Data Reliability

Interpreting Sensor Data

Notes and Reminders

- *Reminder:* HW1 is due Thursday.
- *Reminder:* MPs start next week. If you have not already sent us your group members, no problem; we shall assign them. Last call to send group request in (today):
  - Send email to Yiran zhao97@illinois.edu and CC me zaher@Illinois.edu
  - Use subject line: "CS424 GROUP" (in uppercase letters)

Reliability of Systems with Sensors

- Example: How to compute the probability of successful collision-avoidance when using a camera-based collision avoidance system?

Reliability of Systems with Sensors

- Example: How to compute the probability of successful collision-avoidance when using a camera-based collision avoidance system?
- The system fails if:
  - The software crashes, or
  - The camera fails, or
  - The breaks fail, or
  - The vision algorithm fails to recognize an obstacle (false negative)

Analogy

- System reliability challenge:
  - Building reliable systems from less reliable components
- Data reliability challenge:
  - Making reliable conclusions from less reliable (sensor) data

Systems with Imperfect Sensors

- Cyber-physical systems obtain data about their environment via sensors
- Sensors (or data sources in general) are often imperfect
- The challenge is: how to correctly compute probability of correct/safe system behavior given imperfect sensor readings?

Systems with Imperfect Sensors

- The challenge is: how to correctly compute probability of correct/safe system behavior given imperfect sensor readings?
  - The probability of successful detection may depend on context (conditional probability)
  - There may be multiple sensors involved
  - Sensors may have false positives and false negatives. Those will have different implications on safety

Review: Things You Should Know About Probabilities

- Probability of multiple simultaneous events
  - What are the odds that it rains and my basement floods?
    - Say P(rains) = 0.2. P(flood) = 0.1

Review: Things You Should Know About Probabilities

- Probability of multiple simultaneous events
  - What are the odds that it rains and my basement floods?
  - Answer: It is the odds that "it rains", times the odds that "my basement floods given that it rains":

$$P(\text{rain, flood}) = P(\text{rain})\, P(\text{flood}|\text{rain})$$

Note: $P(\text{flood}|\text{rain})$ is larger than $P(\text{flood})$

Review: Things You Should Know About Probabilities

- $P(A,B) = P(A|B).P(B)$

- Corollary: If events A and B are independent, the odds of them happening together is the product of their individual probabilities.
- $P(A,B) = P(A).P(B)$

Note: This is because $P(A|B) = P(A)$

Review: Probability versus Conditional Probability

- Probability: the odds that something, say X, happens, P(X).
- Conditional probability: the odds that X happens *given* that a certain condition, C, has occurred, P(X|C).
  - This condition may affect the odds.

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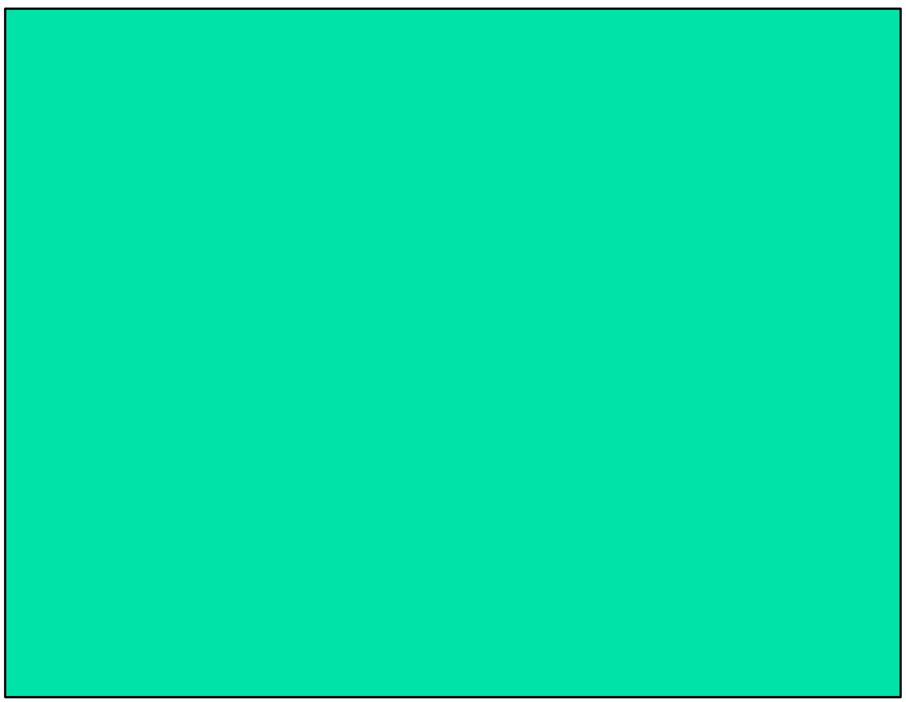
- Conditional probability: the odds that X happens *given* that a certain condition, C, has occurred, P(X|C).
  - This condition may affect the odds.
- Traffic example:
  - P(Accident) may be low
  - P(Accident|Black ice) is a lot higher!

