

# Programming Languages and Compilers (CS 421)

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<https://courses.engr.illinois.edu/cs421/sp2023>

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

# Variants - Syntax (slightly simplified)

- $\text{type } name = C_1 [\text{of } ty_1] \mid \dots \mid C_n [\text{of } ty_n]$
- Introduce a type called *name*
- $(\text{fun } x \rightarrow C_i x) : ty_1 \rightarrow name$
- $C_i$  is called a *constructor*; if the optional type argument is omitted, it is called a *constant*
- Constructors are the basis of almost all pattern matching



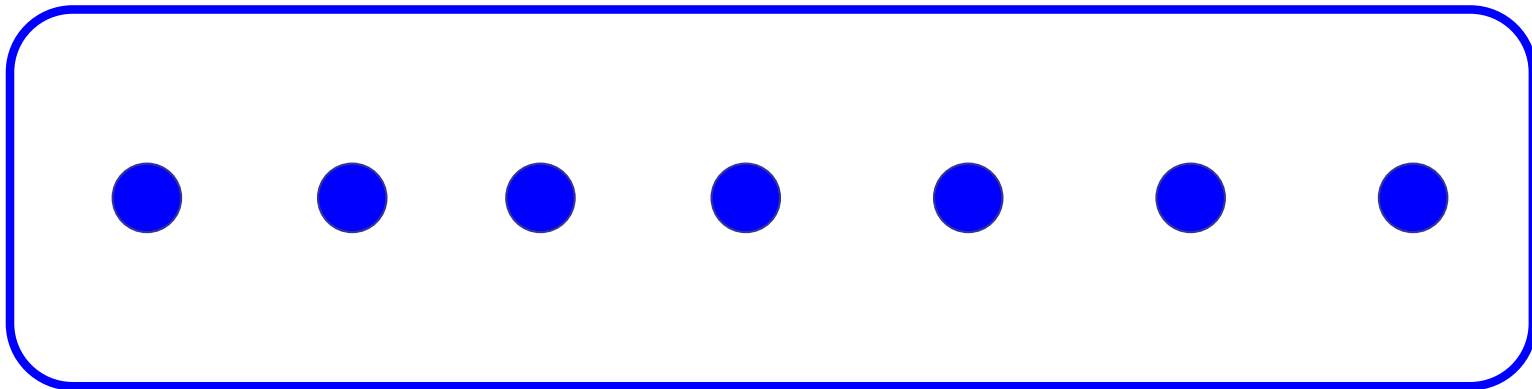
# Data type in Ocaml: lists

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- Frequently used lists in recursive program
- Matched over two structural cases
  - `[]` - the empty list
  - `(x :: xs)` a non-empty list
- Covers all possible lists
- `type 'a list = [] | (::) of 'a * 'a list`
  - Not quite legitimate declaration because of special syntax

# Enumeration Types as Variants

An enumeration type is a collection of distinct values



In C and Ocaml they have an order structure;  
order by order of input



# Enumeration Types as Variants

---

```
# type weekday = Monday | Tuesday | Wednesday  
| Thursday | Friday | Saturday | Sunday;;
```

```
type weekday =
```

```
Monday
```

```
| Tuesday
```

```
| Wednesday
```

```
| Thursday
```

```
| Friday
```

```
| Saturday
```

```
| Sunday
```



# Functions over Enumerations

---

```
# let day_after day = match day with
```

```
    Monday -> Tuesday
```

```
| Tuesday -> Wednesday
```

```
| Wednesday -> Thursday
```

```
| Thursday -> Friday
```

```
| Friday -> Saturday
```

```
| Saturday -> Sunday
```

```
| Sunday -> Monday;;
```

```
val day_after : weekday -> weekday = <fun>
```



# Functions over Enumerations

---

```
# let rec days_later n day =  
  match n with 0 -> day  
  | _ -> if n > 0  
         then day_after (days_later (n - 1) day)  
         else days_later (n + 7) day;;  
val days_later : int -> weekday -> weekday  
= <fun>
```



# Functions over Enumerations

---

```
# days_later 2 Tuesday;;
```

```
- : weekday = Thursday
```

```
# days_later (-1) Wednesday;;
```

```
- : weekday = Tuesday
```

```
# days_later (-4) Monday;;
```

```
- : weekday = Thursday
```





# Problem:

---

```
# type weekday = Monday | Tuesday |  
Wednesday  
| Thursday | Friday | Saturday | Sunday;;  
■ Write function is_weekend : weekday -> bool  
let is_weekend day =
```



## Problem:

---

```
# type weekday = Monday | Tuesday |  
Wednesday  
| Thursday | Friday | Saturday | Sunday;;  
■ Write function is_weekend : weekday -> bool  
let is_weekend day =  
  match day with Saturday -> true  
  | Sunday -> true  
  | _ -> false
```



# Example Enumeration Types

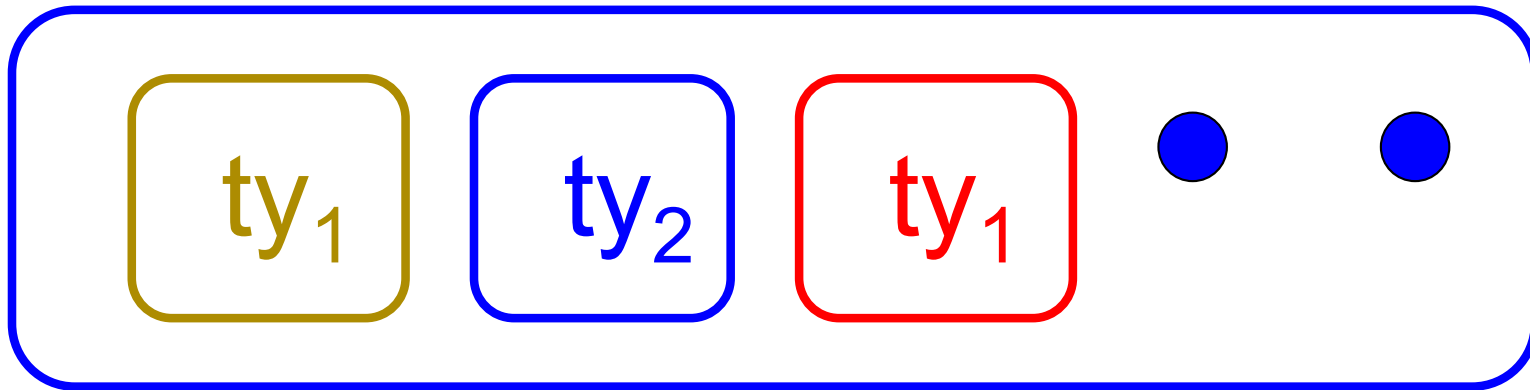
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```
# type bin_op = IntPlusOp | IntMinusOp  
              | EqOp | CommaOp | ConsOp
```

```
# type mon_op = HdOp | TlOp | FstOp  
              | SndOp
```

# Disjoint Union Types

- Disjoint union of types, with some possibly occurring more than once



- We can also add in some new singleton elements



# Disjoint Union Types

---

```
# type id = DriversLicense of int
  | SocialSecurity of int | Name of string;;
type id = DriversLicense of int | SocialSecurity
  of int | Name of string
# let check_id id = match id with
  DriversLicense num ->
    not (List.mem num [13570; 99999])
  | SocialSecurity num -> num < 900000000
  | Name str -> not (str = "John Doe");;
val check_id : id -> bool = <fun>
```



# Problem

---

- Create a type to represent the currencies for US, UK, Europe and Japan



# Problem

---

- Create a type to represent the currencies for US, UK, Europe and Japan

type currency =

  Dollar of int

| Pound of int

| Euro of int

| Yen of int



# Example Disjoint Union Type

---

```
# type const =  
  BoolConst of bool  
| IntConst of int  
| FloatConst of float  
| StringConst of string  
| NilConst  
| UnitConst
```





# Example Disjoint Union Type

---

```
# type const = BoolConst of bool  
| IntConst of int | FloatConst of float  
| StringConst of string | NilConst  
| UnitConst
```

- How to represent 7 as a const?
- Answer: `IntConst 7`



# Polymorphism in Variants

---

- The type `'a option` gives us something to represent non-existence or failure

```
# type 'a option = Some of 'a | None;;
```

```
type 'a option = Some of 'a | None
```

- Used to encode partial functions
- Often can replace the raising of an exception



# Functions producing option

---

```
# let rec first p list =  
  match list with [ ] -> None  
  | (x::xs) -> if p x then Some x else first p xs;;  
val first : ('a -> bool) -> 'a list -> 'a option = <fun>  
# first (fun x -> x > 3) [1;3;4;2;5];;  
- : int option = Some 4  
# first (fun x -> x > 5) [1;3;4;2;5];;  
- : int option = None
```



# Functions over option

---

```
# let result_ok r =  
  match r with None -> false  
  | Some _ -> true;;  
val result_ok : 'a option -> bool = <fun>  
# result_ok (first (fun x -> x > 3) [1;3;4;2;5]);;  
- : bool = true  
# result_ok (first (fun x -> x > 5) [1;3;4;2;5]);;  
- : bool = false
```



# Problem

---

- Write a `hd` and `tl` on lists that doesn't raise an exception and works at all types of lists.



# Problem

---

- Write a `hd` and `tl` on lists that doesn't raise an exception and works at all types of lists.
- `let hd list =`
  - `match list with [] -> None`
  - `| (x::xs) -> Some x`
- `let tl list =`
  - `match list with [] -> None`
  - `| (x::xs) -> Some xs`



# Mapping over Variants

---

```
# let optionMap f opt =  
  match opt with None -> None  
  | Some x -> Some (f x);;  
val optionMap : ('a -> 'b) -> 'a option -> 'b  
  option = <fun>  
# optionMap  
  (fun x -> x - 2)  
  (first (fun x -> x > 3) [1;3;4;2;5]);;  
- : int option = Some 2
```



# Folding over Variants

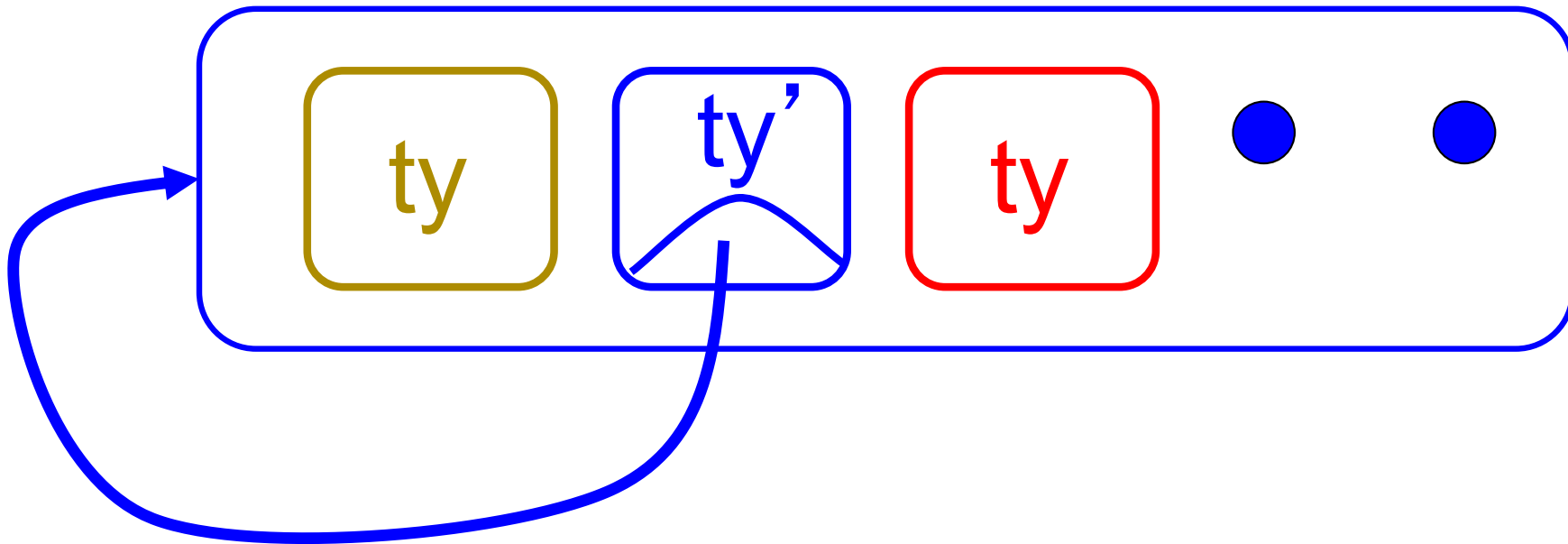
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```
# let optionFold someFun noneVal opt =  
  match opt with None -> noneVal  
  | Some x -> someFun x;;  
val optionFold : ('a -> 'b) -> 'b -> 'a option ->  
  'b = <fun>  
# let optionMap f opt =  
  optionFold (fun x -> Some (f x)) None opt;;  
val optionMap : ('a -> 'b) -> 'a option -> 'b  
  option = <fun>
```



# Recursive Types

- The type being defined may be a component of itself





# Recursive Data Types

---

```
# type int_Bin_Tree =  
  Leaf of int | Node of (int_Bin_Tree *  
    int_Bin_Tree);;
```

```
type int_Bin_Tree = Leaf of int | Node of  
  (int_Bin_Tree * int_Bin_Tree)
```



# Recursive Data Type Values

---

```
# let bin_tree =
```

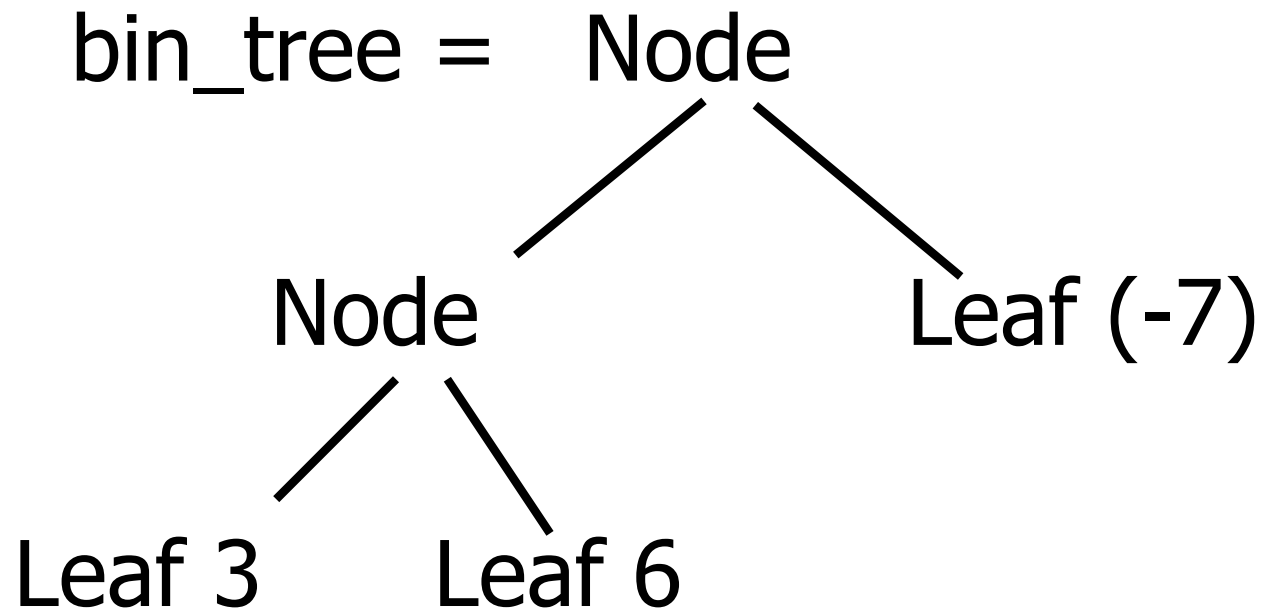
```
Node(Node(Leaf 3, Leaf 6),Leaf (-7));;
```

```
val bin_tree : int_Bin_Tree = Node (Node  
  (Leaf 3, Leaf 6), Leaf (-7))
```



# Recursive Data Type Values

---





# Recursive Functions

---

```
# let rec first_leaf_value tree =  
  match tree with (Leaf n) -> n  
  | Node (left_tree, right_tree) ->  
    first_leaf_value left_tree;;  
val first_leaf_value : int_Bin_Tree -> int =  
  <fun>  
# let left = first_leaf_value bin_tree;;  
val left : int = 3
```



# Recursive Data Types

---

```
# type exp =  
  VarExp of string  
| ConstExp of const  
| MonOpAppExp of mon_op * exp  
| BinOpAppExp of bin_op * exp * exp  
| IfExp of exp * exp * exp  
| AppExp of exp * exp  
| FunExp of string * exp
```



# Recursive Data Types

---

```
# type bin_op = IntPlusOp | IntMinusOp
              | EqOp | CommaOp | ConsOp | ...
# type const = BoolConst of bool | IntConst of int |
...
# type exp = VarExp of string | ConstExp of const
            | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent 6 as an exp?



# Recursive Data Types

---

```
# type bin_op = IntPlusOp | IntMinusOp  
              | EqOp | CommaOp | ConsOp | ...
```

```
# type const = BoolConst of bool | IntConst of int |  
...
```

```
# type exp = VarExp of string | ConstExp of const  
            | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent 6 as an exp?
- Answer: ConstExp (IntConst 6)





# Recursive Data Types

---

```
# type bin_op = IntPlusOp | IntMinusOp  
              | EqOp | CommaOp | ConsOp | ...
```

```
# type const = BoolConst of bool | IntConst of int |  
...
```

```
# type exp = VarExp of string | ConstExp of const  
           | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent (6, 3) as an exp?



# Recursive Data Types

---

```
# type bin_op = IntPlusOp | IntMinusOp  
              | EqOp | CommaOp | ConsOp | ...
```

```
# type const = BoolConst of bool | IntConst of int |  
...
```

```
# type exp = VarExp of string | ConstExp of const  
            | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent (6, 3) as an exp?
- BinOpAppExp (CommaOp, ConstExp (IntConst 6),  
ConstExp (IntConst 3))



# Recursive Data Types

---

```
# type bin_op = IntPlusOp | IntMinusOp  
              | EqOp | CommaOp | ConsOp | ...
```

```
# type const = BoolConst of bool | IntConst of int |  
...
```

```
# type exp = VarExp of string | ConstExp of const  
           | BinOpAppExp of bin_op * exp * exp | ...
```

- How to represent [(6, 3)] as an exp?
- BinOpAppExp (ConsOp, BinOpAppExp (CommaOp, ConstExp (IntConst 6), ConstExp (IntConst 3)), ConstExp NilConst))));;



# Problem

---

```
type int_Bin_Tree = Leaf of int  
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write `sum_tree : int_Bin_Tree -> int`
- Adds all ints in tree

```
let rec sum_tree t =
```



# Problem

---

```
type int_Bin_Tree = Leaf of int
```

```
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write `sum_tree : int_Bin_Tree -> int`
- Adds all ints in tree

```
let rec sum_tree t =
```

```
  match t with Leaf n -> n
```

```
  | Node(t1,t2) -> sum_tree t1 + sum_tree t2
```



# Recursion over Recursive Data Types

---

- ```
# type exp = VarExp of string | ConstExp of const  
| BinOpAppExp of bin_op * exp * exp  
| FunExp of string * exp | AppExp of exp * exp
```
- How to count the number of variables in an exp?



# Recursion over Recursive Data Types

---

```
# type exp = VarExp of string | ConstExp of const  
| BinOpAppExp of bin_op * exp * exp  
| FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =  
  match exp with VarExp x ->  
    | ConstExp c ->  
    | BinOpAppExp (b, e1, e2) ->  
    | FunExp (x,e) ->  
    | AppExp (e1, e2) ->
```



# Recursion over Recursive Data Types

---

```
# type exp = VarExp of string | ConstExp of const  
| BinOpAppExp of bin_op * exp * exp  
| FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =  
  match exp with VarExp x -> 1  
  | ConstExp c -> 0  
  | BinOpAppExp (b, e1, e2) -> varCnt e1 + varCnt e2  
  | FunExp (x,e) -> 1 + varCnt e  
  | AppExp (e1, e2) -> varCnt e1 + varCnt e2
```





# Mapping over Recursive Types

---

```
# let rec ibtreeMap f tree =  
  match tree with (Leaf n) -> Leaf (f n)  
  | Node (left_tree, right_tree) ->  
    Node (ibtreeMap f left_tree,  
          ibtreeMap f right_tree);;  
val ibtreeMap : (int -> int) -> int_Bin_Tree ->  
  int_Bin_Tree = <fun>
```



# Mapping over Recursive Types

---

```
# ibtreeMap ((+) 2) bin_tree;;
```

```
- : int_Bin_Tree = Node (Node (Leaf 5, Leaf  
8), Leaf (-5))
```



# Folding over Recursive Types

---

```
# let rec ibtreeFoldRight leafFun nodeFun tree =  
  match tree with Leaf n -> leafFun n  
  | Node (left_tree, right_tree) ->  
    nodeFun  
      (ibtreeFoldRight leafFun nodeFun left_tree)  
      (ibtreeFoldRight leafFun nodeFun right_tree);;  
val ibtreeFoldRight : (int -> 'a) -> ('a -> 'a -> 'a) ->  
  int_Bin_Tree -> 'a = <fun>
```



# Folding over Recursive Types

---

```
# let tree_sum =  
    ibtreeFoldRight (fun x -> x) (+);;  
val tree_sum : int_Bin_Tree -> int = <fun>  
# tree_sum bin_tree;;  
- : int = 2
```