Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Functions

```
# let plus_two n = n + 2;;
val plus_two : int -> int = <fun>
# plus_two 17;;
- : int = 19
# let plus_two = fun n -> n + 2;;
val plus_two : int -> int = <fun>
# plus_two 14;;
- : int = 16
```

First definition syntactic sugar for second

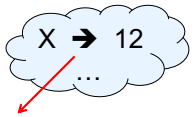
Using a nameless function

```
# (fun x -> x * 3) 5;;   (* An application *)
- : int = 15
# ((fun y -> y +. 2.0), (fun z -> z * 3));;
  (* As data *)
- : (float -> float) * (int -> int) = (<fun>,
  <fun>)
```

Note: in fun v -> exp(v), scope of variable is only the body exp(v)

Values fixed at declaration time

```
# let x = 12;;
val x : int = 12
# let plus_x y = y + x;;
val plus_x : int -> int = <fun>
# plus_x 3;;
```



What is the result?

Values fixed at declaration time

```
# let x = 12;;
val x : int = 12
# let plus_x y = y + x;;
val plus_x : int -> int = <fun>
# plus_x 3;;
- : int = 15
```

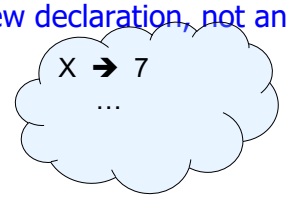

Values fixed at declaration time

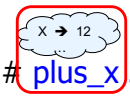
```
# let x = 7;;   (* New declaration, not an
  update *)
val x : int = 7

# plus_x 3;;
```

What is the result this time?

Values fixed at declaration time

```
# let x = 7;;   (* New declaration, not an
  update *)
val x : int = 7
# plus_x 3;;
```





What is the result this time?

Values fixed at declaration time

```
# let x = 7;;   (* New declaration, not an
  update *)
val x : int = 7

# plus_x 3;;
- : int = 15
```


Question

- Observation: Functions are first-class values in this language
- Question: What value does the environment record for a function variable?
- Answer: a closure

Save the Environment!

- A *closure* is a pair of an environment and an association of a formal parameter (the input variables)* with an expression (the function body), written:

  $f \rightarrow < (v1,\ldots,vn) \rightarrow exp, \rho_f >$

- Where ρ_f is the environment in effect when f is defined (if f is a simple function)

* Will come back to the "formal parameter"

Closure for plus_x

- When plus_x was defined, had environment:

  $\rho_{plus\_x} = \{\ldots, x \rightarrow 12, \ldots\}$

- Recall: let plus_x y = y + x

  is really let plus_x = fun y -> y + x

- Closure for fun y -> y + x:

  $< y \rightarrow y + x, \rho_{plus\_x} >$

- Environment just after plus_x defined:

  $\{plus\_x \rightarrow < y \rightarrow y + x, \rho_{plus\_x} >\} + \rho_{plus\_x}$

Now it's your turn

You should be able to complete ACT1

Functions with more than one argument

```
# let add_three x y z = x + y + z;;
val add_three : int -> int -> int -> int = <fun>
# let t = add_three 6 3 2;;
val t : int = 11
# let add_three =
   fun x -> (fun y -> (fun z -> x + y + z));;
val add_three : int -> int -> int -> int = <fun>
```

Again, first syntactic sugar for second

Functions with more than one argument

```
# let add_three x y z = x + y + z;;
val add_three : int -> int -> int -> int = <fun>
```

- What is the value of add_three?
- Let ρ_{add_three} be the environment before the declaration
- Remember:

```
let add_three =
 fun x -> (fun y -> (fun z -> x + y + z));;
```

Value: <x ->fun y -> (fun z -> x + y + z), ρ_{add_three} >

Partial application of functions

```
let add_three x y z = x + y + z;;
```

```
# let h = add_three 5 4;;
val h : int -> int = <fun>
# h 3;;
- : int = 12
# h 7;;
- : int = 16
```