A Visual Interpretation

The universe of all possibilities



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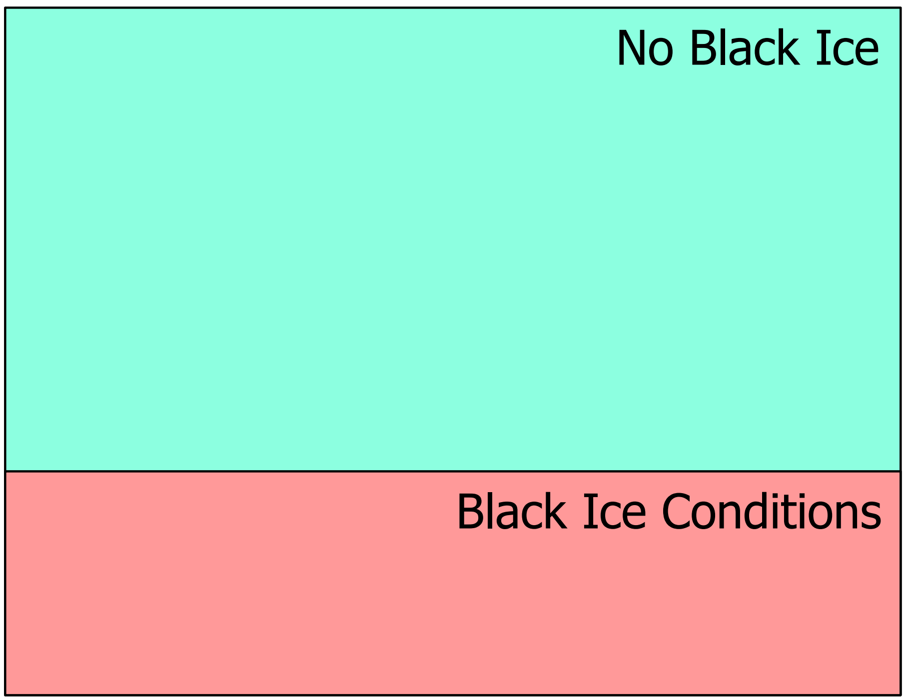

Consider a graphical visualization of probabilities where area represents probability

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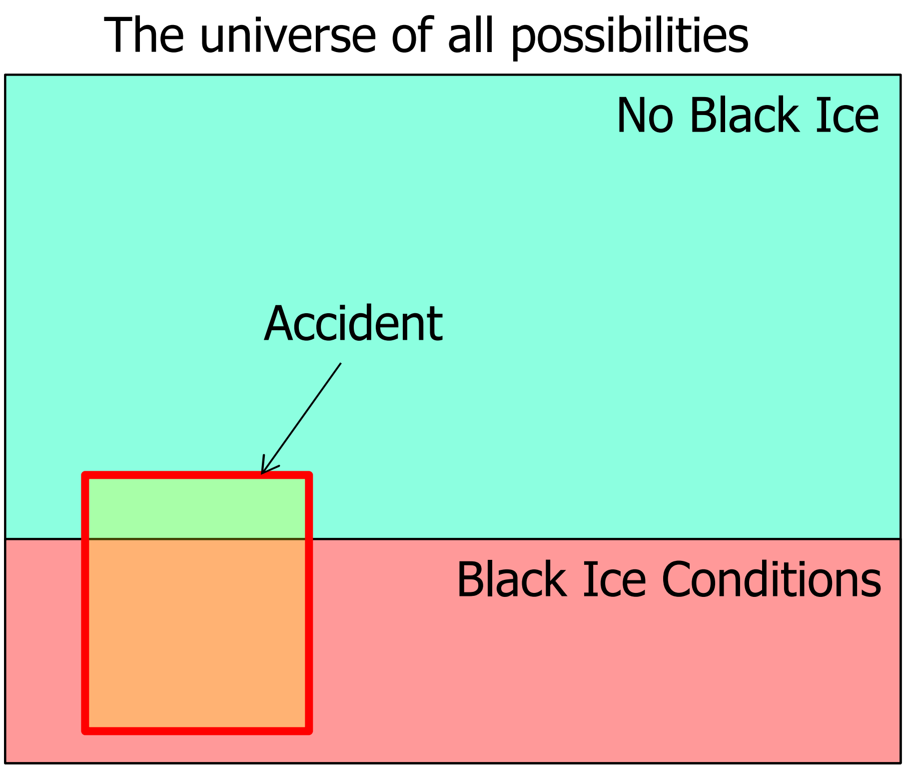

