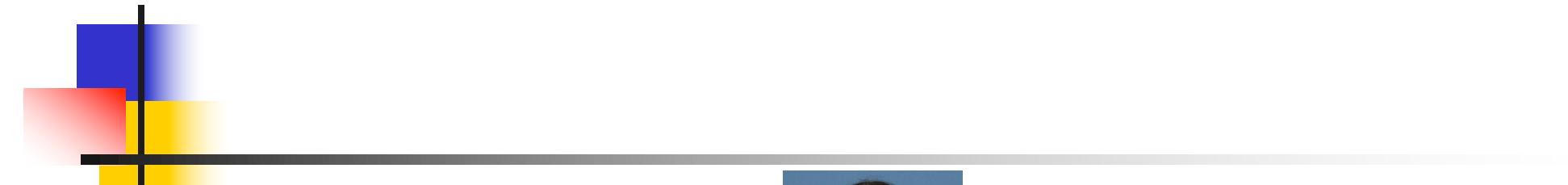


# Programming Languages and Compilers (CS 421)



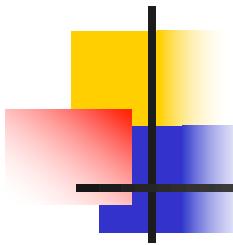
Elsa L Gunter

2112 SC, UIUC



<https://courses.engr.illinois.edu/cs421/sp2023>

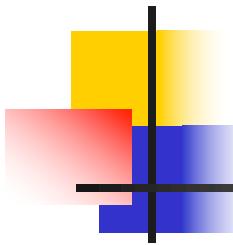
Based in part on slides by Mattox Beckman, as updated  
by Vikram Adve and Gul Agha



# Example: If Then Else Rule

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(if  $x > 5$  then  $y := 2 + 3$  else  $y := 3 + 4$  fi,  
 $\{x \rightarrow 7\}) \Downarrow ?$



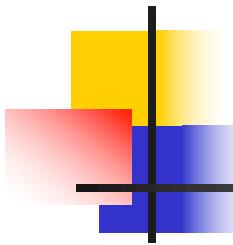
# Example: If Then Else Rule

---

$$(x > 5, \{x \rightarrow 7\}) \Downarrow ?$$

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$$\text{(if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \\ \{x \rightarrow 7\}) \Downarrow ?$$



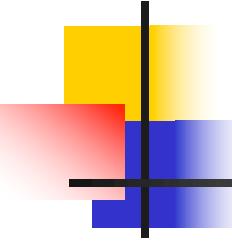
# Example: Arith Relation

? > ? = ?

$$\frac{(x, \{x > 7\}) \Downarrow? \quad (5, \{x > 7\}) \Downarrow?}{(x > 5, \{x > 7\}) \Downarrow?}$$

(if  $x > 5$  then  $y := 2 + 3$  else  $y := 3 + 4$  fi,

$\{x > 7\}) \Downarrow?$



# Example: Identifier(s)

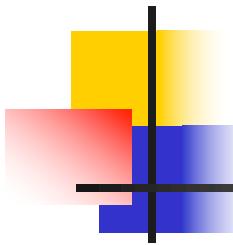
$7 > 5 = \text{true}$

$(x, \{x > 7\}) \Downarrow 7 \quad (5, \{x > 7\}) \Downarrow 5$

$(x > 5, \{x > 7\}) \Downarrow ?$

---

$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,}$   
 $\{x > 7\}) \Downarrow ?$



# Example: Arith Relation

---

$7 > 5 = \text{true}$

$(x, \{x > 7\}) \Downarrow 7 \quad (5, \{x > 7\}) \Downarrow 5$

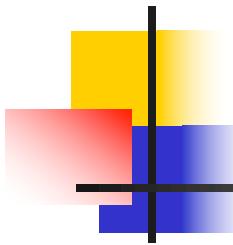
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$(x > 5, \{x > 7\}) \Downarrow \text{true}$

