Data Reliability

Interpreting Sensor Data
Notes and Reminders

- **Reminder:** HW1 is due Thursday.

- **Reminder:** MPs start next week. If you have not already sent us your group members, no problem; we shall assign them. Last call to send group request in (today):
  - Send email to Yiran zhao97@illinois.edu and CC me zaher@Illinois.edu
  - Use subject line: “CS424 GROUP” (in uppercase letters)
Reliability of Systems with Sensors

Example: How to compute the probability of successful collision-avoidance when using a camera-based collision avoidance system?
Reliability of Systems with Sensors

Example: How to compute the probability of successful collision-avoidance when using a camera-based collision avoidance system?

The system fails if:
- The software crashes, or
- The camera fails, or
- The breaks fail, or
- The vision algorithm fails to recognize an obstacle (false negative)
Analogy

- **System reliability challenge:**
  - Building reliable systems from less reliable components

- **Data reliability challenge:**
  - Making reliable conclusions from less reliable (sensor) data
Systems with Imperfect Sensors

- Cyber-physical systems obtain data about their environment via sensors
- Sensors (or data sources in general) are often imperfect
- The challenge is: how to correctly compute probability of correct/safe system behavior given imperfect sensor readings?
Systems with Imperfect Sensors

- The challenge is: how to correctly compute probability of correct/safe system behavior given imperfect sensor readings?
  - The probability of successful detection may depend on context (conditional probability)
  - There may be multiple sensors involved
  - Sensors may have false positives and false negatives. Those will have different implications on safety
Review: Things You Should Know About Probabilities

- Probability of multiple simultaneous events
  - What are the odds that it rains and my basement floods?
    Say \( P(\text{rains}) = 0.2 \). \( P(\text{flood}) = 0.1 \)
Review: Things You Should Know About Probabilities

- Probability of multiple simultaneous events
  - What are the odds that it rains and my basement floods?
  - Answer: It is the odds that “it rains”, times the odds that “my basement floods given that it rains”:
    \[ P(\text{rain, flood}) = P(\text{rain}) \cdot P(\text{flood|rain}) \]

Note: \( P(\text{flood|rain}) \) is larger than \( P(\text{flood}) \)
Review: Things You Should Know About Probabilities

- $P(A, B) = P(A|B).P(B)$

- Corollary: If events $A$ and $B$ are independent, the odds of them happening together is the product of their individual probabilities.
  
  $P(A, B) = P(A).P(B)$

  Note: This is because $P(A|B) = P(A)$
Review: Probability versus Conditional Probability

- Probability: the odds that something, say X, happens, $P(X)$.
- Conditional probability: the odds that X happens *given* that a certain condition, C, has occurred, $P(X|C)$.
  - This condition may affect the odds.
Review: Probability versus Conditional Probability

- Probability: the odds that something, say $X$, happens, $P(X)$.
- Conditional probability: the odds that $X$ happens \textit{given} that a certain condition, $C$, has occurred, $P(X|C)$.
  - This condition may affect the odds.
- Traffic example:
  - $P(\text{Accident})$ may be low
  - $P(\text{Accident}|\text{Black ice})$ is a lot higher!
A Visual Interpretation

The universe of all possibilities
A Visual Interpretation

The universe of all possibilities

Consider a graphical visualization of probabilities where area represents probability.
A Visual Interpretation

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Consider a graphical visualization of probabilities where area represents probability.
A Visual Interpretation

- $P(\text{Accident}) = \frac{(N\&A + B\&A)}{(N+B)}$

The universe of all possibilities

- N: No Black Ice
- B: Black Ice Conditions
- N&A: No accident, no black ice
- B&A: Accident, black ice
A Visual Interpretation

- \( P(\text{Accident}) = \frac{(N \& A + B \& A)}{(N + B)} \)
- \( P(\text{Accident} \mid \text{Ice}) = \frac{(B \& A)}{B} \)
- \( P(\text{Accident} \mid \text{No Ice}) = \frac{(N \& A)}{N} \)
A Visual Interpretation

\[ P(\text{Accident}) = \frac{(N\&A + B\&A)}{(N + B)} \]

\[ P(\text{Accident}|\text{Ice}) = \frac{(B\&A)}{B} \]

\[ P(\text{Accident}|\text{No Ice}) = \frac{(N\&A)}{N} \]