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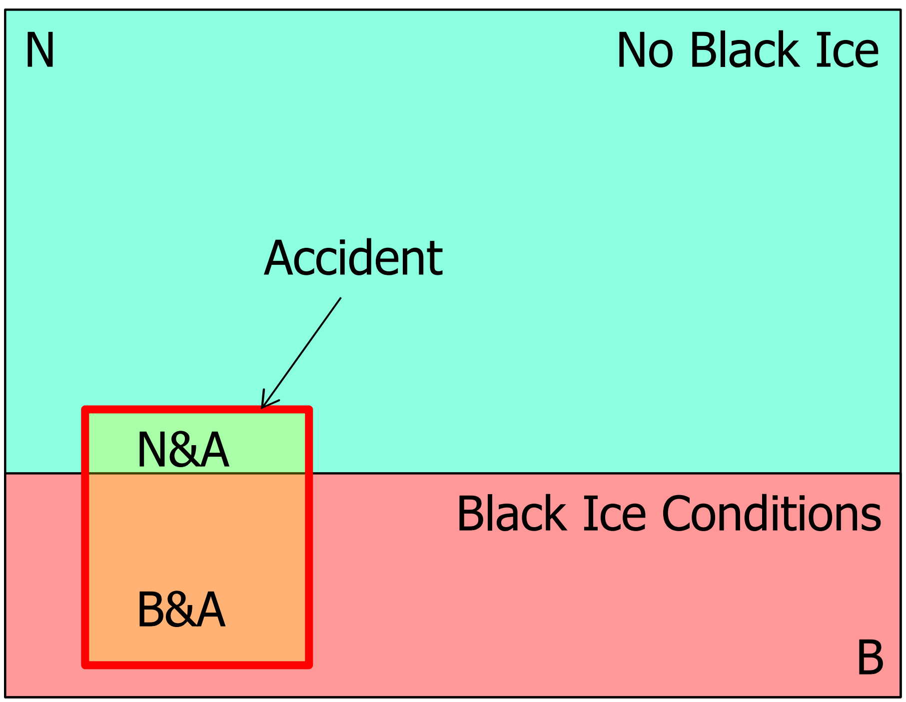

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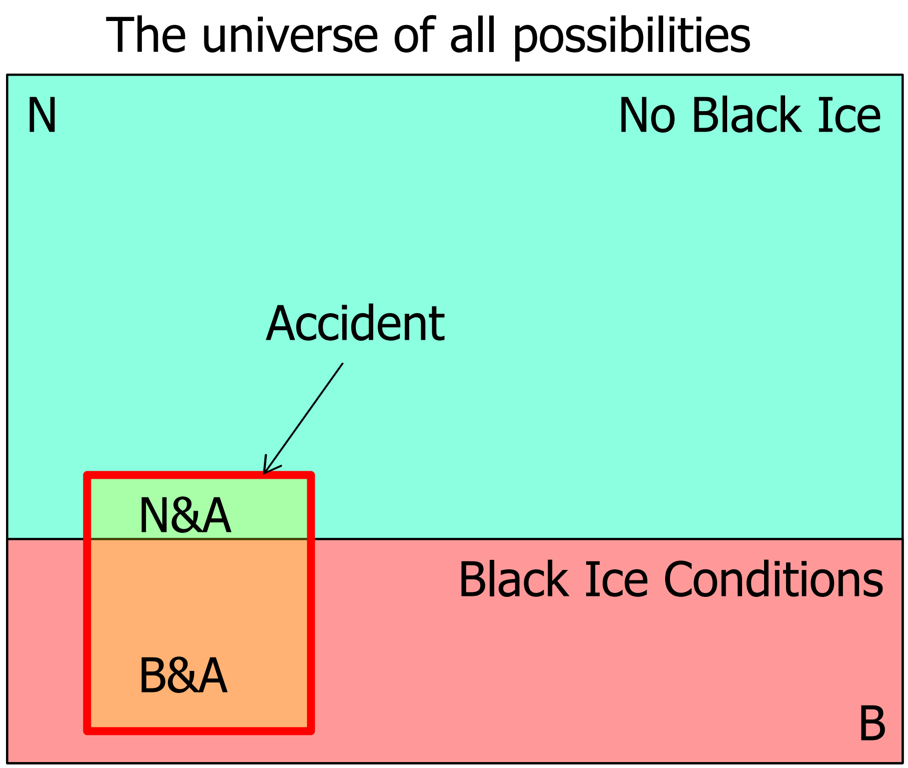

