Programming Languages and Compilers (CS 421)

Elsa L Gunter

2112 SC, UIUC

https://courses.engr.illinois.edu/cs421/sp2023

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

BNF Deriviations

- Given rules

$$\mathbf{X} ::= y\,\mathbf{Z}\,w \text{ and } \mathbf{Z} ::= v$$

we may replace **Z** by *v* to say

$$\mathbf{X} => y\,\mathbf{Z}\,w => y\;v\;w$$

- Sequence of such replacements called *derivation*
- Derivation called *right-most* if always replace the right-most non-terminal

BNF Semantics

- The meaning of a BNF grammar is the set of all strings consisting only of terminals that can be derived from the Start symbol

BNF Derivations

- Start with the start symbol:

<Sum> =>

BNF Derivations

- Pick a non-terminal

<Sum> =>

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= (<Sum>)

<Sum> => <Sum> + <Sum >

=> (<Sum>) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum >

=> (<Sum>) + <Sum>

=> (<Sum> + <Sum>) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum >::= 1

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>

BNF Derivations

- Pick a rule and substitute:
  - <Sum >::= 0

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0

BNF Derivations

- Pick a non-terminal:

<Sum> => <Sum> + <Sum >

=> (<Sum>) + <Sum>

=> (<Sum> + <Sum>) + <Sum>

=> (<Sum> + 1) + <Sum>

=> (<Sum> + 1) + 0

BNF Derivations

- Pick a rule and substitute
  - <Sum> ::= 0

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) 0
=> (0 + 1) + 0

BNF Derivations

- (0 + 1) + 0 is generated by grammar

<Sum> => <Sum> + <Sum >
=> (<Sum>) + <Sum>
=> (<Sum> + <Sum>) + <Sum>
=> (<Sum> + 1) + <Sum>
=> (<Sum> + 1) + 0
=> (0 + 1) + 0

Extended BNF Grammars

- Alternatives: allow rules of from $X ::= y \mid z$
  - Abbreviates $X ::= y$, $X ::= z$
- Options: $X ::= y\,[v]\,z$
  - Abbreviates $X ::= y\,v\,z$, $X ::= y\,z$
- Repetition: $X ::= y\,\{v\}^*\,z$
  - Can be eliminated by adding new nonterminal V and rules $X ::= y\,z$, $X ::= y\,V\,z$, $V ::= v$, $V ::= v\,V$

Parse Trees

- Graphical representation of derivation
- Each node labeled with either non-terminal or terminal
- If node is labeled with a terminal, then it is a leaf (no sub-trees)
- If node is labeled with a non-terminal, then it has one branch for each character in the right-hand side of rule used to substitute for it

Example

- Consider grammar:

  <exp> ::= <factor>

  | <factor> + <factor>

  <factor> ::= <bin>

  | <bin> * <exp>

  <bin> ::= 0 | 1

- Problem: Build parse tree for 1 * 1 + 0 as an <exp>

Example cont.

- 1 * 1 + 0:   <exp>

<exp> is the start symbol for this parse tree

Example cont.

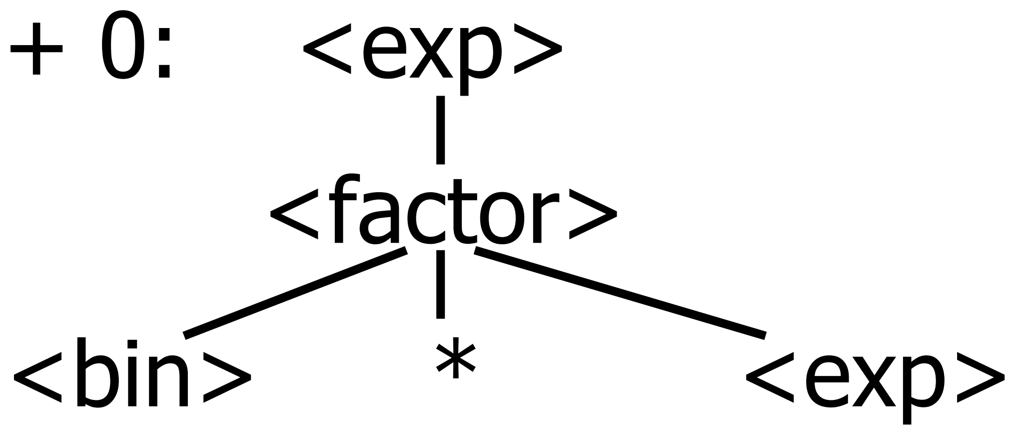
- 1 * 1 + 0: <exp>

  <factor>

Use rule: <exp> ::= <factor>

Example cont.

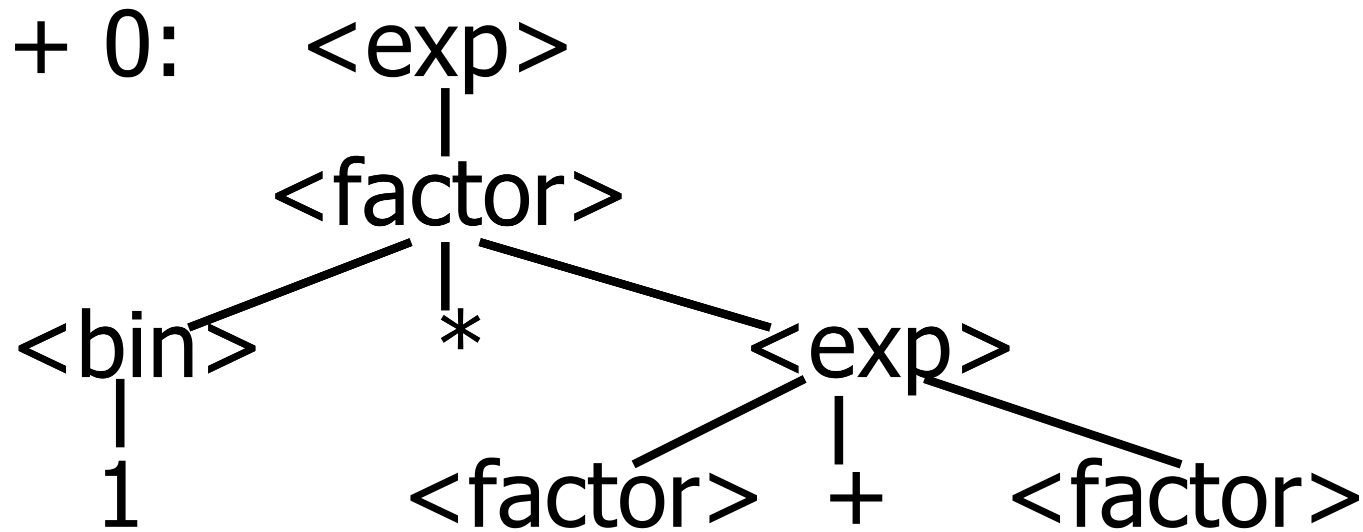
- 1 * 1 + 0:



Use rule: <factor> ::= <bin> * <exp>

Example cont.

- 1 * 1 + 0:

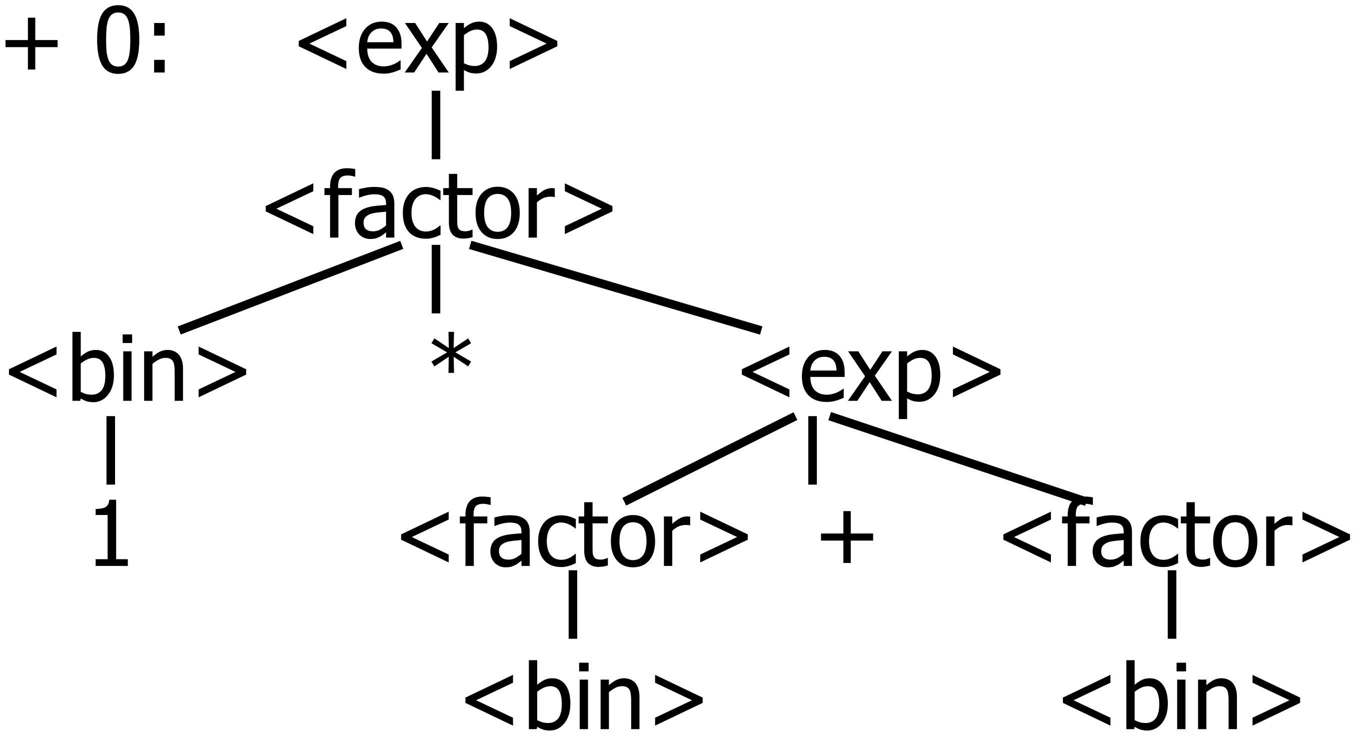


Use rules: <bin> ::= 1 and

<exp> ::= <factor> + <factor>

Example cont.

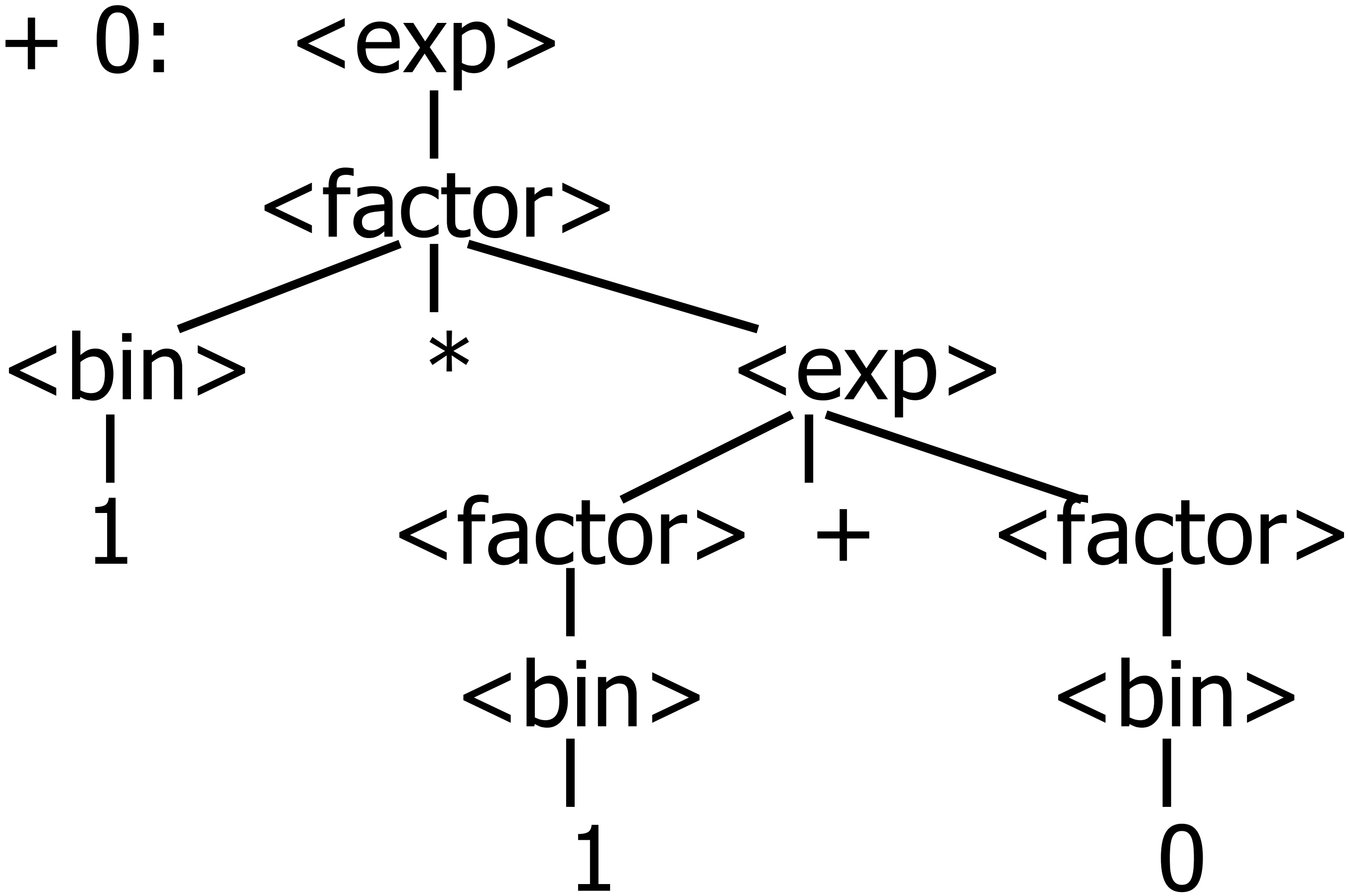
- 1 * 1 + 0:



Use rule: <factor> ::= <bin>

Example cont.

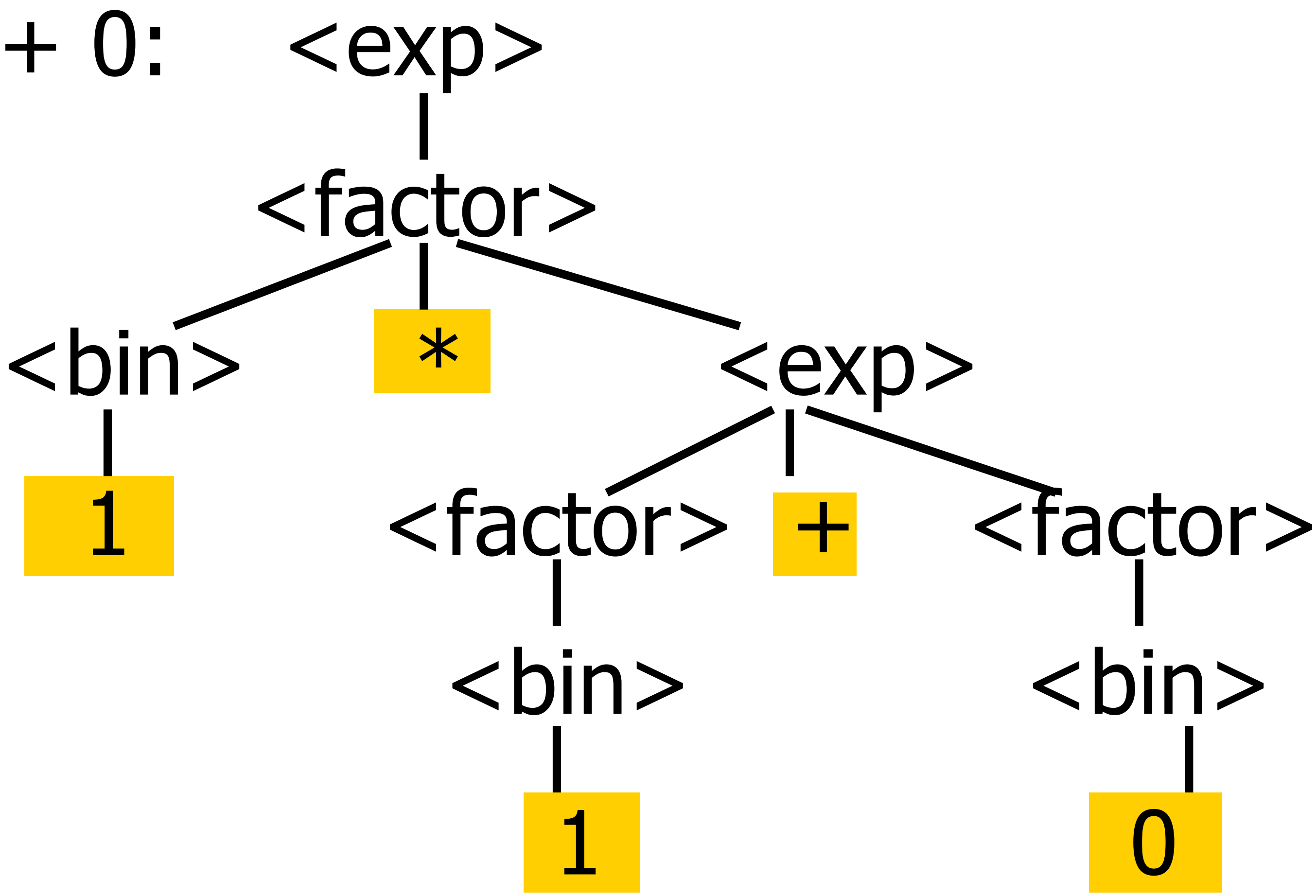
- 1 * 1 + 0:



Use rules: <bin> ::= 1 | 0

Example cont.

- 1 * 1 + 0:



Fringe of tree is string generated by grammar

Your Turn: 1 * 0 + 0 * 1

- <exp>
- / | \
- <fact>   +   <fact>
- / | \   / | \
- <b>   *   <e> <b>   *   <e>

Parse Tree Data Structures

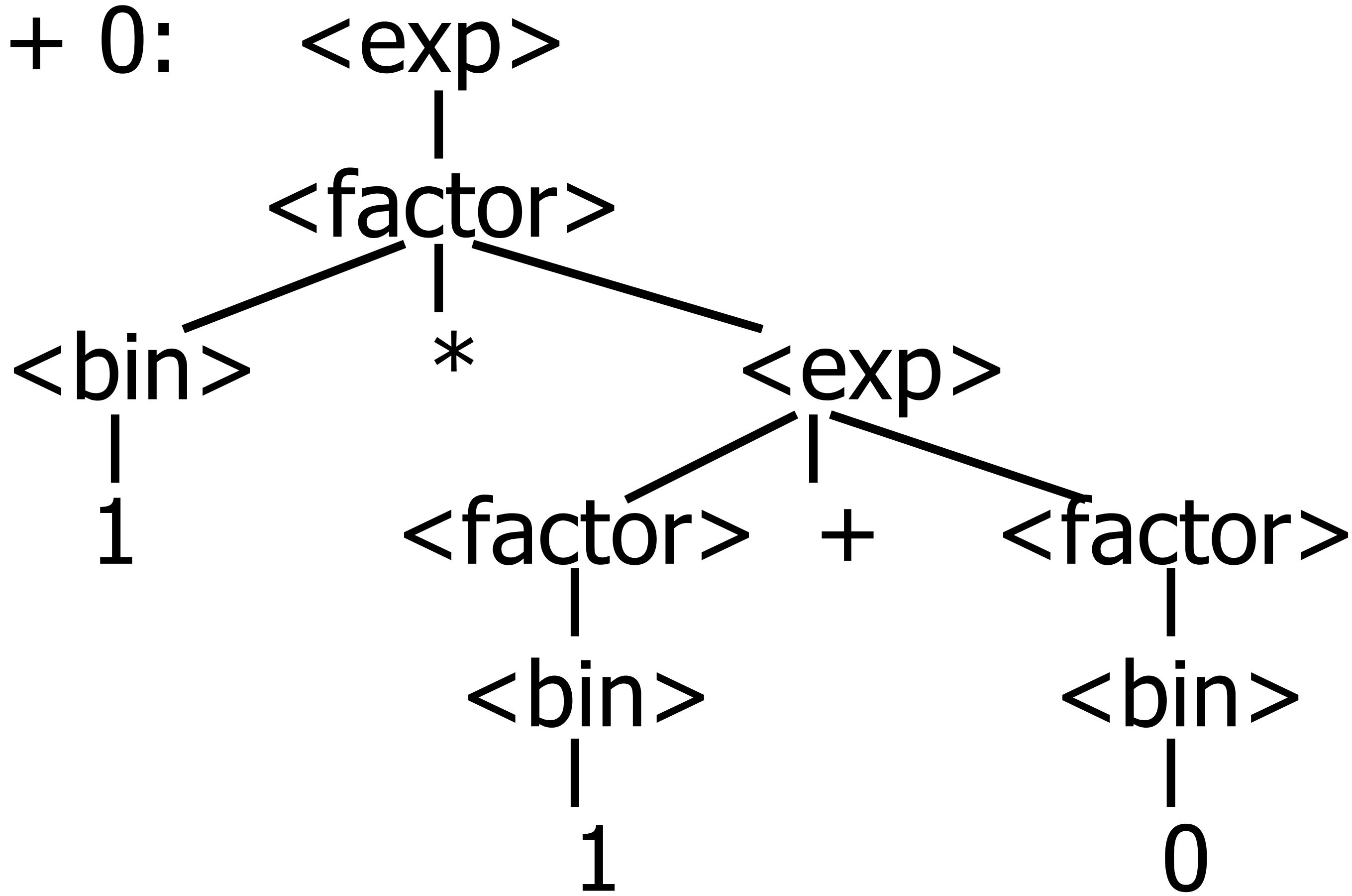
- Parse trees may be represented by OCaml datatypes
- One datatype for each nonterminal
- One constructor for each rule
- Defined as mutually recursive collection of datatype declarations

Example

- Recall grammar:
  <exp> ::= <factor> | <factor> + <factor>
  <factor> ::= <bin> | <bin> * <exp>
  <bin> ::= 0 | 1
- type exp = Factor2Exp of factor
  | Plus of factor * factor
  and factor = Bin2Factor of bin
  | Mult of bin * exp
  and bin = Zero | One

Example cont.

- 1 * 1 + 0:



Example cont.

- Can be represented as

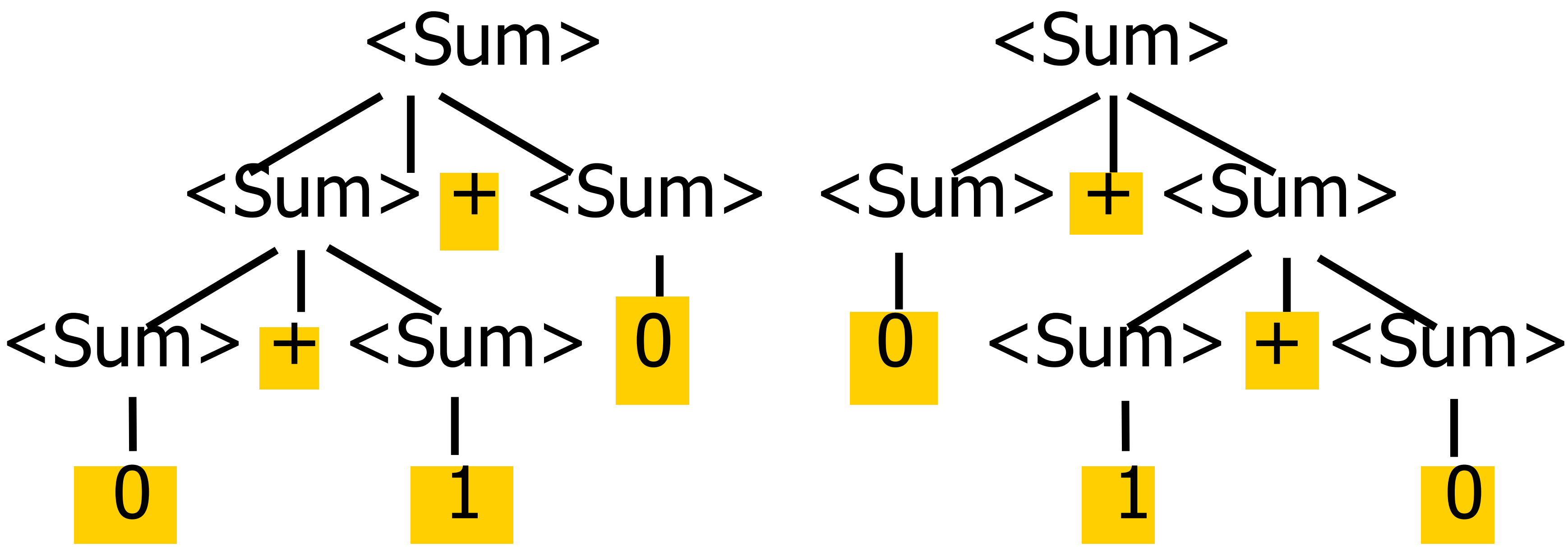
```
Factor2Exp
(Mult(One,
        Plus(Bin2Factor One,
               Bin2Factor Zero)))
```

Ambiguous Grammars and Languages

- A BNF grammar is *ambiguous* if its language contains strings for which there is more than one parse tree
- If all BNF's for a language are ambiguous then the language is *inherently ambiguous*

Example: Ambiguous Grammar

- 0 + 1 + 0



Example

- What is the result for:

$$3 + 4 * 5 + 6$$

Example

- What is the result for:

$$3 + 4 * 5 + 6$$

- Possible answers:
  - $41 = ((3 + 4) * 5) + 6$
  - $47 = 3 + (4 * (5 + 6))$
  - $29 = (3 + (4 * 5)) + 6 = 3 + ((4 * 5) + 6)$
  - $77 = (3 + 4) * (5 + 6)$

Example

- What is the value of:

$$7 - 5 - 2$$

Example

- What is the value of:

$$7 - 5 - 2$$

- Possible answers:
  - In Pascal, C++, SML assoc. left

    $7 - 5 - 2 = (7 - 5) - 2 = 0$
  - In APL, associate to right

    $7 - 5 - 2 = 7 - (5 - 2) = 4$

Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator associativity

- Not the only sources of ambiguity