---

$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,}$

$\{x > 7\}) \Downarrow ?$



# Example: If Then Else Rule

$$7 > 5 = \text{true}$$

$$\underline{(x, \{x > 7\}) \Downarrow 7 \quad (5, \{x > 7\}) \Downarrow 5}$$

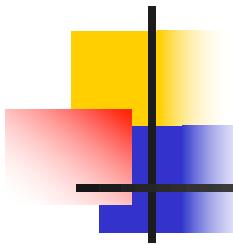
$$\underline{(x > 5, \{x > 7\}) \Downarrow \text{true}}$$

$$\underline{\quad \quad \quad (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi},}$$

$$\quad \quad \quad \{x > 7\}) \Downarrow ?$$

$$\underline{\quad \quad \quad (y := 2 + 3, \{x > 7\})}$$

$$\underline{\quad \quad \quad \Downarrow ?}$$

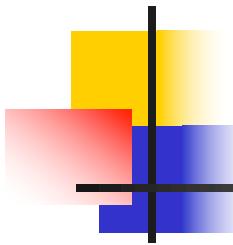


# Example: Assignment

$$\frac{\begin{array}{c} 7 > 5 = \text{true} \\ \hline (x, \{x > 7\}) \Downarrow 7 \quad (5, \{x > 7\}) \Downarrow 5 \end{array}}{(x > 5, \{x > 7\}) \Downarrow \text{true}} \quad \frac{(2+3, \{x > 7\}) \Downarrow ?}{(y := 2 + 3, \{x > 7\})}$$
$$\frac{(if \ x > 5 \ then \ y := 2 + 3 \ else \ y := 3 + 4 \ fi, \ \{x > 7\}) \Downarrow ?}{}$$

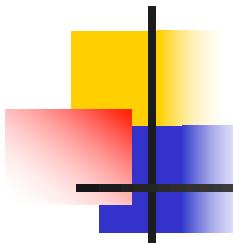
# Example: Arith Op

$$\begin{array}{c} ? + ? = ? \\ \hline (2, \{x > 7\}) \Downarrow ? \quad (3, \{x > 7\}) \Downarrow ? \\ \hline \begin{array}{c} 7 > 5 = \text{true} \\ \hline (x, \{x > 7\}) \Downarrow 7 \quad (5, \{x > 7\}) \Downarrow 5 \end{array} \quad \begin{array}{c} (2+3, \{x > 7\}) \Downarrow ? \\ \hline (y := 2 + 3, \{x > 7\}) \end{array} \\ \hline \begin{array}{c} (x > 5, \{x > 7\}) \Downarrow \text{true} \\ \hline \begin{array}{c} ( \text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\ \{x > 7\} ) \Downarrow ? \end{array} \end{array} \end{array}$$



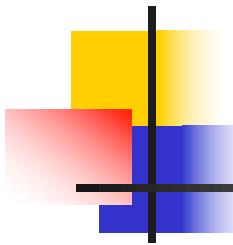
# Example: Numerals

$$\frac{\frac{\frac{2 + 3 = 5}{(2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3}}{\frac{7 > 5 = \text{true}}{(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5}} \quad \frac{(2+3, \{x \rightarrow 7\}) \Downarrow ?}{(y := 2 + 3, \{x \rightarrow 7\})}}{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}} \quad \frac{}{\Downarrow ?}}$$
$$\frac{(if \ x > 5 \ then \ y := 2 + 3 \ else \ y := 3 + 4 \ fi, \{x \rightarrow 7\}) \Downarrow ?}{}$$



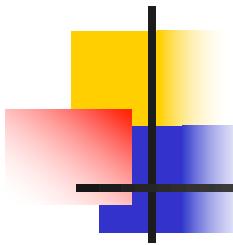
# Example: Arith Op

$$\frac{\frac{\frac{2 + 3 = 5}{(2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3}}{\frac{7 > 5 = \text{true}}{(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5}} \quad \frac{(2+3, \{x \rightarrow 7\}) \Downarrow 5}{(y := 2 + 3, \{x \rightarrow 7\})}}{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}} \quad \Downarrow ?}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\ \{x \rightarrow 7\}) \Downarrow ?}$$



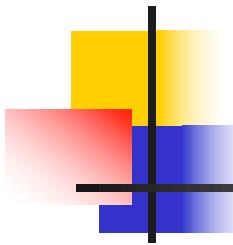
# Example: Assignment

$$\frac{\frac{\frac{2 + 3 = 5}{(2, \{x > 7\}) \Downarrow 2 \quad (3, \{x > 7\}) \Downarrow 3}}{\frac{7 > 5 = \text{true}}{(x, \{x > 7\}) \Downarrow 7 \quad (5, \{x > 7\}) \Downarrow 5}} \quad \frac{(2+3, \{x > 7\}) \Downarrow 5}{(y := 2 + 3, \{x > 7\}) \Downarrow \{x > 7, y > 5\}}}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x > 7\}) \Downarrow ?}$$



# Example: If Then Else Rule

$$\frac{\frac{\frac{2 + 3 = 5}{(2, \{x > 7\}) \Downarrow 2 \quad (3, \{x > 7\}) \Downarrow 3}}{\frac{7 > 5 = \text{true}}{(x, \{x > 7\}) \Downarrow 7 \quad (5, \{x > 7\}) \Downarrow 5}} \quad \frac{(2+3, \{x > 7\}) \Downarrow 5}{(y := 2 + 3, \{x > 7\}) \Downarrow \{x > 7, y > 5\}}}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\ \{x > 7\}) \Downarrow \{x > 7, y > 5\}}$$

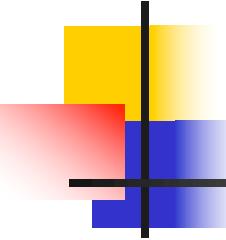


# Let in Command

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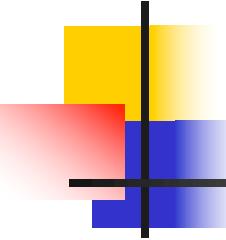
$$\frac{(E, m) \downarrow v \quad (C, m[I \leftarrow v]) \downarrow m'}{(\text{let } I = E \text{ in } C, m) \downarrow m''}$$

Where  $m''(y) = m'(y)$  for  $y \neq I$  and  
 $m''(I) = m(I)$  if  $m(I)$  is defined,  
and  $m''(I)$  is undefined otherwise



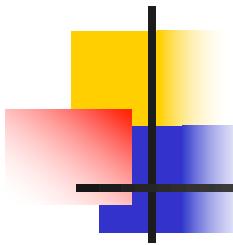
# Example

$$\frac{\begin{array}{c} \underline{(x, \{x > 5\}) \downarrow 5 \quad (3, \{x > 5\}) \downarrow 3} \\ \underline{(x+3, \{x > 5\}) \downarrow 8} \\ (5, \{x > 17\}) \downarrow 5 \quad (x := x + 3, \{x > 5\}) \downarrow \{x > 8\} \end{array}}{(\text{let } x = 5 \text{ in } (x := x + 3), \{x > 17\}) \downarrow ?}$$



# Example

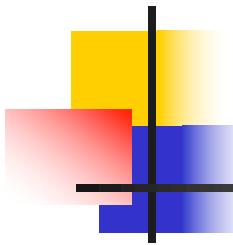
$$\frac{\begin{array}{c} \underline{(x, \{x > 5\}) \downarrow 5 \quad (3, \{x > 5\}) \downarrow 3} \\ \hline (x+3, \{x > 5\}) \downarrow 8 \end{array}}{(5, \{x > 17\}) \downarrow 5 \quad \underline{(x := x + 3, \{x > 5\}) \downarrow \{x > 8\}}} \\ \hline (\text{let } x = 5 \text{ in } (x := x + 3), \{x > 17\}) \downarrow \{x > 17\}$$



# Comment

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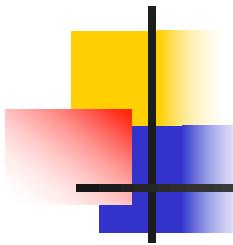
- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explicitly*
- Clash of constructs apparent in awkward semantics



# Interpretation Versus Compilation

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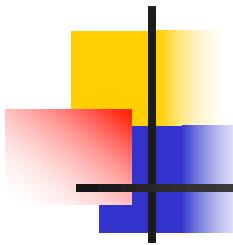
- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed



# Interpreter

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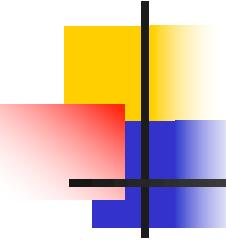
- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations



# Interpreter

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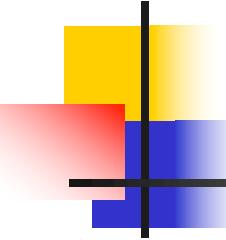
- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next “state”
  - To get final value, put in a loop



# Natural Semantics Example

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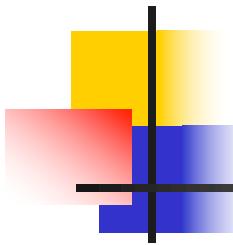
- $\text{compute\_exp}(\text{Var}(v), m) = \text{look\_up } v \text{ in } m$
- $\text{compute\_exp}(\text{Int}(n), \_) = \text{Num}(n)$
- ...
- $\text{compute\_com}(\text{IfExp}(b, c_1, c_2), m) =$   
    if  $\text{compute\_exp}(b, m) = \text{Bool}(\text{true})$   
    then  $\text{compute\_com}(c_1, m)$   
    else  $\text{compute\_com}(c_2, m)$



# Natural Semantics Example

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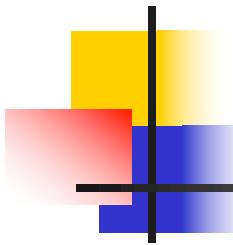
- $\text{compute\_com}(\text{While}(b,c), m) =$   
    if  $\text{compute\_exp}(b,m) = \text{Bool}(\text{false})$   
        then  $m$   
    else  $\text{compute\_com}(\text{While}(b,c), \text{compute\_com}(c,m))$
  
- May fail to terminate - exceed stack limits
- Returns no useful information then



# Transition Semantics

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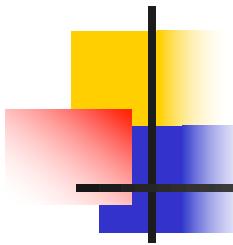
- Form of operational semantics
- Describes how each program construct transforms machine state by *transitions*
- Rules look like
$$(C, m) \rightarrow (C', m') \text{ or } (C, m) \rightarrow m'$$
- $C, C'$  is code remaining to be executed
- $m, m'$  represent the state/store/memory/environment
  - Partial mapping from identifiers to values
  - Sometimes  $m$  (or  $C$ ) not needed
- Indicates exactly one step of computation



# Expressions and Values

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- $C, C'$  used for commands;  $E, E'$  for expressions;  $U, V$  for values
- Special class of expressions designated as *values*
  - Eg 2, 3 are values, but  $2+3$  is only an expression
- Memory only holds values
  - Other possibilities exist

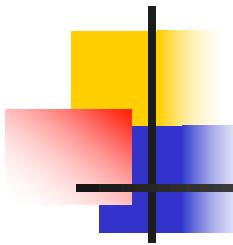


# Evaluation Semantics

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- Transitions successfully stops when E/C is a value/memory
- Evaluation fails if no transition possible, but not at value/memory
- Value/memory is the final *meaning* of original expression/command (in the given state)
- Coarse semantics: final value / memory
- More fine grained: whole transition sequence

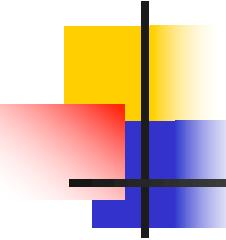
1525 minutes



# Simple Imperative Programming Language

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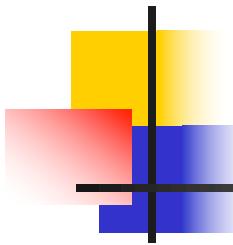
- $I \in Identifiers$
- $N \in Numerals$
- $B ::= \text{true} \mid \text{false} \mid B \& B \mid B \text{ or } B \mid \text{not } B \mid E < E \mid E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- $C ::= \text{skip} \mid C; C \mid I ::= E$   
■  $\mid \text{if } B \text{ then } C \text{ else } C \text{ fi} \mid \text{while } B \text{ do } C \text{ od}$



# Transitions for Expressions

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- Numerals are values
- Boolean values = {true, false}
- Identifiers:  $(I, m) \rightarrow (m(I), m)$



# Boolean Operations:

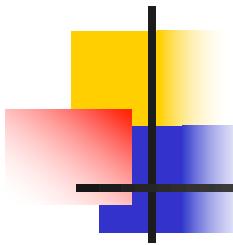
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- Operators: (short-circuit)

$$\frac{(\text{false} \ \& \ B, m) \rightarrow (\text{false}, m) \quad (B, m) \rightarrow (B'', m)}{(\text{true} \ \& \ B, m) \rightarrow (B, m) \quad \overline{(B \ \& \ B', m) \rightarrow (B'' \ \& \ B', m)}}$$

$$\frac{(\text{true or } B, m) \rightarrow (\text{true}, m) \quad (B, m) \rightarrow (B'', m)}{(\text{false or } B, m) \rightarrow (B, m) \quad \overline{(B \text{ or } B', m) \rightarrow (B'' \text{ or } B', m)}}$$

$$\frac{(\text{not true}, m) \rightarrow (\text{false}, m) \quad (B, m) \rightarrow (B', m)}{(\text{not false}, m) \rightarrow (\text{true}, m) \quad \overline{(\text{not } B, m) \rightarrow (\text{not } B', m)}}$$



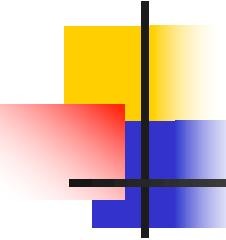
# Relations

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$$\frac{(E, m) \rightarrow (E', m)}{(E \sim E', m) \rightarrow (E' \sim E', m)}$$

$$\frac{(E, m) \rightarrow (E', m)}{(V \sim E, m) \rightarrow (V \sim E', m)}$$

$(U \sim V, m) \rightarrow (\text{true}, m)$  or  $(\text{false}, m)$   
depending on whether  $U \sim V$  holds or not



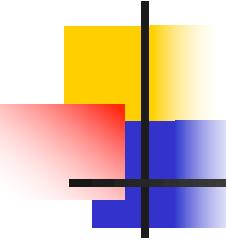
# Arithmetic Expressions

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$$\frac{(E, m) \rightarrow (E'', m)}{(E \text{ op } E', m) \rightarrow (E'' \text{ op } E', m)}$$

$$\frac{(E, m) \rightarrow (E', m)}{(V \text{ op } E, m) \rightarrow (V \text{ op } E', m)}$$

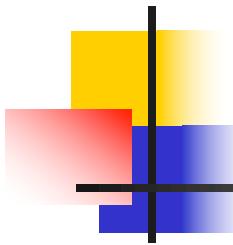
$(U \text{ op } V, m) \rightarrow (N, m)$  where  $N$  is the specified value for  $U \text{ op } V$



# Commands - in English

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- skip means done evaluating
- When evaluating an assignment, evaluate the expression first
- If the expression being assigned is already a value, update the memory with the new value for the identifier
- When evaluating a sequence, work on the first command in the sequence first
- If the first command evaluates to a new memory (ie completes), evaluate remainder with new memory



# Commands

---

$$(\text{skip}, m) \rightarrow m$$

$$\frac{(E, m) \rightarrow (E', m)}{(I := E, m) \rightarrow (I := E', m)}$$

$$(I := V, m) \rightarrow m[I \leftarrow V]$$

$$\frac{(C, m) \rightarrow (C', m')}{(C; C', m) \rightarrow (C'; C', m')} \quad \frac{(C, m) \rightarrow m'}{(C; C', m) \rightarrow (C', m')}$$