- P(Accident) = (N&A+B&A)/(N+B)

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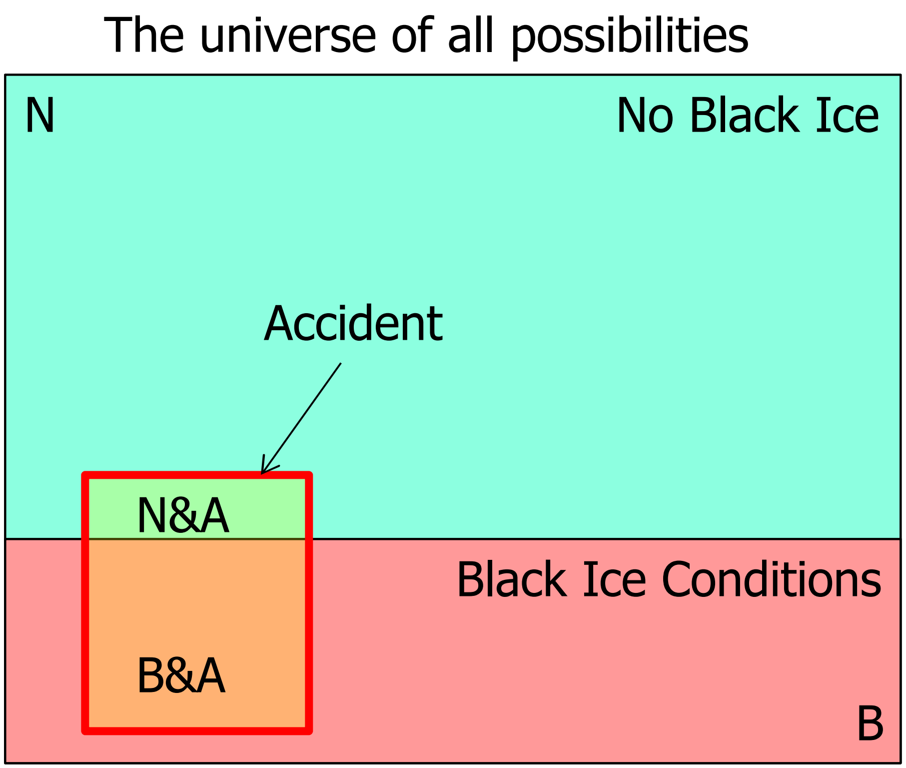

- P(Accident) = (N&A+B&A)/(N+B)
- P(Accident|Ice) = (B&A)/B
- P(Accident|No Ice) = (N&A)/N

