Ex. evaluation by substitution

Let $\mathcal{F}$ denote "`rec fac (fun x -> if x=0 then 1 else x * fac(x-1))`".

```
let fac = 𝓕 in fac 2 ⇓ 2
   𝓕 ⇓ fun x -> if x=0 then 1 else x * 𝓕(x-1)
   (fun x -> if x=0 then 1 else x * 𝓕(x-1)) 2 ⇓ 2
      fun x -> if x=0 then 1 else x*𝓕(x-1) ⇓ fun x -> if x=0 then 1 else x*𝓕(x-1)
      2 ⇓ 2
      if 2=0 then 1 else 2 * 𝓕(2-1) ⇓ 2
         2=0 ⇓ False
            2 ⇓ 2
            0 ⇓ 0
         2 * 𝓕(2-1) ⇓ 2
            2 ⇓ 2
            𝓕(2-1) ⇓ 1
               𝓕 ⇓ fun x -> if x=0 then 1 else x * 𝓕(x-1)
               2-1 ⇓ 1
                  2 ⇓ 2
                  1 ⇓ 1
               if 1=0 then 1 else 1 * 𝓕(1-1) ⇓ 1
                  1=0 ⇓ false
                     1 ⇓ 1
```

```
   0 ⇓ 0
1 * ℱ(1-1) ⇓ 1
   1 ⇓ 1
   ℱ(1-1) ⇓ 1
      ℱ ⇓ fun x -> if x=0 then 1 else x * ℱ(x-1)
      (1-1) ⇓ 0
         1 ⇓ 1
         1 ⇓ 1
      if 0=0 then 1 else 0 * ℱ(0-1) ⇓ 1
         0=0 ⇓ true
            0 ⇓ 0
            0 ⇓ 0
         1 ⇓ 1
```

Evaluation in environment model

Let $\mathcal{F}$ denote "`rec fac (fun x -> if x=0 then 1 else x * fac(x-1))`".

```
let fac = 
```
$\mathcal{F}$ `in fac 2`, $\emptyset \Downarrow 2$

$\mathcal{F}, \emptyset \Downarrow < \mathcal{F}, \emptyset >$

`fac 2`, $\rho_1 \Downarrow 2$ $\quad (\rho_1 = \{fac \mapsto < \mathcal{F}, \emptyset >\})$

`fac`, $\rho_1 \Downarrow < \mathcal{F}, \emptyset >$

`2`, $\rho_1 \Downarrow 2$

`if x=0 then 1 else x * fac(x-1)`, $\rho_2 \Downarrow 2$ $\quad (\rho_2 = \rho_1[\mathsf{x} \mapsto 2])$

`x=0`, $\rho_2 \Downarrow$ False

`x`, $\rho_2 \Downarrow 2$

`0`, $\rho_2 \Downarrow 0$

`x * fac(x-1)`, $\rho_2 \Downarrow 2$

`x`, $\rho_2 \Downarrow 2$

`fac(x-1)`, $\rho_2 \Downarrow 1$

`fac`, $\rho_2 \Downarrow < \mathcal{F}, \emptyset >$

`x-1`, $\rho_2 \Downarrow 1$

`x`, $\rho_2 \Downarrow 2$

`1`, $\rho_2 \Downarrow 1$

`if x=0 then 1 else x * fac(x-1)`, $\rho_3 \Downarrow 1$ $\quad (\rho_3 = \rho_1[\mathsf{x} \mapsto 1])$

`x=0`, $\rho_3 \Downarrow$ false

`x`, $\rho_3 \Downarrow 1$

0, $\rho_3 \Downarrow 0$

`x * fac(x-1)`, $\rho_3 \Downarrow 1$

`x`, $\rho_3 \Downarrow 1$

`fac(x-1)`, $\rho_3 \Downarrow 1$

`fac`, $\rho_3 \Downarrow < \mathcal{F}, \emptyset >$

`x-1`, $\rho_3 \Downarrow 0$

`x`, $\rho_3 \Downarrow 1$

`1`, $\rho_3 \Downarrow 1$

`if x=0 then 1 else x * fac(x-1)`, $\rho_4 \Downarrow 1$ $\quad (\rho_4 = \rho_1[\mathsf{x} \mapsto 0])$

`x=0`, $\rho_4 \Downarrow$ true

`x`, $\rho_4 \Downarrow 0$

`0`, $\rho_4 \Downarrow 0$

`1`, $\rho_4 \Downarrow 1$