2 ⇓ 2   1 ⇓ 1

1=1 ⇓ true   1 ⇓ 1

C = if 1=1 then 1 else 1*(rec f = ...)(1-1) ⇓ 1

2 ⇓ 2   1 ⇓ 1

(rec f = ...) ⇓ (fun n -> if n=1 then 1 else n*(rec f = ...)(n-1))   (2-1) ⇓ 1   C

2 ⇓ 2   (rec f = fun n -> if n=1 then 1 else n*f(n-1))(2-1) ⇓ 1

B = 2*(rec f = fun n -> if n=1 then 1 else n*f(n-1))(2-1) ⇓ 2

2 ⇓ 2   1 ⇓ 1

2=1 ⇓ false   B

(fun n -> ...) ⇓(fun n -> ...)   2 ⇓ 2   if 2=1 then 1 else 2*(rec f = ...)(2-1) ⇓ 2

A = (fun n -> if n=1 then 1 else n*(rec f = ...)(n-1))2 ⇓ 2 (fun f->f 2) ⇓ (fun f->f 2)   (rec f = fun n -> …) ⇓ (fun n -> if n=1 then 1 else n*(rec f = …)(n-1))   **A**

(fun f->f 2)(rec f = fun n -> if n=1 then 1 else n*f(n-1)) ⇓ 2

let rec f = fun n -> if n=1 then 1 else n*f(n-1) in f(2) ⇓ 2

This example shows that η and η' in the App rule of OS_{clo} may be different. These two environments are in **blue** color.

$$\frac{\dfrac{}{\{x:5,\ y:3\},\ x \Downarrow 5} \qquad \dfrac{}{\{x:5,\ y:3\},\ y \Downarrow 3}}{\mathbf{E} = \{x:5,\ y:3\},\ x+y \Downarrow 8}$$

$$\frac{\dfrac{}{\mathbf{\{x: <(fun\ y \rightarrow x+y),\{x:5\}>\}},\ x \Downarrow <(fun\ y \rightarrow x+y),\mathbf{\{x:5\}}>} \qquad \dfrac{}{\{x: <(fun\ y \rightarrow x+y),\{x:5\}>\},\ 3 \Downarrow 3} \qquad \mathbf{E}}{D = \{x: <(fun\ y \rightarrow x+y),\{x:5\}>\},\ x(3) \Downarrow 8}$$

$$\frac{\dfrac{}{\{x:5\},\ (fun\ x \rightarrow x(3)) \Downarrow <(fun\ x \rightarrow x(3)),\ \{x:5\}>} \qquad \dfrac{}{\{x:5\},\ (fun\ y \rightarrow x+y) \Downarrow <(fun\ y \rightarrow x+y),\{x:5\}>} \qquad \mathbf{D}}{\{x:5\},\ (fun\ x \rightarrow x(3))(fun\ y \rightarrow x+y) \Downarrow 8}$$