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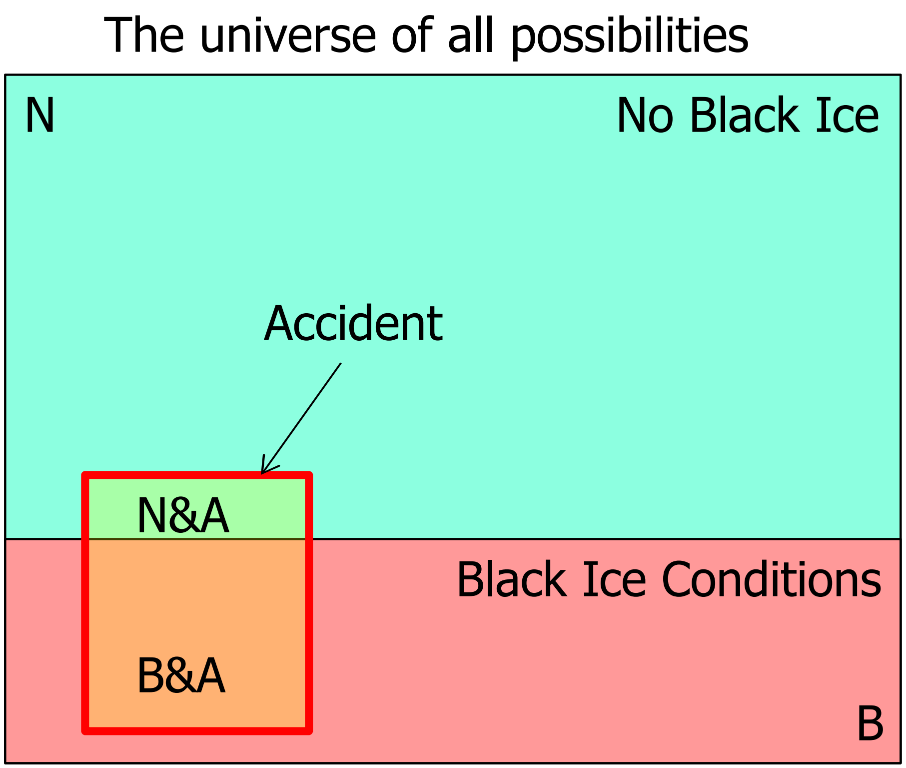

- P(Accident) = (N&A+B&A)/(N+B)
- P(Accident|Ice) = (B&A)/B
- P(Accident|No Ice) = (N&A)/N

Question: If P(Accident|Ice) = 0.1, P(Accident|No-Ice) = 0.02, P(Ice) = 0.2, what are the odds of accidents in general, P(Accident)?

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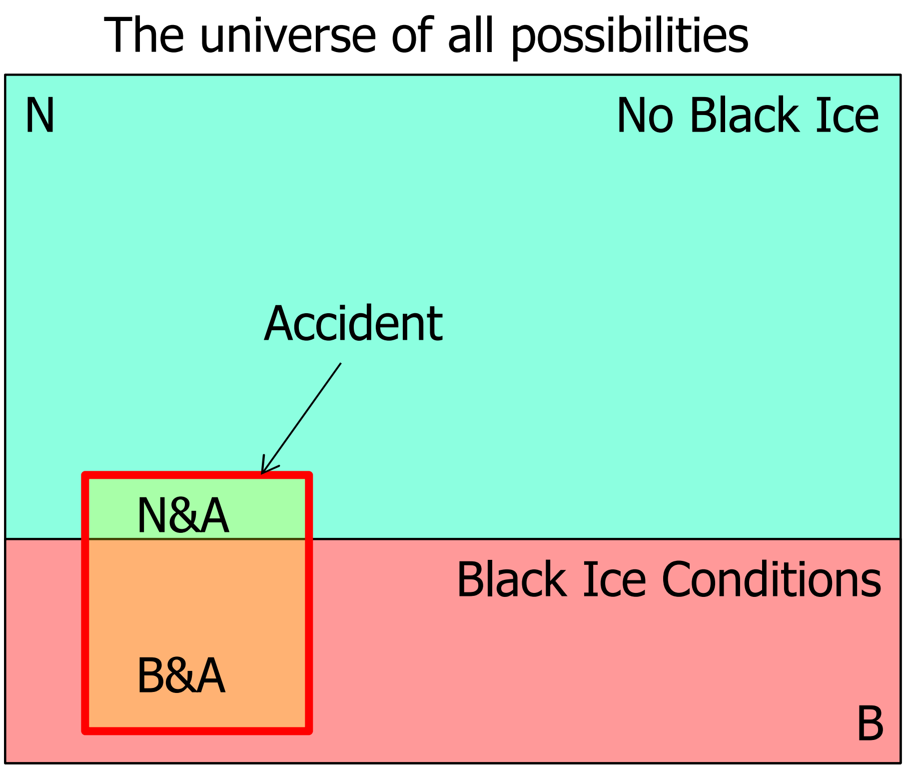


