Full proof trees for the examples in lecture 22

$$\dfrac{\dfrac{}{\emptyset \vdash \texttt{fst} : (\texttt{int} * \texttt{bool}) \to \texttt{int}} \qquad \dfrac{\dfrac{}{\emptyset \vdash 3 : \texttt{int}} \qquad \dfrac{}{\emptyset \vdash \texttt{true} : \texttt{bool}}}{\emptyset \vdash (3, \texttt{true}) : \texttt{int} * \texttt{bool}}}{\emptyset \vdash \texttt{fst}\ (3, \texttt{true}) : \texttt{int}}$$

$$\texttt{fst}\ (3, \texttt{true}) : \texttt{int} :$$

$$\texttt{let f} = \texttt{fun x} \to \texttt{x 0 in f (fun y} \to \texttt{y} + \texttt{1}) : \texttt{int} :$$

$$\dfrac{\dfrac{}{\emptyset \vdash \texttt{fun x} \to \texttt{x 0} : (\texttt{int} \to \alpha) \to \alpha}\ \text{left branch} \qquad \dfrac{}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha] \vdash \texttt{f (fun y} \to \texttt{y} + \texttt{1}) : \texttt{int}}\ \text{right branch}}{\emptyset \vdash \texttt{let f} = \texttt{fun x} \to \texttt{x 0 in f (fun y} \to \texttt{y} + \texttt{1}) : \texttt{int}}$$

Left branch:

$$\dfrac{\dfrac{\dfrac{}{\emptyset[\texttt{x} : \texttt{int} \to \alpha] \vdash \texttt{x} : \texttt{int} \to \alpha} \qquad \dfrac{}{\emptyset[\texttt{x} : \texttt{int} \to \alpha] \vdash \texttt{0} : \texttt{int}}}{\emptyset[\texttt{x} : \texttt{int} \to \alpha] \vdash \texttt{x 0} : \alpha}}{\emptyset \vdash \texttt{fun x} \to \texttt{x 0} : (\texttt{int} \to \alpha) \to \alpha}$$

Right branch:

$$\dfrac{\dfrac{}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha] \vdash \texttt{f} : (\texttt{int} \to \texttt{int}) \to \texttt{int}} \qquad \dfrac{\dfrac{\ldots}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha, \texttt{y} : \texttt{int}] \vdash \texttt{y} + \texttt{1} : \texttt{int}}}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha] \vdash \texttt{fun y} \to \texttt{y} + \texttt{1} : \texttt{int} \to \texttt{int}}}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha] \vdash \texttt{f (fun y} \to \texttt{y} + \texttt{1}) : \texttt{int}}$$

$$\texttt{let f} = \texttt{fun x} \to \texttt{x 0 in (f (fun y} \to \texttt{y} + \texttt{1}), \texttt{f (fun n} \to [\texttt{n}])) : \texttt{int} * \texttt{int list} :$$

$$\dfrac{\dfrac{\text{left branch above}}{\emptyset \vdash \texttt{fun x} \to \texttt{x 0} : ...} \qquad \dfrac{\dfrac{\text{right branch above}}{\emptyset[\texttt{f} : \forall\alpha....] \vdash \texttt{f (fun y} \to \texttt{y} + \texttt{1}) : \texttt{int}} \qquad \dfrac{\text{third branch}}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha] \vdash \texttt{f (fun n} \to [\texttt{n}]) : \texttt{int list}}}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha] \vdash (\texttt{f (fun y} \to \texttt{y} + \texttt{1}), \texttt{f (fun n} \to [\texttt{n}])) : \texttt{int} * \texttt{int list}}}{\emptyset \vdash \texttt{let f} = \texttt{fun x} \to \texttt{x 0 in (f (fun y} \to \texttt{y} + \texttt{1}), \texttt{f (fun n} \to [\texttt{n}])) : \texttt{int} * \texttt{int list}}$$

Third branch:

$$\dfrac{\dfrac{}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha] \vdash \texttt{f} : (\texttt{int} \to \texttt{int list}) \to \texttt{int list}} \qquad \dfrac{\dfrac{...}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha, \texttt{n} : \texttt{int}] \vdash [\texttt{n}] : \texttt{int list}}}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha] \vdash \texttt{fun n} \to [\texttt{n}] : \texttt{int} \to \texttt{int list}}}{\emptyset[\texttt{f} : \forall\alpha.(\texttt{int} \to \alpha) \to \alpha] \vdash \texttt{f (fun n} \to [\texttt{n}]) : \texttt{int list}}$$