Partial application of functions

```
let add_three x y z = x + y + z;;
```

```
# let h = add_three 5 4;;
val h : int -> int = <fun>
# h 3;;
- : int = 12
# h 7;;
- : int = 16
```

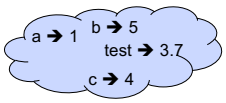
- Partial application also called *sectioning*

Functions as arguments

```
# let thrice f x = f (f (f x));;
val thrice : ('a -> 'a) -> 'a -> 'a = <fun>
# let g = thrice plus_two;;
val g : int -> int = <fun>
# g 4;;
- : int = 10
# thrice (fun s -> "Hi! " ^ s) "Good-bye!";;
- : string = "Hi! Hi! Hi! Good-bye!"
```

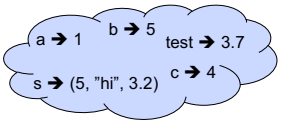

Tuples as Values

// ρ_7 = {c → 4, test → 3.7,
a → 1, b → 5}



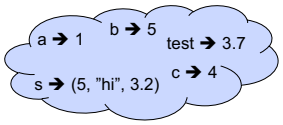

```
# let s = (5,"hi",3.2);;
val s : int * string * float = (5, "hi", 3.2)
```

// ρ_8 = {s → (5, "hi", 3.2),
c → 4, test → 3.7,
a → 1, b → 5}

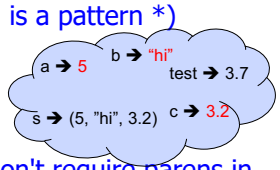


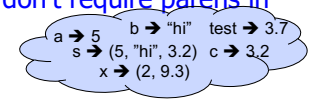
Pattern Matching with Tuples

/ ρ_8 = {s → (5, "hi", 3.2),
c → 4, test → 3.7,
a → 1, b → 5}




```
# let (a,b,c) = s;;  (* (a,b,c) is a pattern *)
val a : int = 5
val b : string = "hi"
val c : float = 3.2
# let x = 2, 9.3;; (* tuples don't require parens in
  Ocaml *)
val x : int * float = (2, 9.3)
```





Nested Tuples

```
#  (*Tuples can be nested *)
let d = ((1,4,62),("bye",15),73.95);;
val d : (int * int * int) * (string * int) * float =
  ((1, 4, 62), ("bye", 15), 73.95)
#  (*Patterns can be nested *)
let (p,(st,_),_) = d;; (* _ matches all, binds nothing
  *)
val p : int * int * int = (1, 4, 62)
val st : string = "bye"
```


Functions on tuples

```
# let plus_pair (n,m) = n + m;;
val plus_pair : int * int -> int = <fun>
# plus_pair (3,4);;
- : int = 7
# let double x = (x,x);;
val double : 'a -> 'a * 'a = <fun>
# double 3;;
- : int * int = (3, 3)
# double "hi";;
- : string * string = ("hi", "hi")
```


Match Expressions

```
# let triple_to_pair triple =
  match triple
  with (0, x, y) -> (x, y)
  | (x, 0, y) -> (x, y)
  | (x, y, _) -> (x, y);;
val triple_to_pair : int * int * int -> int * int =
  <fun>
```

- Each clause: pattern on left, expression on right
- Each x, y has scope of only its clause
- Use first matching clause

Closure for plus_pair

- Assume ρ_{plus_pair} was the environment just before plus_pair defined
- Closure for plus_pair:

  $<(n,m) \rightarrow n + m, \rho_{plus\_pair}>$
- Environment just after plus_pair defined:

  $\{plus\_pair \rightarrow <(n,m) \rightarrow n + m, \rho_{plus\_pair}>\}$

  $+ \rho_{plus\_pair}$

Save the Environment!

- A *closure* is a pair of an environment and an association of a pattern (e.g. (v1,…,vn) giving the input variables) with an expression (the function body), written:

  $< (v1,\ldots,vn) \rightarrow exp, \rho >$
- Where ρ is the environment in effect when the function is defined (for a simple function)