Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

CPS for Higher Order Functions

- In CPS, every procedure / function takes a continuation to receive its result
- Procedures passed as arguments take continuations
- Procedures returned as results take continuations
- CPS version of higher-order functions must expect input procedures to take continuations

Example: all

```
#let rec all (p, l) = match l with [] -> true
    | (x :: xs) -> let b = p x in
        if b then all (p, xs) else false
val all : ('a -> bool) -> 'a list -> bool = <fun>
```

- What is the CPS version of this?

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#let rec allk (pk, l) k =
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#let rec allk (pk, l) k = match l with [] ->  k true
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Example: all

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#let rec allk (pk, l) k = match l with [] ->  k true
 | (x :: xs) -> pk x
          (fun b -> if b then                else
    )
```


Example: all

```
#let rec all (p, l) = match l with [] -> true
    | (x :: xs) -> let b = p x in
        if b then all (p, xs) else false
val all : ('a -> bool) -> 'a list -> bool = <fun>
```

- What is the CPS version of this?

```
#let rec allk (pk, l) k = match l with [] ->  k true
 | (x :: xs) -> pk x
          (fun b -> if b then allk (pk, xs) k else k
false)
val allk : ('a -> (bool -> 'b) -> 'b) * 'a list ->
(bool -> 'b) -> 'b = <fun>
```


Terminology: Review

- A function is in Direct Style when it returns its result back to the caller.
- A function is in Continuation Passing Style when it, and every function call in it, passes its result to another function.
- A Tail Call occurs when a function returns the result of another function call without any more computations (eg tail recursion)
- Instead of returning the result to the caller, we pass it forward to another function giving the computation after the call.

CPS Transformation

- Step 1: Add continuation argument to any function definition:
  - let f arg = e ⇒ let f arg k = e
  - Idea: Every function takes an extra parameter saying where the result goes
- Step 2: A simple expression in tail position should be passed to a continuation instead of returned:
  - return a ⇒ k a
  - Assuming a is a constant or variable.
  - “Simple” = “No available function calls.”

CPS Transformation

- Step 3: Pass the current continuation to every function call in tail position
  - return f arg ⇒ f arg k
  - The function "isn't going to return," so we need to tell it where to put the result.

CPS Transformation

- Step 4: Each function call not in tail position needs to be converted to take a new continuation (containing the old continuation as appropriate)
  - return op (f arg) ⇒ f arg (fun r -> k(op r))
  - op represents a primitive operation
  - return g(f arg) ⇒ f arg (fun r-> g r k)

Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                  (* rule 1 *)
```


Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                  (* rule 1 *)

           k false (* rule 2 *)


         k true (* rule 2 *)
```


Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                  (* rule 1 *)

      k false (* rule 2 *)


    k true (* rule 2 *)
      memk (y, xs) k (* rule 3 *)
```


Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                  (* rule 1 *)

           k false (* rule 2 *)

eqk (x, y)
 (fun b ->   b (* rule 4 *)
      k true (* rule 2 *)
        memk (y, xs) (* rule 3 *)
```

Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
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let rec memk (y,lst) k =
                        (* rule 1 *)

           k false (* rule 2 *)

 eqk (x, y)
  (fun b ->if b (* rule 4 *)
then k true (* rule 2 *)
   else memk (y, xs) (* rule 3 *)
```


Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
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After:

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let rec memk (y,lst) k =
                        (* rule 1 *)
match lst with
| [ ] -> k false (* rule 2 *)
| x :: xs ->
 eqk (x, y)
  (fun b ->if b (* rule 4 *)
then k true (* rule 2 *)
   else memk (y, xs) k (* rule 3 *)
```


Example

Before:

```
let rec mem (y,lst) =
match lst with
  [ ] -> false
| x :: xs ->
  if (x = y)
   then true
   else mem(y,xs);;
```

After:

```
let rec memk (y,lst) k =
                        (* rule 1 *)
match lst with
| [ ] -> k false (* rule 2 *)
| x :: xs ->
 eqk (x, y)
  (fun b ->if b (* rule 4 *)
then k true (* rule 2 *)
   else memk (y, xs) k (* rule 3 *)
```


Example

Before:

```
let rec add_list lst =
match lst with
  [ ] -> 0
| 0 :: xs -> add_list xs
| x :: xs -> (+) x
   (add_list xs);;
```

After:

```
let rec add_listk lst k =
                        (* rule 1 *)
match lst with
| [ ] -> k 0 (* rule 2 *)
| 0 :: xs -> add_listk xs k
                          (* rule 3 *)
| x :: xs -> add_listk xs
           (fun r -> k ((+) x r));;
                       (* rule 4 *)
```


Extra Material

Other Uses for Continuations

- CPS designed to preserve order of evaluation
- Continuations used to express order of evaluation
- Can be used to change order of evaluation
- Implements:
  - Exceptions and exception handling
  - Co-routines
  - (pseudo, aka green) threads

Exceptions - Example

```
# exception Zero;;
exception Zero
# let rec list_mult_aux list =
    match list with [ ] -> 1
    | x :: xs ->
     if x = 0 then raise Zero
               else x * list_mult_aux xs;;
val list_mult_aux : int list -> int = <fun>
```


Exceptions - Example

```
# let list_mult list =
   try list_mult_aux list with Zero -> 0;;
val list_mult : int list -> int = <fun>
# list_mult [3;4;2];;
- : int = 24
# list_mult [7;4;0];;
- : int = 0
# list_mult_aux [7;4;0];;
Exception: Zero.
```


Exceptions

- When an exception is raised
  - The current computation is aborted
  - Control is “thrown” back up the call stack until a matching handler is found
  - All the intermediate calls waiting for a return values are thrown away

Implementing Exceptions

```
# let multkp (m, n) k =
   let r = m * n in
    (print_string "product result: ";
     print_int r; print_string "\n";
     k r);;
val multkp : int ( int -> (int -> 'a) -> 'a =
  <fun>
```


Implementing Exceptions

```
# let rec list_multk_aux list k kexcp =
    match list with [ ] -> k 1
    | x :: xs -> if x = 0 then  kexcp  0
      else list_multk_aux xs
          (fun r -> multkp (x, r) k) kexcp;;
val list_multk_aux : int list -> (int -> 'a) -> (int -> 'a)
   -> 'a = <fun>
# let rec list_multk list k = list_multk_aux list  k  k;;
val list_multk : int list -> (int -> 'a) -> 'a = <fun>
```


Implementing Exceptions

```
# list_multk [3;4;2] report;;
product result: 2
product result: 8
product result: 24
24
- : unit = ()
# list_multk [7;4;0] report;;
0
- : unit = ()
```

End of Extra Material

Data type in Ocaml: lists

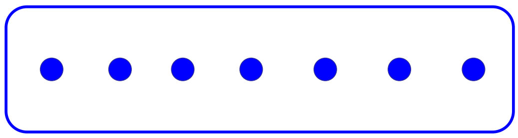
- Frequently used lists in recursive program
- Matched over two structural cases
  - [ ] - the empty list
  - (x :: xs) a non-empty list
- Covers all possible lists
- type 'a list = [ ] | (::) of 'a * 'a list
  - Not quite legitimate declaration because of special syntax

Variants - Syntax (slightly simplified)

- type *name* = C_1 [of ty_1] | . . . | C_n [of ty_n]
- Introduce a type called *name*
- (fun x -> C_i x) : ty_1 -> *name*
- C_i is called a *constructor*; if the optional type argument is omitted, it is called a *constant*
- Constructors are the basis of almost all pattern matching

Enumeration Types as Variants

An enumeration type is a collection of distinct values



In C and Ocaml they have an order structure; order by order of input

Enumeration Types as Variants

```
# type weekday = Monday | Tuesday | Wednesday
  | Thursday | Friday | Saturday | Sunday;;
type weekday =
    Monday
  | Tuesday
  | Wednesday
  | Thursday
  | Friday
  | Saturday
  | Sunday
```


Functions over Enumerations

```
# let day_after day = match day with
     Monday -> Tuesday
   | Tuesday -> Wednesday
   | Wednesday -> Thursday
   | Thursday -> Friday
   | Friday -> Saturday
   | Saturday -> Sunday
   | Sunday -> Monday;;
val day_after : weekday -> weekday = <fun>
```

Functions over Enumerations

```
# let rec days_later n day =
   match n with 0 -> day
   | _ -> if n > 0
        then day_after (days_later (n - 1) day)
       else days_later (n + 7) day;;
val days_later : int -> weekday -> weekday
  = <fun>
```


Functions over Enumerations

```
# days_later 2 Tuesday;;
- : weekday = Thursday
# days_later (-1) Wednesday;;
- : weekday = Tuesday
# days_later (-4) Monday;;
- : weekday = Thursday
```


Problem:

```
# type weekday = Monday | Tuesday |
  Wednesday
  | Thursday | Friday | Saturday | Sunday;;
```

- Write function is_weekend : weekday -> bool

```
let is_weekend day =
```


Problem:

```
# type weekday = Monday | Tuesday |
  Wednesday
  | Thursday | Friday | Saturday | Sunday;;
```

- Write function is_weekend : weekday -> bool

```
let is_weekend day =
    match day with Saturday -> true
     | Sunday -> true
     | _ -> false
```

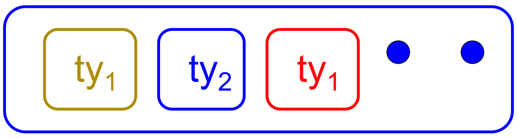

Example Enumeration Types

```
# type bin_op = IntPlusOp |  IntMinusOp
        |  EqOp | CommaOp | ConsOp

# type mon_op = HdOp | TlOp | FstOp
        | SndOp
```


Disjoint Union Types

- Disjoint union of types, with some possibly occurring more than once



- We can also add in some new singleton elements

Disjoint Union Types

```
# type id = DriversLicense of int
  | SocialSecurity of int | Name of string;;
type id = DriversLicense of int | SocialSecurity
  of int | Name of string
# let check_id id = match id with
     DriversLicense num ->
      not (List.mem num [13570; 99999])
    | SocialSecurity num -> num < 900000000
    | Name str -> not (str = "John Doe");;
 val check_id : id -> bool = <fun>
```


Problem

- Create a type to represent the currencies for US, UK, Europe and Japan

Problem

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```
type currency =
   Dollar of int
 | Pound of int
 | Euro of int
 | Yen of int
```


Example Disjoint Union Type

```
# type const =
     BoolConst of bool
   | IntConst of int
   | FloatConst of float
   | StringConst of string
   | NilConst
   | UnitConst
```


Example Disjoint Union Type

```
# type const = BoolConst of bool
    | IntConst of int | FloatConst of float
    | StringConst of string  | NilConst
    | UnitConst
```

- How to represent 7 as a const?
- Answer: IntConst 7

Polymorphism in Variants

- The type 'a option is gives us something to represent non-existence or failure

```
# type 'a option = Some of 'a | None;;
type 'a option = Some of 'a | None
```

- Used to encode partial functions
- Often can replace the raising of an exception

Functions producing option

```
# let rec first p list =
    match list with [ ] -> None
    | (x::xs) -> if p x then Some x else first p xs;;
val first : ('a -> bool) -> 'a list -> 'a option = <fun>
# first (fun x -> x > 3) [1;3;4;2;5];;
- : int option = Some 4
# first (fun x -> x > 5) [1;3;4;2;5];;
- : int option = None
```


Functions over option

```
# let result_ok r =
   match r with None -> false
   | Some _ -> true;;
val result_ok : 'a option -> bool = <fun>
# result_ok (first (fun x -> x > 3) [1;3;4;2;5]);;
- : bool = true
# result_ok (first (fun x -> x > 5) [1;3;4;2;5]);;
- : bool = false
```


Problem

- Write a hd and tl on lists that doesn't raise an exception and works at all types of lists.

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- Write a hd and tl on lists that doesn't raise an exception and works at all types of lists.

- let hd list =
  ```
      match list with [] -> None
       | (x::xs) -> Some x
  ```
- let tl list =
  ```
      match list with [] -> None
       | (x::xs) -> Some xs
  ```


Mapping over Variants

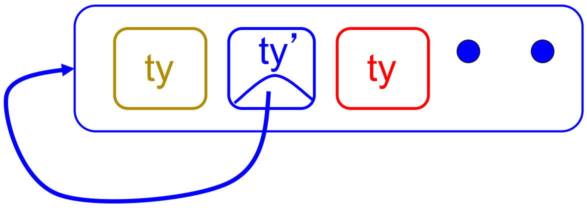
```
# let optionMap f opt =
    match opt with None -> None
    | Some x -> Some (f x);;
val optionMap : ('a -> 'b) -> 'a option -> 'b
   option = <fun>
# optionMap
  (fun x -> x - 2)
  (first (fun x -> x > 3) [1;3;4;2;5]);;
- : int option = Some 2
```


Folding over Variants

```
# let optionFold someFun noneVal opt =
    match opt with None -> noneVal
    | Some x -> someFun x;;
val optionFold : ('a -> 'b) -> 'b -> 'a option ->
   'b = <fun>
# let optionMap f opt =
    optionFold (fun x -> Some (f x)) None opt;;
val optionMap : ('a -> 'b) -> 'a option -> 'b
   option = <fun>
```

Recursive Types

- The type being defined may be a component of itself



Recursive Data Types

```
# type int_Bin_Tree =
 Leaf of int | Node of (int_Bin_Tree *
   int_Bin_Tree);;

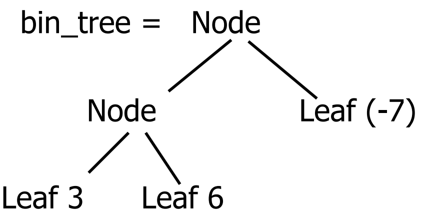
type int_Bin_Tree = Leaf of int | Node of
  (int_Bin_Tree * int_Bin_Tree)
```


Recursive Data Type Values

```
# let bin_tree =
 Node(Node(Leaf 3, Leaf 6),Leaf (-7));;

val bin_tree : int_Bin_Tree = Node (Node
  (Leaf 3, Leaf 6), Leaf (-7))
```


Recursive Data Type Values



Recursive Functions

```
# let rec first_leaf_value tree =
   match tree with (Leaf n) -> n
   | Node (left_tree, right_tree) ->
    first_leaf_value left_tree;;
val first_leaf_value : int_Bin_Tree -> int =
  <fun>
# let left = first_leaf_value bin_tree;;
val left : int = 3
```