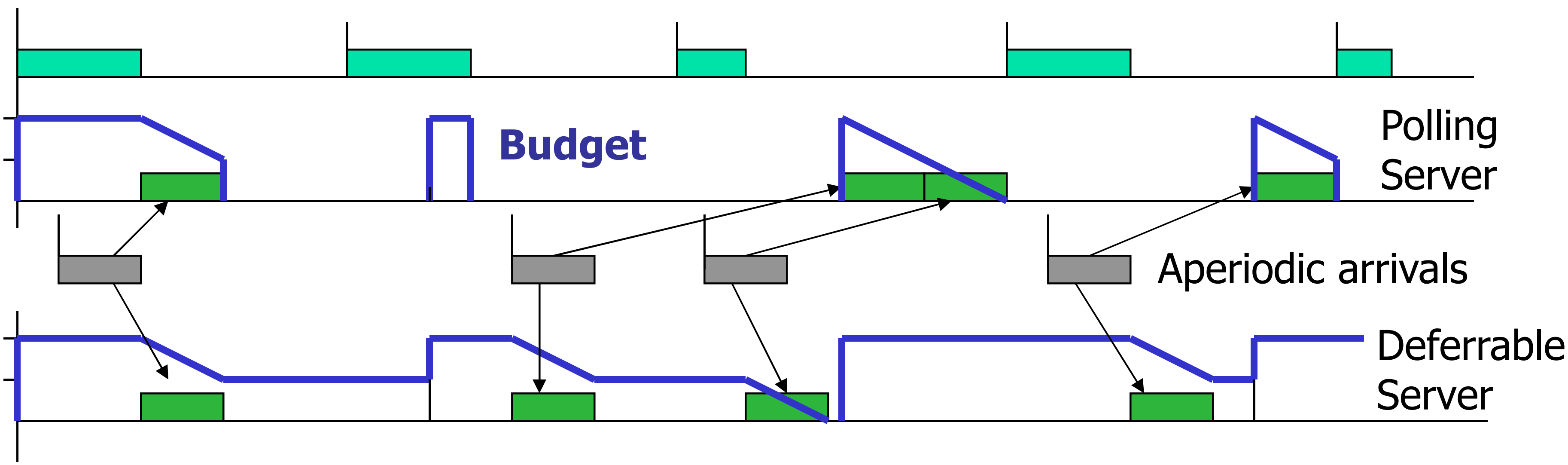
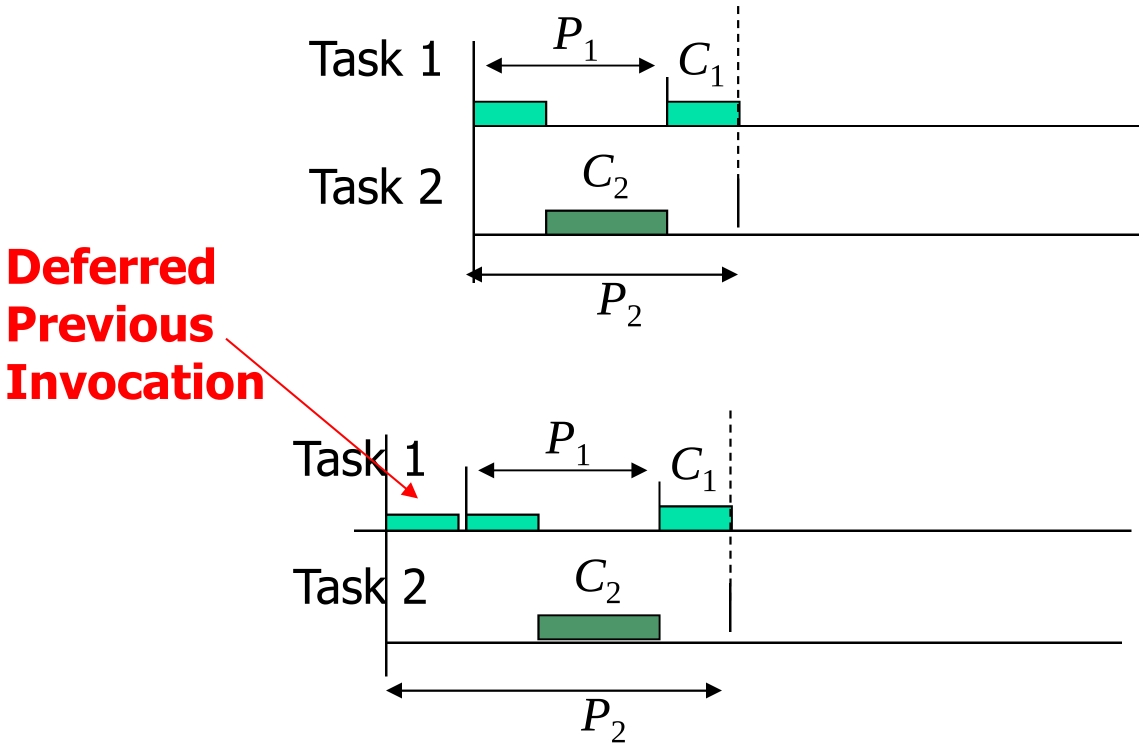


Deferrable Server

- Keeps the balance of the budget until the end of the period
- Example (continued)



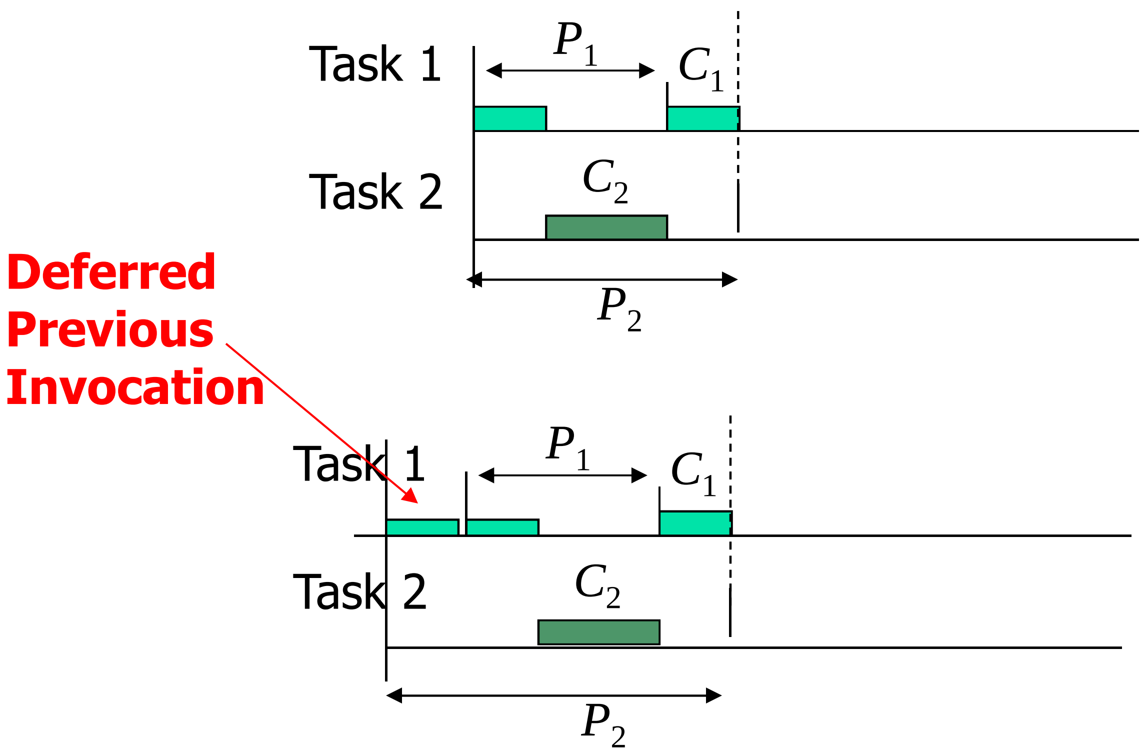
Worst-Case Scenario

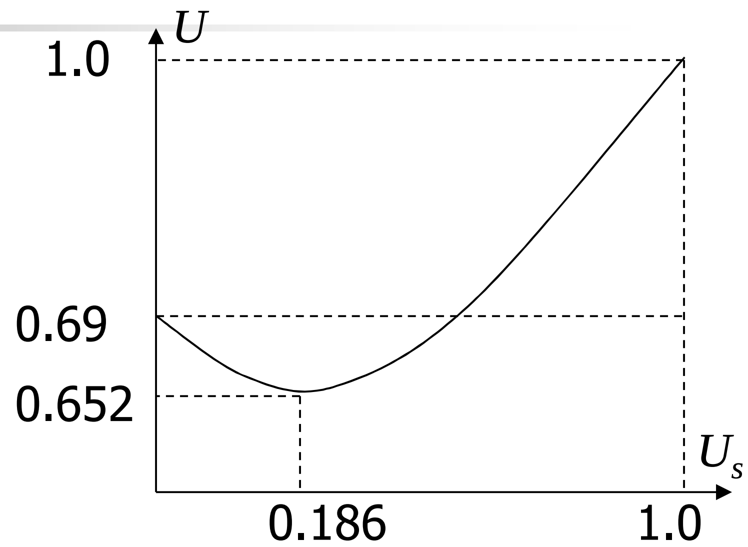


$$U_p \leq \ln\left(\frac{U_s + 2}{2U_s + 1}\right)$$

Exercise: Derive the utilization bound for a deferrable server plus one periodic task

Worst-Case Scenario





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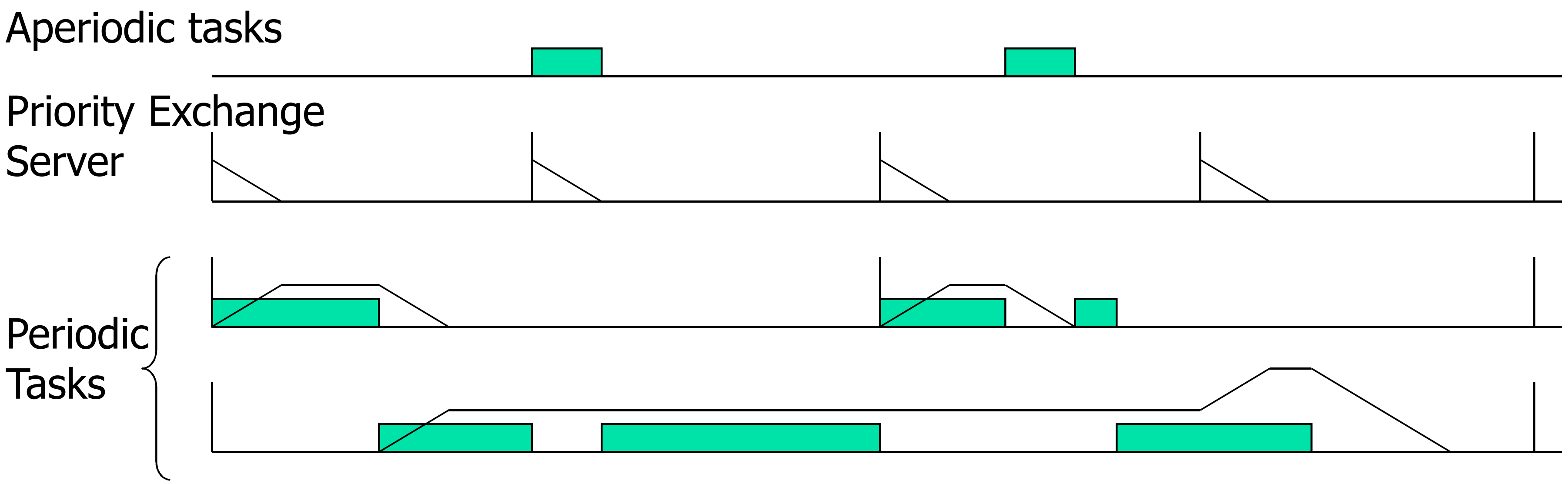
Exercise: Derive the utilization bound for a deferrable server plus one periodic task

Priority Exchange Server

- Like the deferrable server, it keeps the budget until the end of server period
- Unlike the deferrable server the priority slips over time: When not used the priority is exchanged for that of the executing periodic task

Priority Exchange Server

Example



Priority Exchange Server

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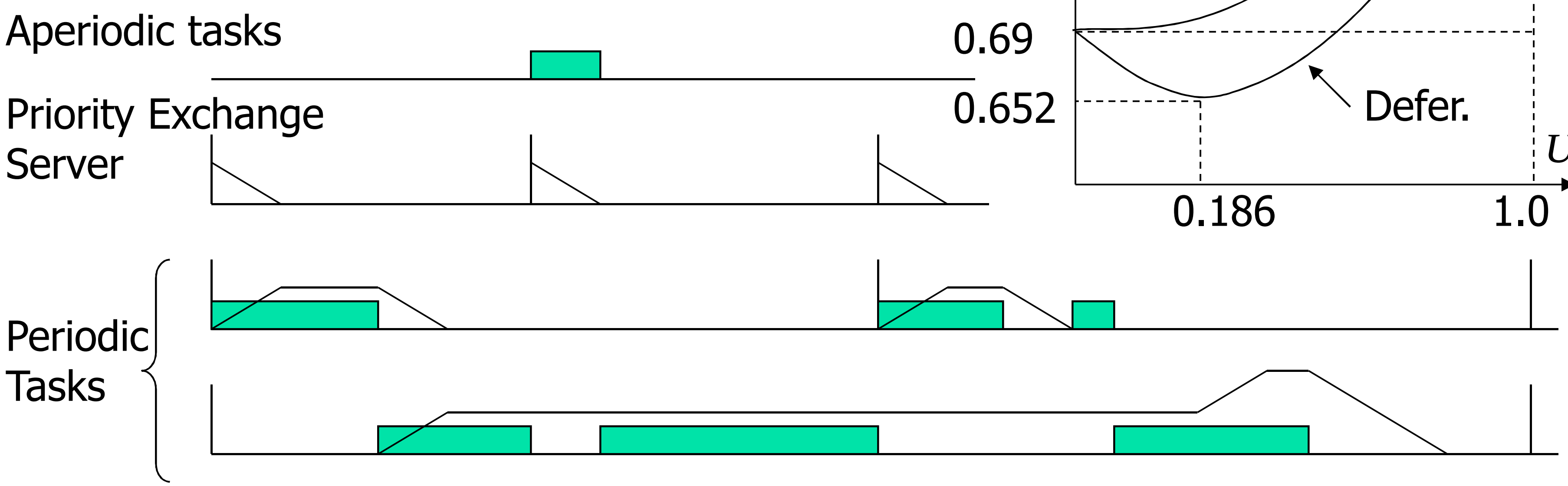

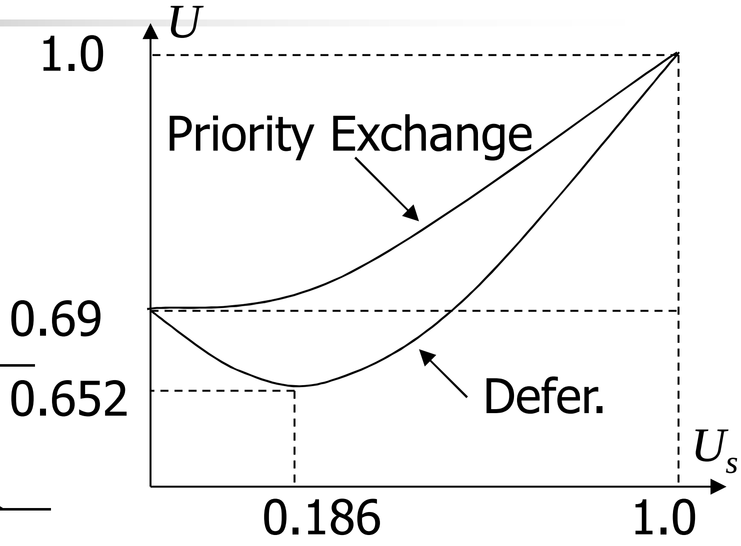

$$U_p \le \ln\left(\frac{2}{U_s + 1}\right)$$

Sporadic Server

- Server is said to be *active* if it is in the *running* or *ready* queue, otherwise it is *idle*.
- When an aperiodic task comes and the budget is not zero, the server becomes active
- Every time the server becomes *active*, say at t_A, it sets replenishment time one period into the future, $t_A + P_s$ (but does not decide on replenishment amount).
- When the server becomes idle, say at t_I, set replenishment amount to capacity consumed in $[t_A, t_I]$

$$U_p \leq \ln\left(\frac{2}{U_s + 1}\right)$$

Slack Stealing Server

- Compute a slack function A(t_s, t_f) that says how much total slack is available
- Admit aperiodic tasks while slack is not exceeded

Putting It All Together

- Covered so far:
  - System reliability
  - Data reliability
  - Timeliness
- Design problem: Design a “safe” robot

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