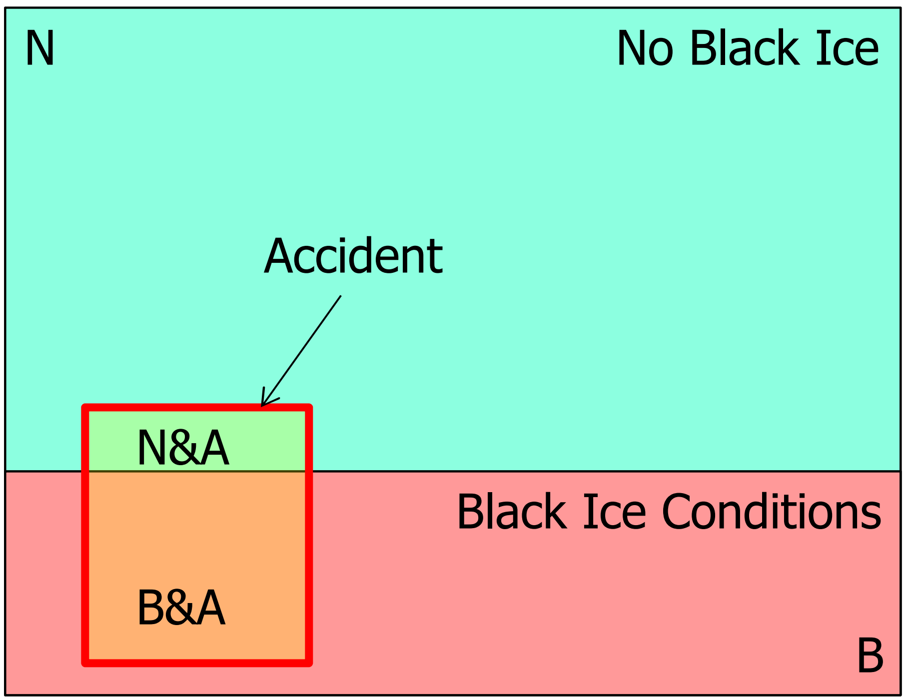
- (B&A)/B = 0.1
- (N&A)/N = 0.02
- B/(B+N) = 0.2
- (N&A+B&A)/(N+B)?

Question: If P(Accident|Ice) = 0.1, P(Accident|No-Ice) = 0.02, P(Ice) = 0.2, what are the odds of accidents in general, P(Accident)?

Review: Total Probability Theorem

$P(\text{Something}) = P(\text{Something}|X)\, P(X) + P(\text{Something}|\bar{X})\, P(\bar{X})$

The universe of all possibilities



- P(Accident) =
  0.1 * 0.2
  + 0.02 * 0.8
  = 0.032

Question: If P(Accident|Ice) = 0.1, P(Accident|No-Ice) = 0.02, P(Ice) = 0.2, what are the odds of accidents in general, P(Accident)?

Example: Probability versus Conditional Probability

- A man is accused of murdering his battered wife. The lawyer says that only 2% of men who batter their wives actually end up killing them, so the odds that this is a murder are very low.
- Is this argument statistically valid? If so, explain why (mathematically). If not, why not?

Example: Probability versus Conditional Probability

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- The relevant statistic is: *given that* a battered wife is murdered, what are the odds that the husband did it? (This happens to be 50%)

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Conditional probability

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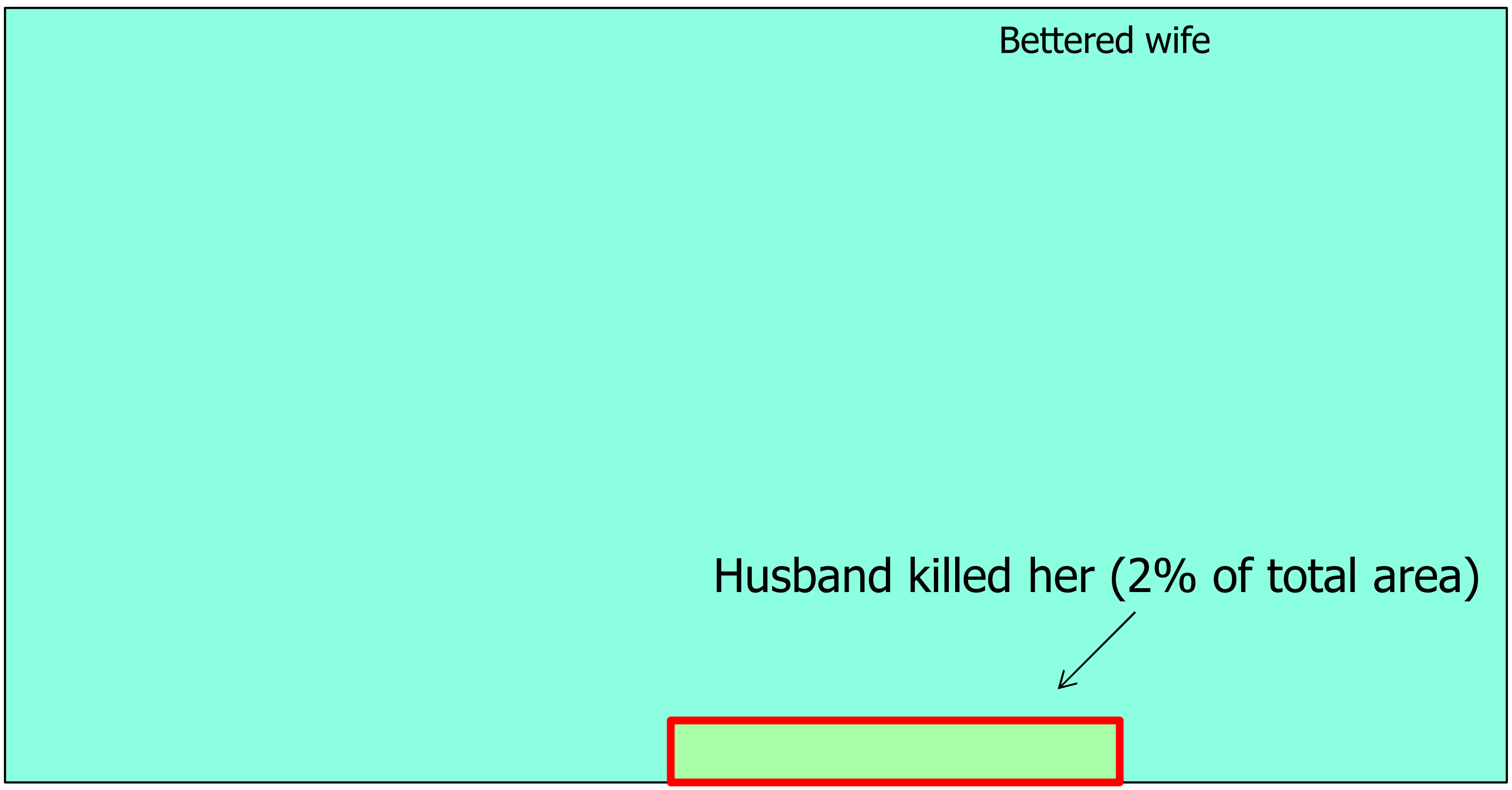

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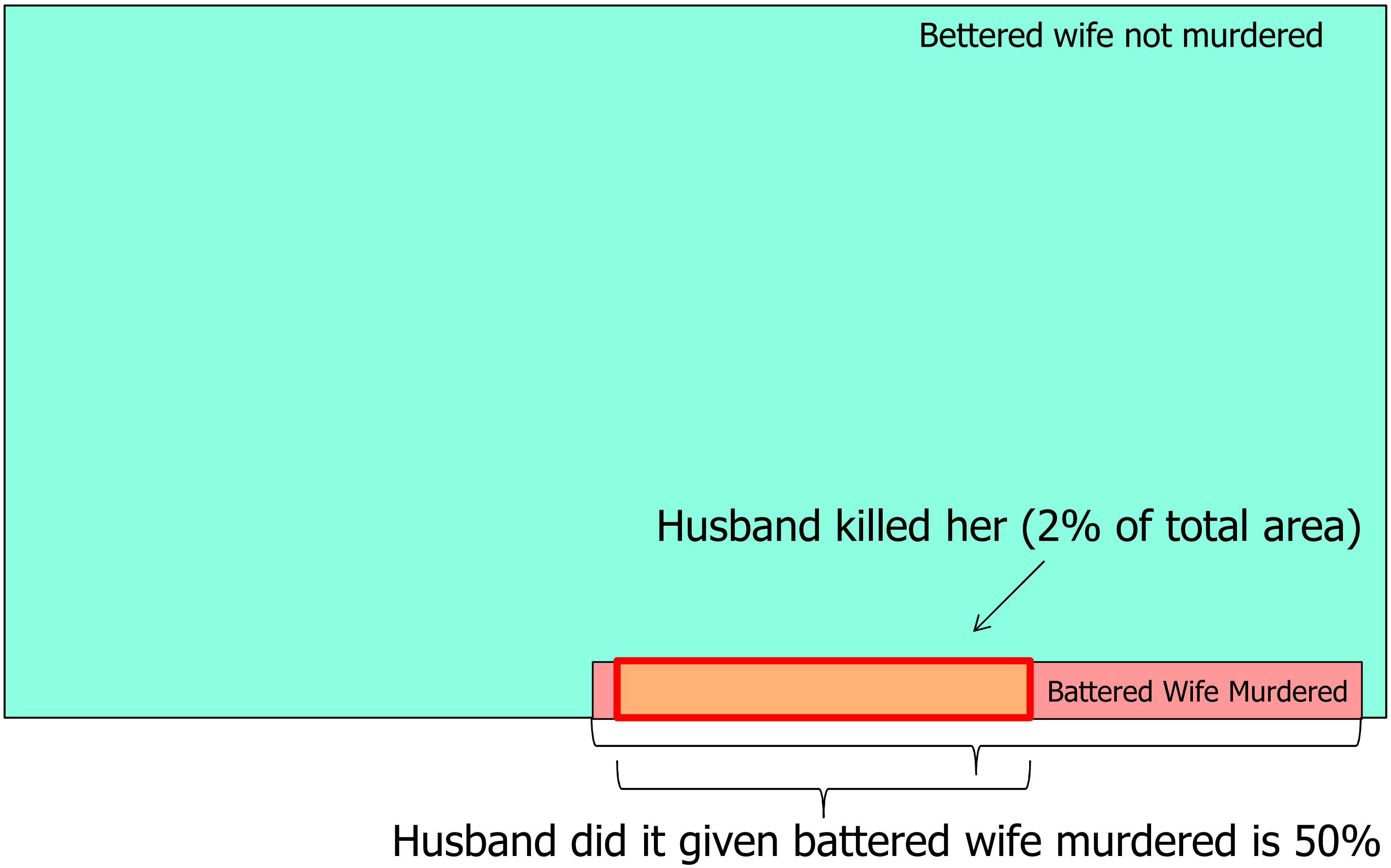

Example: Intrusion Detection

- A motion alarm is used to detect unauthorized access to a warehouse after hours. The motion sensor is mounted near the only entrance to the warehouse. If a burglar enters the building, there is a 99% chance that the burglar triggers the motion alarm.
- At 9pm, on September 16th, 2013, the alarm was set off. What are the odds that a burglar is in the building?

Example: Asteroid Collision with Earth

- If a major Asteroid collides with Earth in St. Louis, traffic on I-57 will be backed up.

- On August 21st, 2017, there was a big back-up on I-57. What are the odds that a major Asteroid collided with Earth?

Example: Asteroid Collision with Earth

- If a major Asteroid collides with Earth in St. Louis (A), traffic on I-57 will be backed up (B).
  - P (B|A) = P (Backup given Asteroid) = 1

- On August 21st, 2017, there was a big back-up on I-57 (B). What are the odds that a major Asteroid collided with Earth in St. Louis (A)?
  - P(A|B) = P (Asteroid given Backup) = ?

Factors to Consider

P (Asteroid given Backup) ?

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- How often do major asteroids hit earth?
  - P(A) = P (Asteroid) = ?
  - The less often it happens, the less likely it is that a traffic jam is attributed to an asteroid.
- How often traffic backs up on I-57
  - P(B) = P (Backup) = ?
  - The more often this happens the less likely it is to be an indicator of asteroid collision

Factors to Consider

P (Asteroid given Backup) ?

- How often do major asteroids hit earth?
  - P(A) = P (Asteroid) = ?
  - The less often it happens, the less likely it is that a traffic jam is attributed to an asteroid.
- How often traffic backs up on I-57
  - P(B) = P (Backup) = ?
  - The more often this happens the less likely it is to be an indicator of asteroid collision
- $P(A|B) = P(B|A). P(A)/P(B)$ ← Bayes Theorem

Factors to Consider

P (Asteroid given Backup) ?

- P(A) = P (Asteroid) = 0.00001
- P(B) = P (Backup) = 0.01
- P(B|A) = 1

- P(A|B) = P(B|A). P(A)/P(B) = 0.001

Review of Important Theorems

- Total Probability Theorem:

  $P(A) = P(A|C_1)\ P(C_1) + \ldots + P(A|C_n)\ P(C_n)$

  where $C_1, \ldots, C_n$ cover the space of all possibilities

- Bayes Theorem:

  $P(A|B) = P(B|A).\ P(A)/P(B)$

- Other: $P(A,B) = P(A|B)\ P(B)$