Data Reliability
Interpreting Sensor Data
Review of Important Theorems

- **Total Probability Theorem:**
  \[ P(A) = P(A|C_1) \cdot P(C_1) + \ldots + P(A|C_n) \cdot P(C_n) \]
  where \( C_1, \ldots, C_n \) cover the space of all possibilities

- **Bayes Theorem:**
  \[ P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \]

- **Other:**
  \[ P(A,B) = P(A|B) \cdot P(B) \]
A motion alarm is used to detect unauthorized access to a warehouse after hours. The motion sensor is mounted near the only entrance to the warehouse. If a burglar enters the building, there is a 99% chance that the burglar triggers the motion alarm.

At 9pm, on September 16th, 2013, the alarm was set off. What are the odds that a burglar is in the building?
Intrusion Detection, Again

- A motion alarm is used to detect unauthorized access to a warehouse after hours. The motion sensor is mounted near the only entrance to the warehouse. If a burglar enters the building, there is a 99% chance that the burglar triggers the motion alarm.

- At 9pm, on September 16^{th}, 2013, the alarm was set off. What are the odds that a burglar is in the building?

- Assume the alarm goes off about 3 days a year and burglaries happen about once a year
Intrusion Detection, Again
A Second Sensor

- In the intrusion detection example, assume that there is a vibration sensor on the floor that detects footsteps. If a burglar enters the building, there is a 90% chance that the vibration sensor will fire. If the vibration sensor fires, what are the odds that there is a burglar? Assume that the vibration sensor fires 10 times a year.
A Second Sensor
Two Sensor Example

- In the intrusion detection example, what are the odds of burglary if both sensors fire?
- \( P(\text{Burg}|A, \text{Vib}) = ? \)

Remember: If burglar enters, motion alarm fires 99% of the time and vibration alarm fires 90% of the time. Burglaries occur once a year, motion alarm fires 3 times a year, and vibration alarm fires 10 times a year.
Two Sensor Example

- In the intrusion detection example, what are the odds of burglary if both sensors fire?
  - $P(\text{Burg}|A, \text{Vib}) = ?$
  - $P(\text{B}|A,V) = P(A,V|B) \frac{P(B)}{P(A,V)}$

Remember: If burglar enters, motion alarm fires 99% of the time and vibration alarm fires 90% of the time. Burglaries occur once a year, motion alarm fires 3 times a year, and vibration alarm fires 10 times a year.
In the intrusion detection example, what are the odds of burglary if both sensors fire?

\[ P(\text{Burg}|A, \text{Vib}) = ? \]

\[ P(B|A,V) = P(A,V|B) \, P(B)/P(A,V) \]

Now what?

Is it OK to say \( P(A,V|B) = P(A|B)P(V|B) \)?
Is it OK to say \( P(A,V) = P(A)P(V) \)?
Independence versus Conditional Independence

- John and Sally follow Mike on Twitter.
- When Mike tweets something, John retweets it with a 50% probability. Sally retweets it with a 30% probability.
- Are John’s and Sally’s tweets independent?
Independence versus Conditional Independence

- John and Sally follow Mike on Twitter.
- When Mike tweets something, John re-tweets it with a 50% probability. Sally re-tweets it with a 30% probability.
- Are John’s and Sally’s tweets independent?
  - No. However, given that Mike says something, their decisions to re-tweet it are independent (conditional independence)
In the intrusion detection example, what are the odds of burglary if both sensors fire?

\[ P \left( \text{Burg}|A, \text{Vib} \right) = ? \]

\[ P \left( B|A,V \right) = P(A,V|B) \frac{P(B)}{P(A,V)} \]

Now what?

OK to say \[ P(A,V|B) = P(A|B)P(V|B) \]

\[ P(A,V) = P(A)P(V)? \]
In the intrusion detection example, what are the odds of burglary if both sensors fire?

P (Burg|A, Vib) = ?

P (B|A,V) = P(A,V|B) P(B)/P(A,V) where
P(A,V) = P(A,V|B) P(B) + P(A,V|B) P(B)
and P(A,V|B) = P(A|B)P(V|B)
Two Sensor Example

In the intrusion detection example, what are the odds of burglary if both sensors fire?

- \( P(\text{Burg}|A, \text{Vib}) = ? \)
- \( P(B|A,V) = P(A,V|B) \frac{P(B)}{P(A,V)} \) where
  \( P(A,V) = P(A,V|B) P(B) + P(A,V|\overline{B}) P(\overline{B}) \)
  and \( P(A,V|B) = P(A|B)P(V|B) \)
  \( P(A,V|\overline{B}) = P(A|\overline{B})P(V|\overline{B}) \)

Remember: If burglar enters, motion alarm fires 99% of the time and vibration alarm fires 90% of the time. Burglaries occur once a year, motion alarm fires 3 times a year, and vibration alarm fires 10 times a year.
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Two Sensor Example
A Robotic Design Example

- A robot has a camera that detects obstacles with probability 70%, a bump sensor that detects imminent collisions with a probability of 99.9% (when an obstacle is 1 inch away), and a cliff sensor that detects imminent falls off a cliff with a probability of 99.9% (when the cliff is 1 inch away). The robot has breaks that can stop it within 0.1 second. The mission is to deliver supplies from point A to point B, safely.
  - What are safety-critical requirements?
  - What are mission-critical (i.e., performance) requirements?
  - What is a safe state?
  - How to ensure well-formed dependencies?
  - What is a safe speed for the robot?
  - Is the algorithm that computes speed based on preferred arrival time and route safety-critical or mission-critical?
A Robotic Design Example

Notes: