## Data Reliability

Interpreting Sensor Data

## Review of Important <br> Theorems

- Total Probability Theorem:
$P(A)=P\left(A \mid C_{1}\right) P\left(C_{1}\right)+\ldots+P\left(A \mid C_{n}\right) P\left(C_{n}\right)$
where $C_{1}, \ldots, C_{n}$ cover the space of all possibilities
- Bayes Theorem:

$$
P(A \mid B)=P(B \mid A) \cdot P(A) / P(B)
$$

- Other: $P(A, B)=P(A \mid B) P(B)$


## Intrusion Detection, Again

- A motion alarm is used to detect unauthorized access to a warehouse after hours. The motion sensor is mounted near the only entrance to the warehouse. If a burglar enters the building, there is a $99 \%$ chance that the burglar triggers the motion alarm.
- At 9pm, on September 16 ${ }^{\text {th }}, 2013$, the alarm was set off. What are the odds that a burglar is in the building?


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- Assume the alarm goes off about 3 days a year and burglaries happen about once a year


## Intrusion Detection, Again

## A Second Sensor

- In the intrusion detection example, assume that there is a vibration sensor on the floor that detects footsteps. If a burglar enters the building, there is a $90 \%$ chance that the vibration sensor will fire. If the vibration sensor fires, what are the odds that there is a burglar? Assume that the vibration sensor fires 10 times a year


## A Second Sensor

## Two Sensor Example

- In the intrusion detection example, what are the odds of burglary if both sensors fire?
- $P($ Burg $\mid A, V i b)=$ ?

Remember: If burglar enters, motion alarm fires $99 \%$ of the time and vibration alarm fires $90 \%$ of the time. Burglaries occur once a year, motion alarm fires 3 times a year, and vibration alarm fires 10 times a year.

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Now what?
Is it OK to say $P(A, V \mid B)=P(A \mid B) P(V \mid B)$ ?
Is it OK to say $\mathrm{P}(\mathrm{A}, \mathrm{V})=\mathrm{P}(\mathrm{A}) \mathrm{P}(\mathrm{V})$ ?

## Independence versus Conditional Independence

- John and Sally follow Mike on Twitter.
- When Mike tweets something, John retweets it with a $50 \%$ probability. Sally retweets it with a $30 \%$ probability.
- Are John's and Sally's tweets independent?


## Independence versus Conditional Independence

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- When Mike tweets something, John retweets it with a $50 \%$ probability. Sally retweets it with a $30 \%$ probability.
- Are John's and Sally's tweets independent?
- No. However, given that Mike says something, their decisions to re-tweet it are independent (conditional independence)

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\mathrm{P}(\mathrm{~A}, \mathrm{~V} \mid \overline{\mathrm{B}})=\mathrm{P}(\mathrm{~A} \mid \overline{\mathrm{B}}) \mathrm{P}(\mathrm{~V} \mid \overline{\mathrm{B}})
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## Two Sensor Example

## A Robotic Design Example

- A robot has a camera that detects obstacles with probability $70 \%$, a bump sensor that detects imminent collisions with a probability of 99.9\% (when an obstacle is 1 inch away), and a cliff sensor that detects imminent falls off a cliff with a probability of $99.9 \%$ (when the cliff is 1 inch away). The robot has breaks that can stop it within 0.1 second. The mission is to deliver supplies from point $A$ to point $B$, safely.
- What are safety-critical requirements?
- What are mission-critical (i.e., performance) requirements?
- What is a safe state?
- How to ensure well-formed dependencies?
- What is a safe speed for the robot?
- Is the algorithm that computes speed based on preferred arrival time and route safety-critical or mission-critical?


## A Robotic Design Example

- Notes:

