

Programming Languages and Compilers (CS 421)

Elsa L Gunter
2112 SC, UIUC



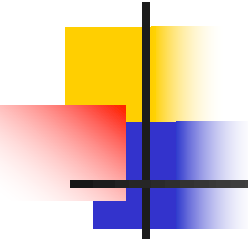
<https://courses.engr.illinois.edu/cs421/sp2023>

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha



LR Parsing

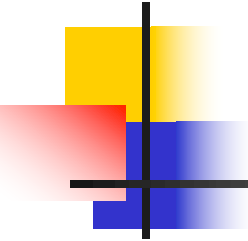
- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

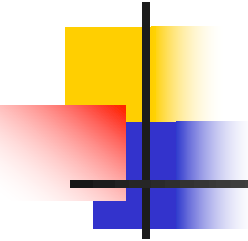
$= \bullet (0 + 1) + 0$ shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$\begin{aligned} &= (\bullet 0 + 1) + 0 && \text{shift} \\ &= \bullet (0 + 1) + 0 && \text{shift} \end{aligned}$$



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$\Rightarrow (0 \bullet + 1) + 0$$

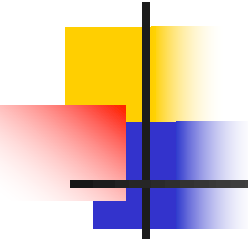
$$= (\bullet 0 + 1) + 0$$

$$= \bullet (0 + 1) + 0$$

reduce

shift

shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$$= (\langle \text{Sum} \rangle \bullet + 1) + 0$$

shift

$$\Rightarrow (0 \bullet + 1) + 0$$

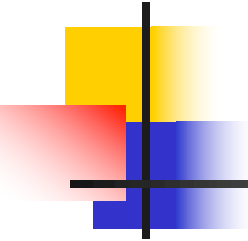
reduce

$$= (\bullet 0 + 1) + 0$$

shift

$$= \bullet (0 + 1) + 0$$

shift



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= \langle \text{Sum} \rangle \bullet + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$

$= \langle \text{Sum} \rangle + \bullet 0$ shift
 $= \langle \text{Sum} \rangle \bullet + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow$
 $\Rightarrow \langle \text{Sum} \rangle + 0 \bullet$ reduce
 $= \langle \text{Sum} \rangle + \bullet 0$ shift
 $= \langle \text{Sum} \rangle \bullet + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle) \bullet + 0$ reduce
 $= (\langle \text{Sum} \rangle \bullet) + 0$ shift
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet) + 0$ reduce
 $\Rightarrow (\langle \text{Sum} \rangle + 1 \bullet) + 0$ reduce
 $= (\langle \text{Sum} \rangle + \bullet 1) + 0$ shift
 $= (\langle \text{Sum} \rangle \bullet + 1) + 0$ shift
 $\Rightarrow (0 \bullet + 1) + 0$ reduce
 $= (\bullet 0 + 1) + 0$ shift
 $= \bullet (0 + 1) + 0$ shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle$	$\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●	reduce
	$\Rightarrow \langle \text{Sum} \rangle + 0$	●	reduce
	$= \langle \text{Sum} \rangle +$	● 0	shift
	$= \langle \text{Sum} \rangle$	● + 0	shift
	$\Rightarrow (\langle \text{Sum} \rangle)$	● + 0	reduce
	$= (\langle \text{Sum} \rangle$	●) + 0	shift
	$\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●) + 0	reduce
	$\Rightarrow (\langle \text{Sum} \rangle + 1$	●) + 0	reduce
	$= (\langle \text{Sum} \rangle +$	● 1) + 0	shift
	$= (\langle \text{Sum} \rangle$	● + 1) + 0	shift
	$\Rightarrow (0$	● + 1) + 0	reduce
	$= ($	● 0 + 1) + 0	shift
	$=$	● (0 + 1) + 0	shift

Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle$	●	\Rightarrow	$\langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●	reduce	
		\Rightarrow	$\langle \text{Sum} \rangle + 0$	●	reduce	
		=	$\langle \text{Sum} \rangle +$	●	0	shift
		=	$\langle \text{Sum} \rangle$	●	+ 0	shift
		\Rightarrow	$(\langle \text{Sum} \rangle)$	●	+ 0	reduce
		=	$(\langle \text{Sum} \rangle$	●) + 0	shift
		\Rightarrow	$(\langle \text{Sum} \rangle + \langle \text{Sum} \rangle$	●) + 0	reduce
		\Rightarrow	$(\langle \text{Sum} \rangle + 1$	●) + 0	reduce
		=	$(\langle \text{Sum} \rangle +$	●	1) + 0	shift
		=	$(\langle \text{Sum} \rangle$	●	+ 1) + 0	shift
		\Rightarrow	$(0$	●	+ 1) + 0	reduce
		=	$($	●	0 + 1) + 0	shift
		=	●	$(0 + 1) + 0$	shift	



Example

$$(0 + 1) + 0$$





Example

$$(\quad 0 \quad + \quad 1 \quad) \quad + \quad 0$$



4/5/23



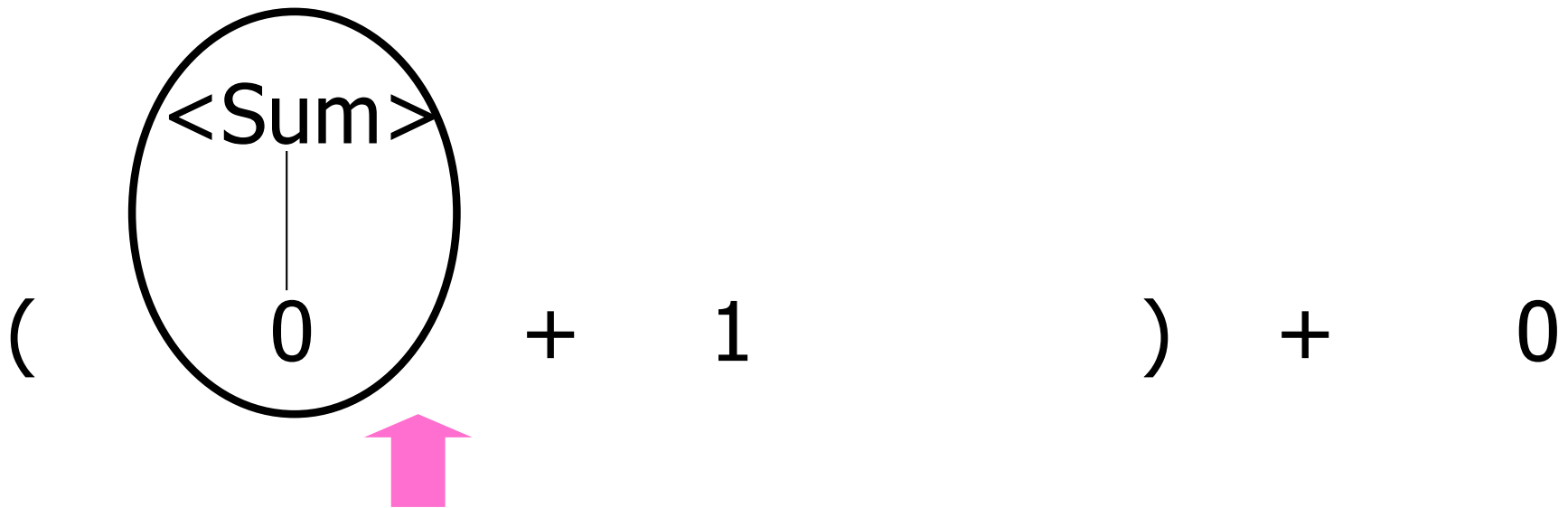
Example

(0 + 1) + 0



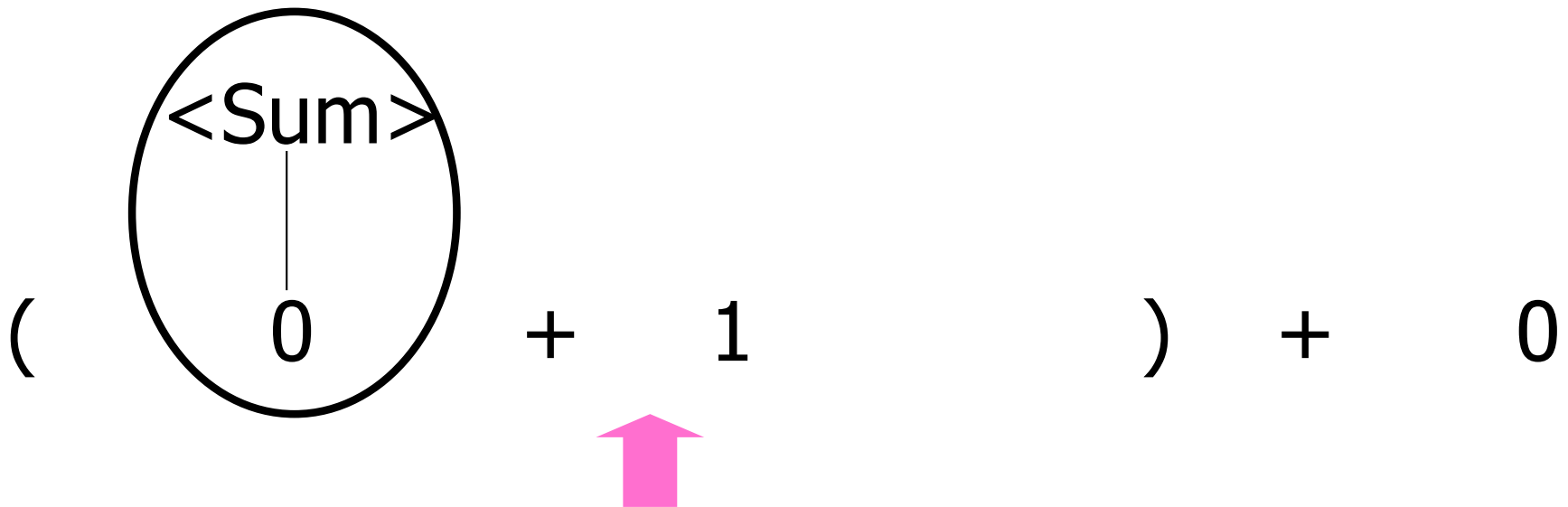


Example





Example



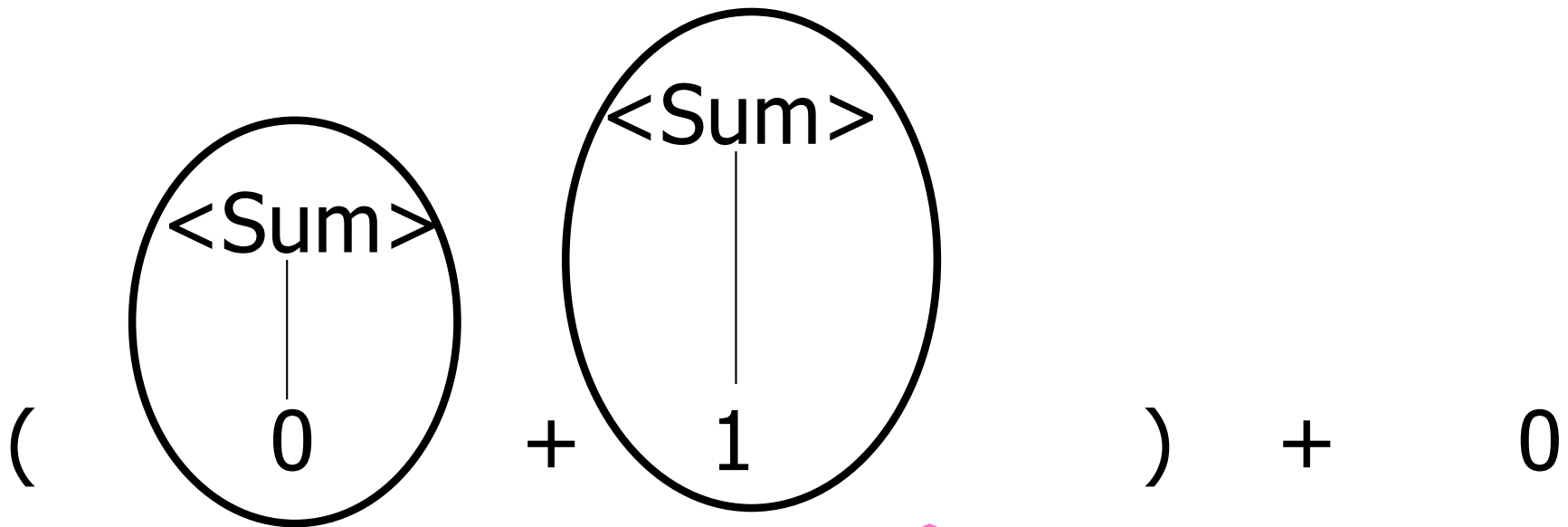


Example

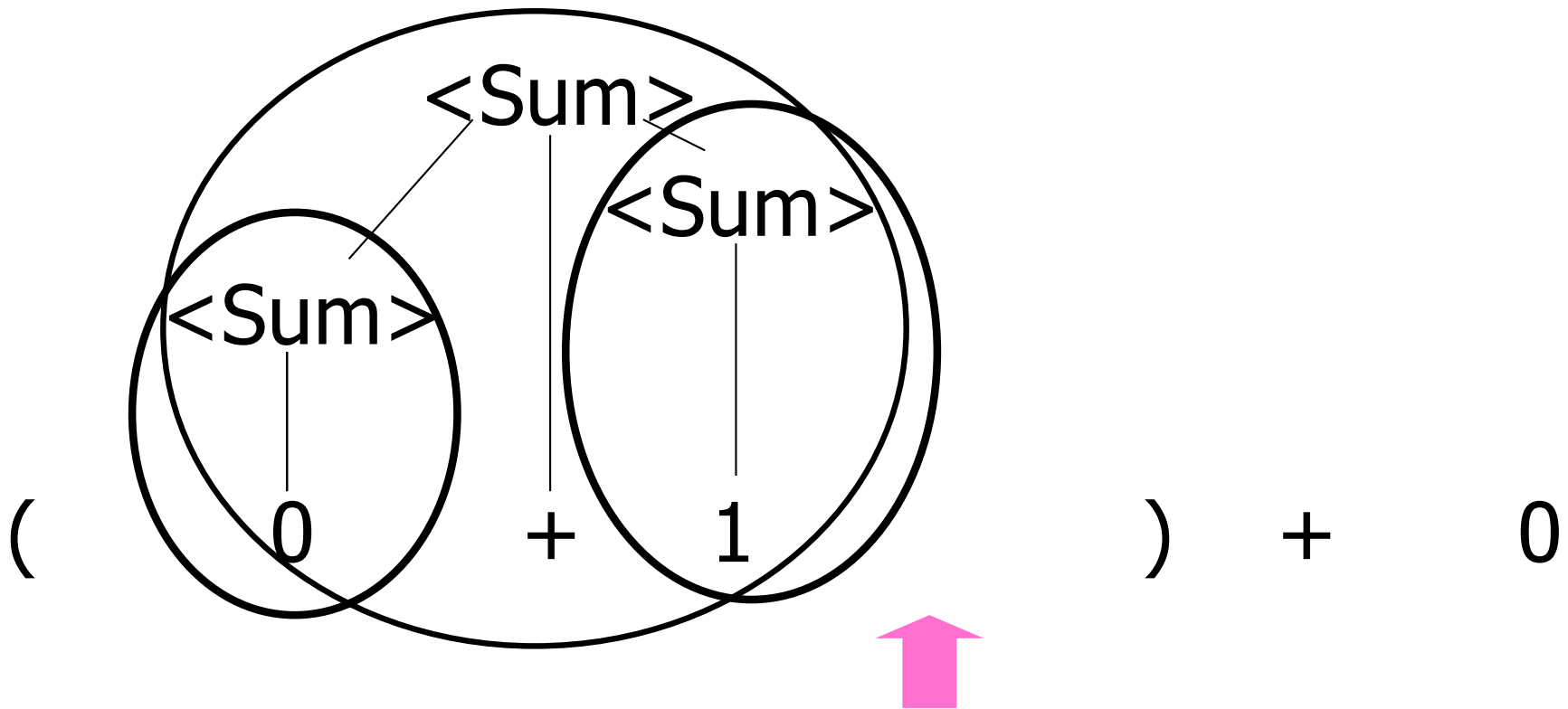




Example

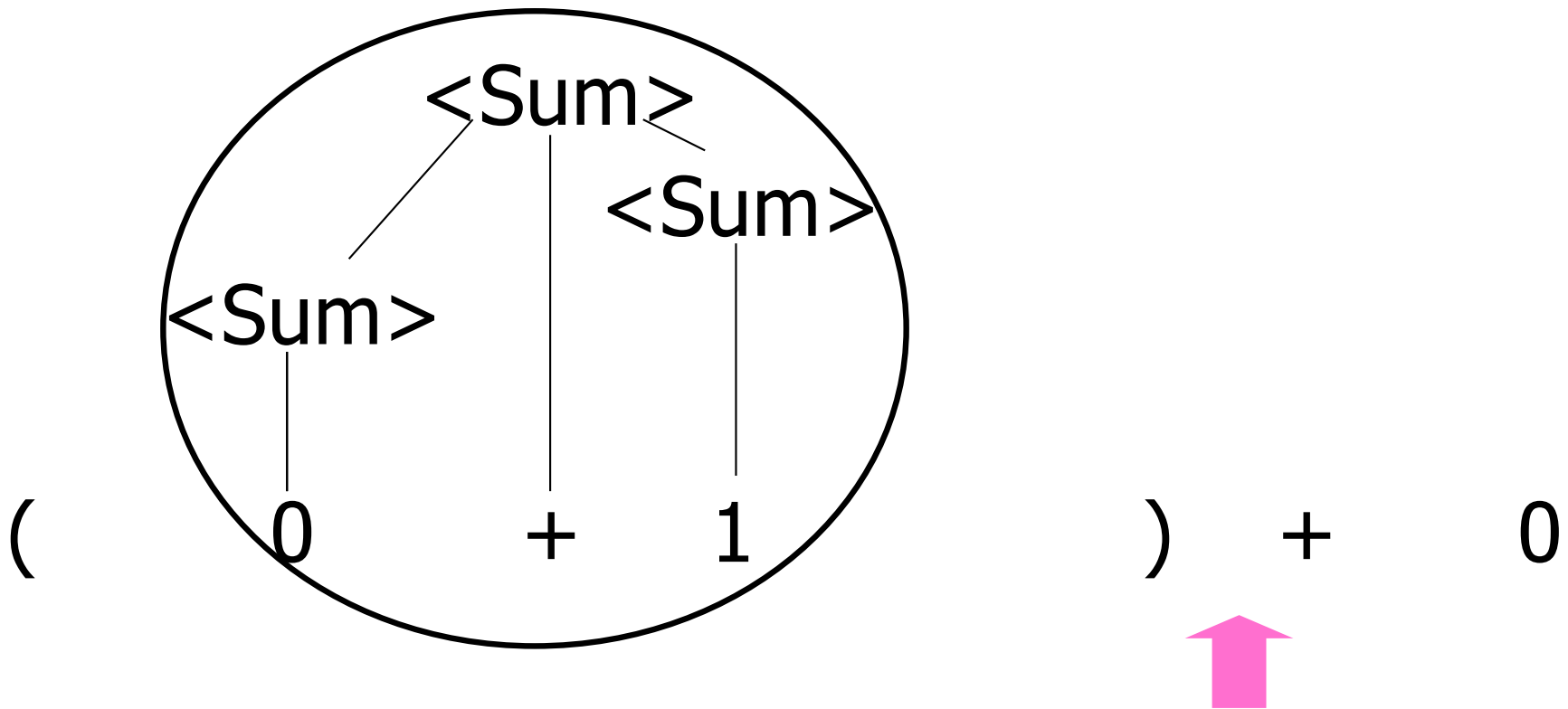


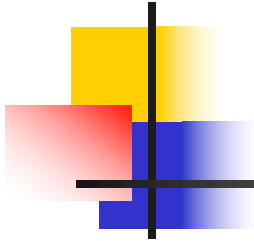
Example



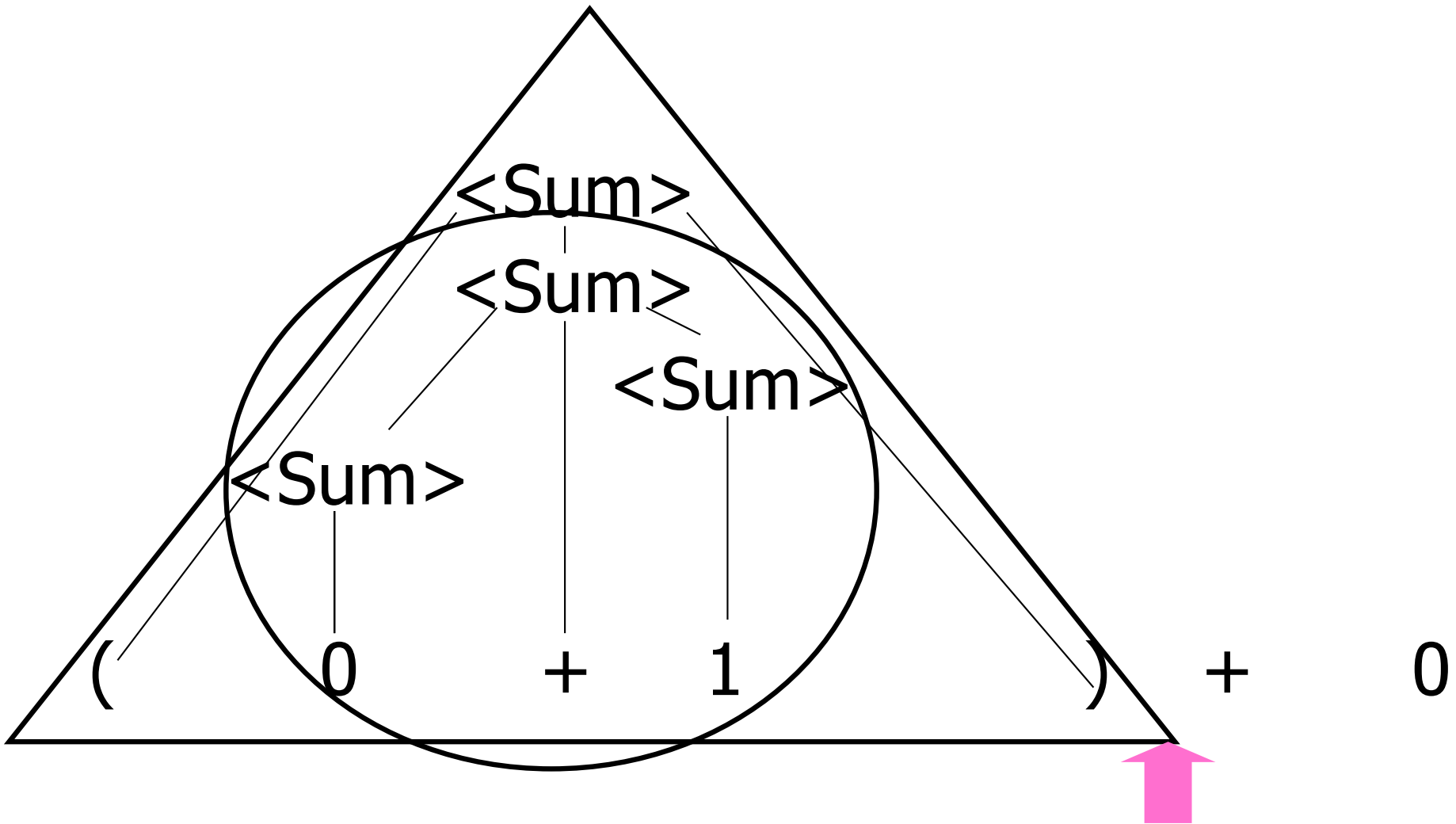


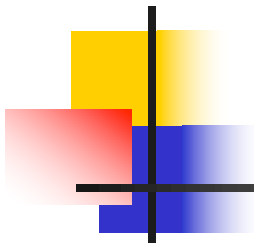
Example



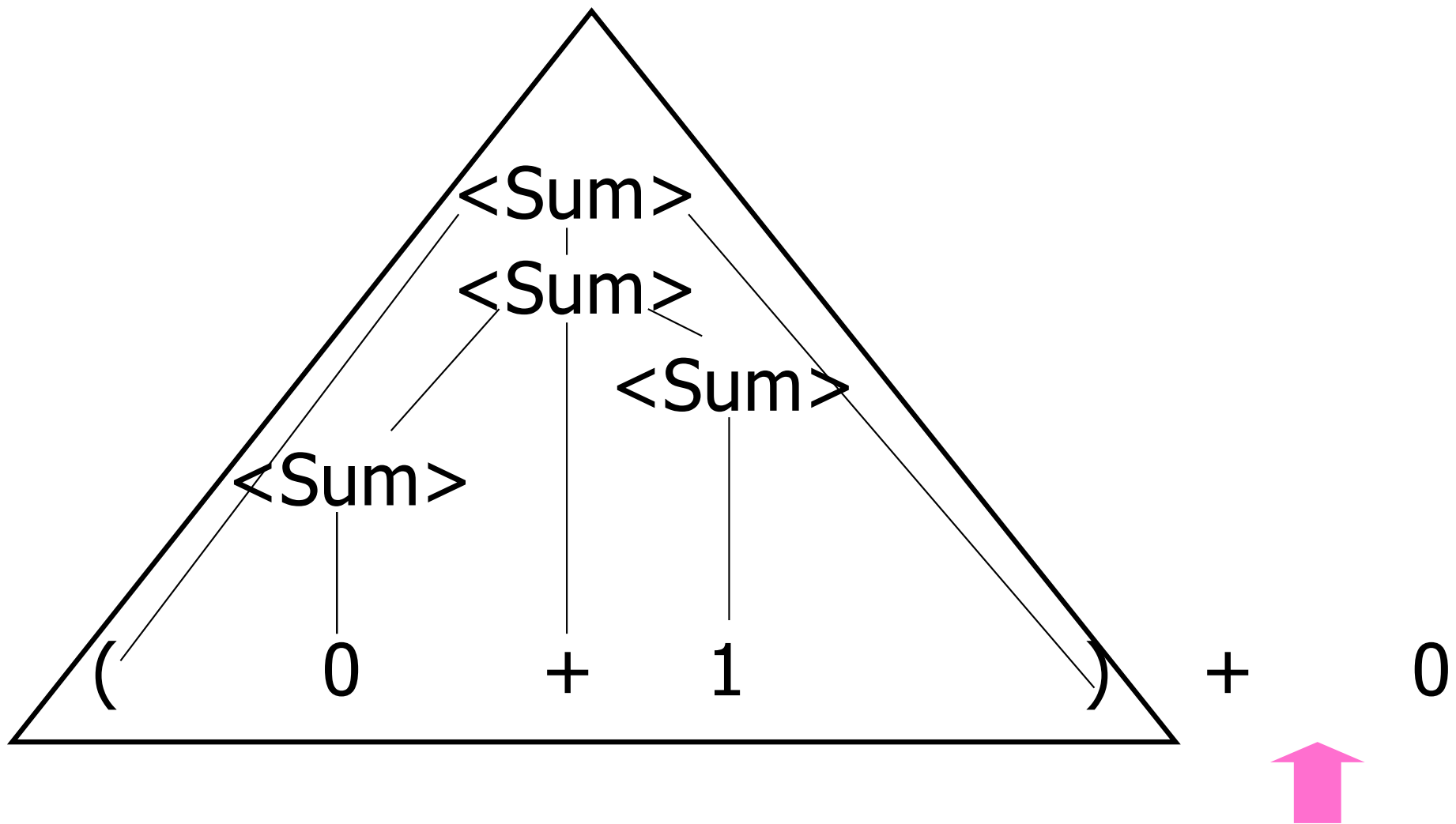


Example



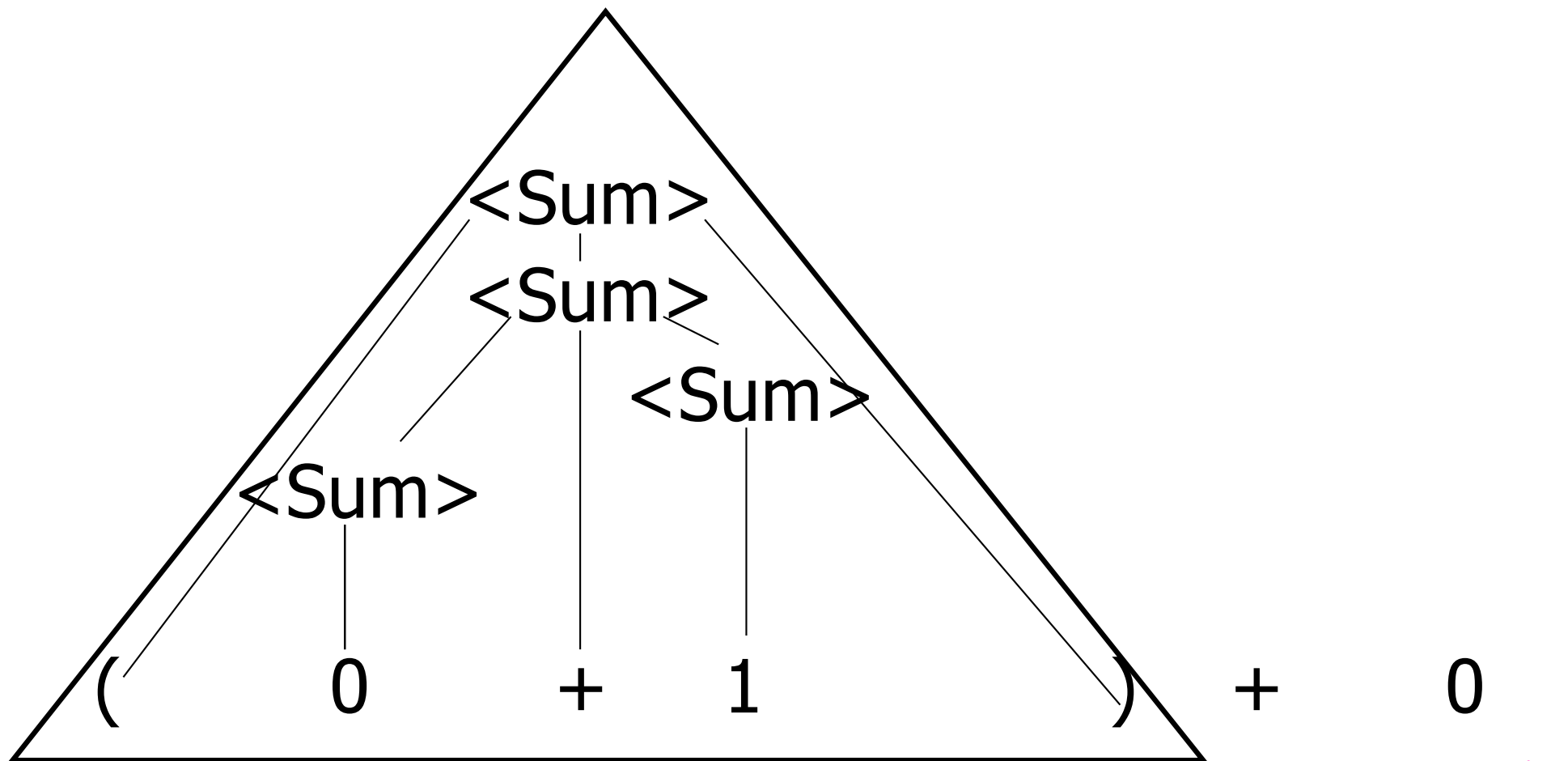


Example



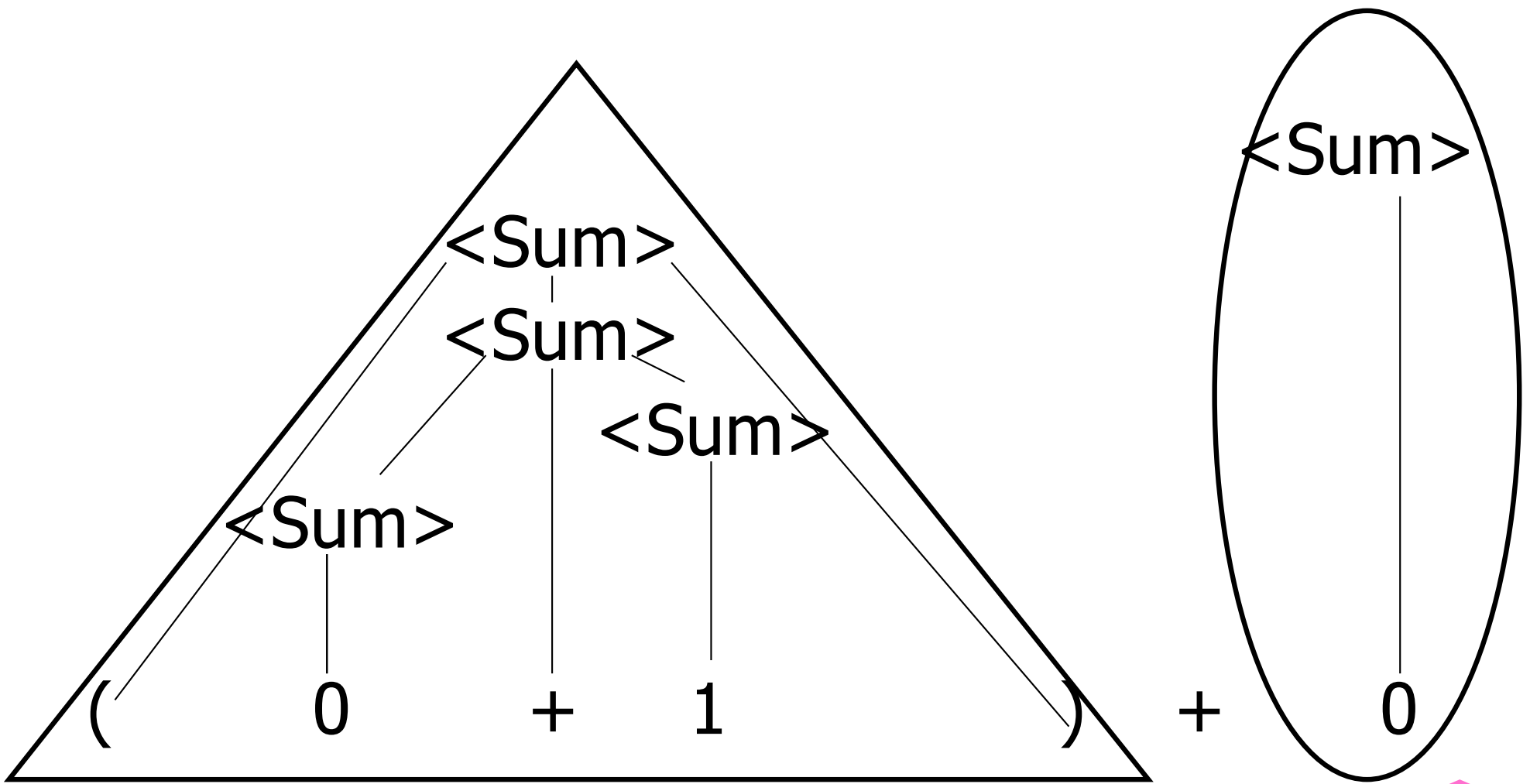


Example



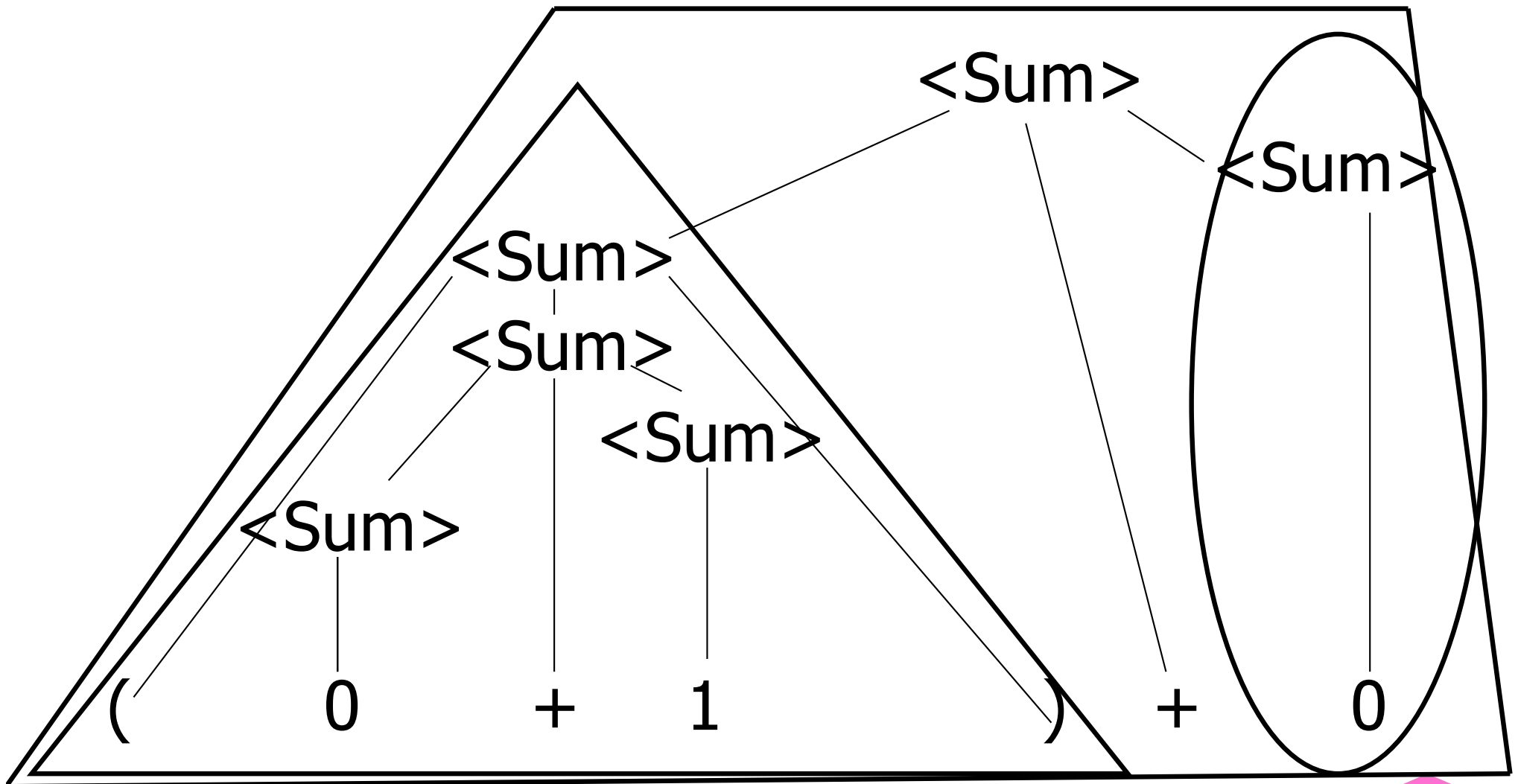


Example



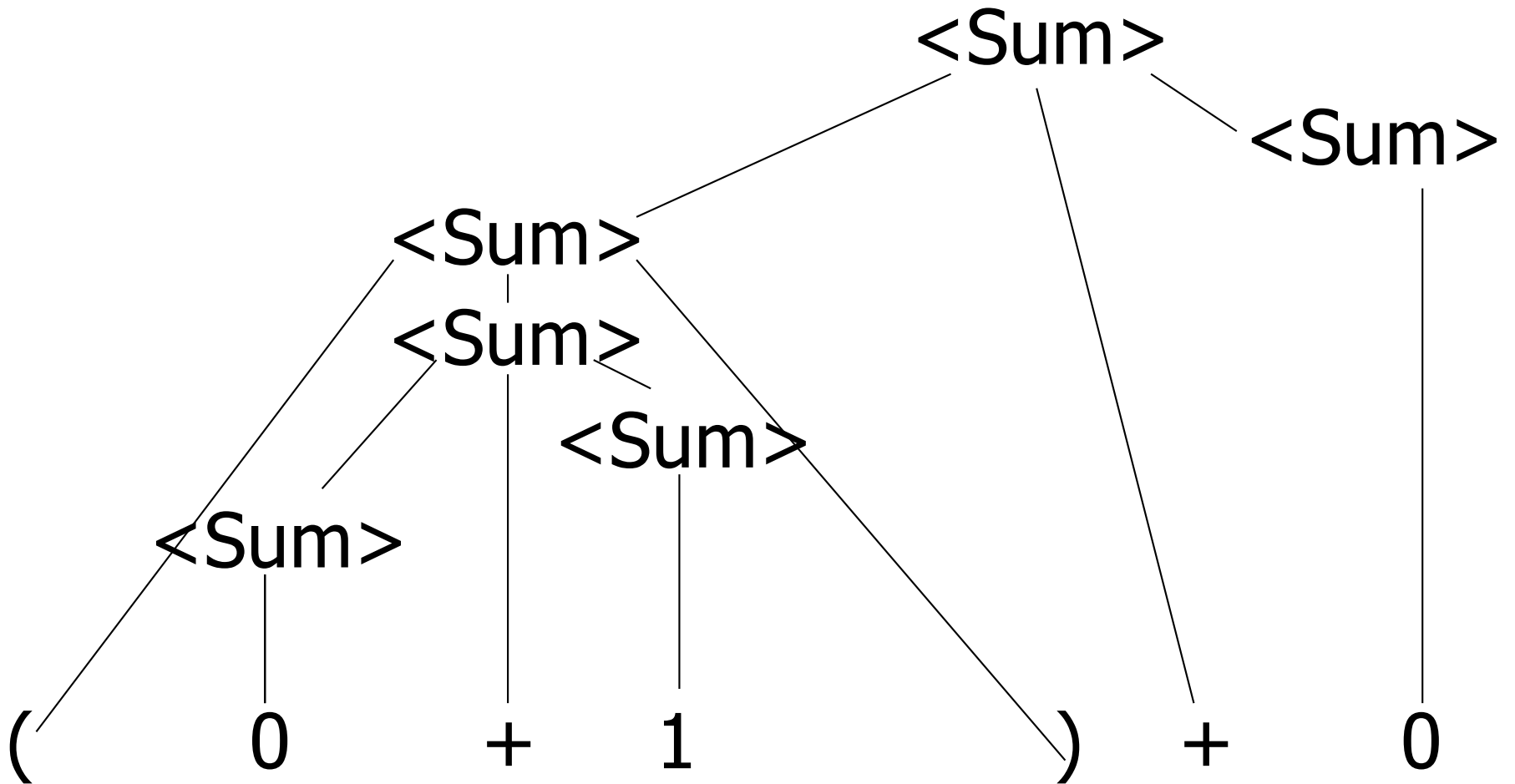


Example





Example





LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
 - This is the hardest part, we omit here
 - Rows labeled by states
 - For Action, columns labeled by terminals and “end-of-tokens” marker
 - (more generally strings of terminals of fixed length)
 - For Goto, columns labeled by non-terminals



Action and Goto Tables

- Given a state and the next input, Action table says either
 - **shift** and go to state n , or
 - **reduce** by production k (explained in a bit)
 - **accept** or **error**
- Given a state and a non-terminal, Goto table says
 - go to state m



LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals



LR(i) Parsing Algorithm

0. Insure token stream ends in special “end-of-tokens” symbol
1. Start in state 1 with an empty stack
2. Push **state**(1) onto stack
- 3. Look at next i tokens from token stream ($toks$) (don't remove yet)
4. If top symbol on stack is **state**(n), look up action in Action table at ($n, toks$)



LR(i) Parsing Algorithm

5. If action = **shift** m ,

- a) Remove the top token from token stream and push it onto the stack
- b) Push **state**(m) onto stack
- c) Go to step 3



LR(i) Parsing Algorithm

6. If action = **reduce** k where production k is
 $E ::= u$
- a) Remove $2 * \text{length}(u)$ symbols from stack (u and all the interleaved states)
 - b) If new top symbol on stack is **state**(m), look up new state p in $\text{Goto}(m, E)$
 - c) Push E onto the stack, then push **state**(p) onto the stack
 - d) Go to step 3



LR(i) Parsing Algorithm

7. If action = **accept**

- Stop parsing, return success

8. If action = **error**,

- Stop parsing, return failure



Adding Synthesized Attributes

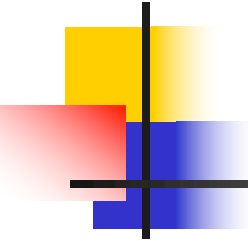
- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
 - gather the recorded attributes from each non-terminal popped from stack
 - Compute new attribute for non-terminal pushed onto stack



Shift-Reduce Conflicts

- **Problem:** can't decide whether the action for a state and input character should be **shift** or **reduce**
- Usually caused by lack of associativity or precedence information in grammar
 - Can be that the grammar needs the parser to look at more than the next token

1425 minutes



Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle)$
 $\mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\bullet 0 + 1 + 0$ shift
-> $0 \bullet + 1 + 0$ reduce
-> $\langle \text{Sum} \rangle \bullet + 1 + 0$ shift
-> $\langle \text{Sum} \rangle + \bullet 1 + 0$ shift
-> $\langle \text{Sum} \rangle + 1 \bullet + 0$ reduce
-> $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet + 0$



Example - cont

- **Problem:** shift or reduce?
- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative



Reduce - Reduce Conflicts

- **Problem:** can't decide between two different rules to reduce by
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors



Example

■ $S ::= A \mid aB$ $A ::= abc$ $B ::= bc$

● abc shift

a ● bc shift

ab ● c shift

abc ●

■ Problem: reduce by $B ::= bc$ then by S
 $::= aB$, or by $A ::= abc$ then $S ::= A$?