**Question:** If \( P(\text{Accident}|\text{Ice}) = 0.1 \), \( P(\text{Accident}|\text{No-Ice}) = 0.02 \), \( P(\text{Ice}) = 0.2 \), what are the odds of accidents in general, \( P(\text{Accident}) \)?
The universe of all possibilities

- \((B&A)/B = 0.1\)
- \((N&A)/N = 0.02\)
- \(B/(B+N) = 0.2\)
- \((N&A+B&A)/(N+B)\)?
Review: Total Probability Theorem

P(Something) = P(Something|X) P(X) + P(Something|X) P(X)

The universe of all possibilities

P(Accident) = 0.1 * 0.2 + 0.02 * 0.8 = 0.032

Question: If P(Accident|Ice) = 0.1, P(Accident|No-Ice) = 0.02, P(Ice) = 0.2, what are the odds of accidents in general, P(Accident)?
A man is accused of murdering his battered wife. The lawyer says that only 2% of men who batter their wives actually end up killing them, so the odds that this is a murder are very low.

Is this argument statistically valid? If so, explain why (mathematically). If not, why not?
Example: Probability versus Conditional Probability

- A man is accused of murdering his battered wife. The lawyer says that only 2% of men who batter their wives actually end up killing them, so the odds that this is a murder are very low.

- The relevant statistic is: *given that* a battered wife is murdered, what are the odds that the husband did it? (This happens to be 50%)
A man is accused of murdering his battered wife. The lawyer says that only 2% of men who batter their wives actually end up killing them, so the odds that this is a murder are very low.

The relevant statistic is: *given that* a battered wife is murdered, what are the odds that the husband did it? (This happens to be 50%)
A Visual Interpretation

The universe of all possibilities

Bettered wife

Husband killed her (2% of total area)
A Visual Interpretation

The universe of all possibilities

Husband did it given battered wife murdered is 50%

Battered wife not murdered

Husband killed her (2% of total area)
Example: Intrusion Detection

- A motion alarm is used to detect unauthorized access to a warehouse after hours. The motion sensor is mounted near the only entrance to the warehouse. If a burglar enters the building, there is a 99% chance that the burglar triggers the motion alarm.

- At 9pm, on September 16th, 2013, the alarm was set off. What are the odds that a burglar is in the building?
Example: Asteroid Collision with Earth

- If a major Asteroid collides with Earth in St. Louis, traffic on I-57 will be backed up.

- On August 21\textsuperscript{st}, 2017, there was a big back-up on I-57. What are the odds that a major Asteroid collided with Earth?
Example: Asteroid Collision with Earth

- If a major Asteroid collides with Earth in St. Louis (A), traffic on I-57 will be backed up (B).
  - \( P(B|A) = P(\text{Backup given Asteroid}) = 1 \)

- On August 21\(^{st}\), 2017, there was a big back-up on I-57 (B). What are the odds that a major Asteroid collided with Earth in St. Louis (A)?
  - \( P(A|B) = P(\text{Asteroid given Backup}) = ? \)
Factors to Consider
P (Asteroid given Backup) ?
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P (Asteroid given Backup) ?

- How often do major asteroids hit earth?
  - \( P(A) = P(\text{Asteroid}) = ? \)
  - The less often it happens, the less likely it is that a traffic jam is attributed to an asteroid.

- How often traffic backs up on I-57
  - \( P(B) = P(\text{Backup}) = ? \)
  - The more often this happens the less likely it is to be an indicator of asteroid collision.
Factors to Consider
P (Asteroid given Backup) ?

- How often do major asteroids hit earth?
  - P(A) = P (Asteroid) = ?
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- How often traffic backs up on I-57
  - P(B) = P (Backup) = ?
  - The more often this happens the less likely it is to be an indicator of asteroid collision

\[ P(A|B) = P(B|A) \cdot \frac{P(A)}{P(B)} \]  

Bayes Theorem
Factors to Consider

P (Asteroid given Backup) ?

- \( P(A) = P (\text{Asteroid}) = 0.00001 \)
- \( P(B) = P (\text{Backup}) = 0.01 \)
- \( P(B|A) = 1 \)

\[ P(A|B) = P(B|A) \cdot \frac{P(A)}{P(B)} = 0.001 \]
Review of Important Theorems

- Total Probability Theorem:
  \[ P(A) = P(A|C_1) P(C_1) + \ldots + P(A|C_n) P(C_n) \]
  where \( C_1, \ldots, C_n \) cover the space of all possibilities

- Bayes Theorem:
  \[ P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \]

- Other: \( P(A,B) = P(A|B) \cdot P(B) \)