

# Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

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## Polymorphic Example

- Assume additional constants and monadic and binary operators:
- $hd : \forall \alpha. \alpha \text{ list} \rightarrow \alpha$  (monadic)
- $tl : \forall \alpha. \alpha \text{ list} \rightarrow \alpha \text{ list}$  (monadic)
- $is\_empty : \forall \alpha. \alpha \text{ list} \rightarrow \text{bool}$  (monadic)
- $(::) : \forall \alpha. \alpha \rightarrow \alpha \text{ list} \rightarrow \alpha \text{ list}$  (binary)
- $[] : \forall \alpha. \alpha \text{ list}$  (constant)

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## Polymorphic Example

- Show:

$$\frac{?}{\{ \} \text{- let rec length = } \\ \text{fun l -> if is\_empty l then 0} \\ \text{else 1 + length (tl l)} \\ \text{in length (2 :: []) + length(true :: []) : int}}$$

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## Polymorphic Example: Let Rec Rule

- Show: (1) (2)
- $$\frac{\{ \text{length} : \alpha \text{ list} \rightarrow \text{int} \} \quad \{ \text{length} : \forall \alpha. \alpha \text{ list} \rightarrow \text{int} \} \\ \text{- fun l -> ...} \quad \text{- length (2 :: []) +} \\ \text{: } \alpha \text{ list} \rightarrow \text{int} \quad \text{length(true :: []) : int}}{\{ \} \text{- let rec length =} \\ \text{fun l -> if is\_empty l then 0} \\ \text{else 1 + length (tl l)} \\ \text{in length (2 :: []) + length(true :: []) : int}}$$

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## Polymorphic Example (1)

- Show:

$$\frac{?}{\{ \text{length} : \alpha \text{ list} \rightarrow \text{int} \} \text{-} \\ \text{fun l -> if is\_empty l then 0} \\ \text{else 1 + length (tl l)} \\ \text{: } \alpha \text{ list} \rightarrow \text{int}}$$

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## Polymorphic Example (1): Fun Rule

- Show: (3)
- $$\frac{\{ \text{length} : \alpha \text{ list} \rightarrow \text{int}, l : \alpha \text{ list} \} \text{-} \\ \text{if is\_empty l then 0} \\ \text{else 1 + length (tl l) : int}}{\{ \text{length} : \alpha \text{ list} \rightarrow \text{int} \} \text{-} \\ \text{fun l -> if is\_empty l then 0} \\ \text{else 1 + length (tl l)} \\ \text{: } \alpha \text{ list} \rightarrow \text{int}}$$

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### Polymorphic Example (3)

- Let  $\Gamma = \{\text{length} : \alpha \text{ list} \rightarrow \text{int}, l : \alpha \text{ list}\}$
- Show

$$\frac{?}{\Gamma \vdash \text{if is\_empty } l \text{ then } 0 \text{ else } 1 + \text{length } (\text{tl } l) : \text{int}}$$

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### Polymorphic Example (3):IfThenElse

- Let  $\Gamma = \{\text{length} : \alpha \text{ list} \rightarrow \text{int}, l : \alpha \text{ list}\}$
- Show

$$\frac{\frac{(4)}{\Gamma \vdash \text{is\_empty } l : \text{bool}} \quad \frac{(5)}{\Gamma \vdash 0 : \text{int}} \quad \frac{(6)}{\Gamma \vdash 1 + \text{length } (\text{tl } l) : \text{int}}}{\Gamma \vdash \text{if is\_empty } l \text{ then } 0 \text{ else } 1 + \text{length } (\text{tl } l) : \text{int}}$$

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### Polymorphic Example (4)

- Let  $\Gamma = \{\text{length} : \alpha \text{ list} \rightarrow \text{int}, l : \alpha \text{ list}\}$
- Show

$$\frac{?}{\Gamma \vdash \text{is\_empty } l : \text{bool}}$$

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### Polymorphic Example (4):MonOpAP

- Let  $\Gamma = \{\text{length} : \alpha \text{ list} \rightarrow \text{int}, l : \alpha \text{ list}\}$
  - Show
- By MonOpApp since  $\alpha \text{ list} \rightarrow \text{bool}$  is instance  $\{\alpha \rightarrow \alpha\}$  of the type of `is_empty`:  $\forall \alpha. \alpha \text{ list} \rightarrow \text{bool}$

$$\frac{\frac{?}{\Gamma \vdash l : \alpha \text{ list}}}{\Gamma \vdash \text{is\_empty } l : \text{bool}}$$

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### Polymorphic Example (4)

- Let  $\Gamma = \{\text{length} : \alpha \text{ list} \rightarrow \text{int}, l : \alpha \text{ list}\}$
  - Show
- By Var

$$\frac{\frac{\text{Var}}{\Gamma \vdash l : \alpha \text{ list}}}{\Gamma \vdash \text{is\_empty } l : \text{bool}}$$

- This finishes (4)

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### Polymorphic Example (5):Const

- Let  $\Gamma = \{\text{length} : \alpha \text{ list} \rightarrow \text{int}, l : \alpha \text{ list}\}$
  - Show
- By Const Rule

$$\frac{}{\Gamma \vdash 0 : \text{int}}$$

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### Polymorphic Example (6): BinOp

- Let  $\Gamma = \{\text{length} : \alpha \text{ list} \rightarrow \text{int}, l : \alpha \text{ list}\}$
- Show

$$\frac{\text{By Const} \quad \frac{\Gamma \vdash 1 : \text{int} \quad \text{App} \quad \frac{\text{By Variable} \quad \frac{\Gamma \vdash \text{length} : \alpha \text{ list} \rightarrow \text{int} \quad (7)}{\Gamma \vdash (\text{tl } l) : \alpha \text{ list}}}{\Gamma \vdash \text{length } (\text{tl } l) : \text{int}}}{\Gamma \vdash 1 + \text{length } (\text{tl } l) : \text{int}}}{\Gamma \vdash 1 : \text{int}}$$

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### Polymorphic Example (7): MonOp

- Let  $\Gamma = \{\text{length} : \alpha \text{ list} \rightarrow \text{int}, l : \alpha \text{ list}\}$
- Show

By MonOpApp since  $\alpha \text{ list} \rightarrow \alpha \text{ list}$  is instance  $\{\alpha \rightarrow \alpha\}$  of the type of tl:  $\forall \alpha. \alpha \text{ list} \rightarrow \alpha \text{ list}$

$$\frac{\Gamma \vdash l : \alpha \text{ list}}{\Gamma \vdash \text{tl } l : \text{bool}} \quad ?$$

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### Polymorphic Example: (2) by BinOp

- Let  $\Gamma' = \{\text{length} : \forall \alpha. \alpha \text{ list} \rightarrow \text{int}\}$
- Show:

$$\frac{\Gamma' \vdash \text{length } (2 :: []) : \text{int} \quad \Gamma' \vdash \text{length } (\text{true} :: []) : \text{int}}{\Gamma' \vdash \text{length } (2 :: []) + \text{length } (\text{true} :: []) : \text{int}} \quad (8) \quad (9)$$

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### Polymorphic Example: (8)AppRule

- Let  $\Gamma' = \{\text{length} : \forall \alpha. \alpha \text{ list} \rightarrow \text{int}\}$
- Show:

$$\frac{\Gamma' \vdash \text{length} : \text{int list} \rightarrow \text{int} \quad \Gamma' \vdash (2 :: []) : \text{int list}}{\Gamma' \vdash \text{length } (2 :: []) : \text{int}} \quad ? \quad ?$$

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### Polymorphic Example: (8)AppRule

- Let  $\Gamma' = \{\text{length} : \forall \alpha. \alpha \text{ list} \rightarrow \text{int}\}$
  - Show:
- By Var since  $\text{int list} \rightarrow \text{int}$  is instance  $\{\alpha \rightarrow \text{int}\}$  of  $\forall \alpha. \alpha \text{ list} \rightarrow \text{int}$  (by  $\{\alpha \rightarrow \text{int}\}$ )

$$\frac{\text{Var} \quad \Gamma' \vdash \text{length} : \text{int list} \rightarrow \text{int} \quad (10) \quad \Gamma' \vdash (2 :: []) : \text{int list}}{\Gamma' \vdash \text{length } (2 :: []) : \text{int}}$$

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### Polymorphic Example: (10)BinOpRule

- Let  $\Gamma' = \{\text{length} : \forall \alpha. \alpha \text{ list} \rightarrow \text{int}\}$
- Show:
- By BinOp since  $::$  has type  $\forall \alpha. \alpha \rightarrow \alpha \text{ list} \rightarrow \alpha \text{ list}$  (by  $\{\alpha \rightarrow \text{int}\}$ )

$$\frac{\text{Const} \quad \Gamma' \vdash 2 : \text{int} \quad \Gamma' \vdash [] : \text{int list}}{\Gamma' \vdash (2 :: []) : \text{int list}} \quad ?$$

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### Polymorphic Example: (10)BinOpRule

- Let  $\Gamma' = \{\text{length} : \forall \alpha. \alpha \text{ list} \rightarrow \text{int}\}$
- Show:
- By Const since  $\text{int list}$  is instance of  $\forall \alpha. \alpha \text{ list}$  (by  $\{\alpha \rightarrow \text{int}\}$ )

$$\frac{\frac{\text{Const}}{\Gamma' \vdash 2 : \text{int}} \quad \frac{\text{Const}}{\Gamma' \vdash [] : \text{int list}}}{\Gamma' \vdash (2 :: []) : \text{int list}}$$

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### Polymorphic Example: (9)AppRule

- Let  $\Gamma' = \{\text{length} : \forall \alpha. \alpha \text{ list} \rightarrow \text{int}\}$
- Show:

$$\frac{\frac{?}{\Gamma' \vdash \text{length} : \text{bool list} \rightarrow \text{int}} \quad \frac{?}{\Gamma' \vdash (\text{true} :: []) : \text{bool list}}}{\Gamma' \vdash \text{length} (\text{true} :: []) : \text{int}}$$

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### Polymorphic Example: (9)AppRule

- Let  $\Gamma' = \{\text{length} : \forall \alpha. \alpha \text{ list} \rightarrow \text{int}\}$
- Show:
- Var since  $\text{bool list} \rightarrow \text{int}$  is instance of  $\forall \alpha. \alpha \text{ list} \rightarrow \text{int}$  (by  $\{\alpha \rightarrow \text{bool}\}$ )

$$\frac{\frac{\Gamma' \vdash \text{length} : \text{bool list} \rightarrow \text{int}}{\Gamma' \vdash \text{length} (\text{true} :: []) : \text{int}} \quad \frac{(11)}{\Gamma' \vdash (\text{true} :: []) : \text{bool list}}}{\Gamma' \vdash \text{length} (\text{true} :: []) : \text{int}}$$

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### Polymorphic Example: (11)BinOpRule

- Let  $\Gamma' = \{\text{length} : \forall \alpha. \alpha \text{ list} \rightarrow \text{int}\}$
- Show:
- By BinOp since  $::$  has type  $\forall \alpha. \alpha \rightarrow \alpha \text{ list} \rightarrow \alpha \text{ list}$  (by  $\{\alpha \rightarrow \text{bool}\}$ )

$$\frac{\frac{\text{Const}}{\Gamma' \vdash \text{true} : \text{bool}} \quad \frac{?}{\Gamma' \vdash [] : \text{bool list}}}{\Gamma' \vdash (\text{true} :: []) : \text{bool list}}$$

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### Polymorphic Example: (10)BinOpRule

- Let  $\Gamma' = \{\text{length} : \forall \alpha. \alpha \text{ list} \rightarrow \text{int}\}$
- Show:
- By Const since  $\text{bool list}$  is instance of  $\forall \alpha. \alpha \text{ list}$  (by  $\{\alpha \rightarrow \text{bool}\}$ )

$$\frac{\frac{\text{Const}}{\Gamma' \vdash \text{true} : \text{bool}} \quad \frac{\text{Const}}{\Gamma' \vdash [] : \text{bool list}}}{\Gamma' \vdash (\text{true} :: []) : \text{bool list}}$$

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