Polymorphism and references

- **Prove the following judgment:**

```
∅ ⊢ let i = fun x -> x
      in let fp = ref i in (fp := not;  (!fp) 5) : int
```

Note: This type judgment is provable in our system, but it should not be, because it has a run-time type error. OCaml uses a slightly different type system, which fixes this problem. The point of this example is to show the flaw in our system.

Proof is on next slide. We actually give the proof for an explicitly-typed version of the term, because it makes things clearer (imo).

$\emptyset \vdash$ let $i : \alpha \to \alpha$ = fun $x : \alpha \to x$

in let $fp : (\beta \to \beta)$ ref = ref $i[\beta \to \beta]$

in $(fp[(\text{bool} \to \text{bool})\text{ref}] := \text{not};\ (!\ fp[(\text{int} \to \text{int})\text{ref}])\ 5) : \text{int}$

$\emptyset \vdash$ fun $x : \alpha \to x$: $\alpha \to \alpha$

$\{x : \alpha\} \vdash x : \alpha$

$\{i : \forall\alpha.\alpha \to \alpha\} \vdash$ let $fp \ldots$: int

$\{i : \forall\alpha.\alpha \to \alpha\} \vdash$ ref $i[\beta \to \beta]$: $(\beta \to \beta)$ ref

$\{i : \forall\alpha.\alpha \to \alpha\} \vdash$ ref : $(\beta \to \beta) \to (\beta \to \beta)$ ref $\quad (((\beta \to \beta) \to \beta \to \beta)\text{ref} \le \forall\alpha.\alpha \to \alpha\text{ref})$

$\{i : \forall\alpha.\alpha \to \alpha\} \vdash i[\beta \to \beta] : \beta \to \beta \quad (\beta \to \beta \le \forall\alpha.\alpha \to \alpha)$

$\Gamma_1 \quad \boxed{\{i : \forall\alpha.\alpha \to \alpha,\ fp : \forall\beta.(\beta \to \beta)\text{ref}\}} \vdash (fp[(\text{bool} \to \text{bool})\text{ref}] \ldots : \text{int}$

$\Gamma_1 \vdash fp[(\text{bool} \to \text{bool})\text{ref}] := \text{not} : \text{unit}$

$\Gamma_1 \vdash fp[(\text{bool} \to \text{bool})\text{ref}] : (\text{bool} \to \text{bool})\text{ref} \quad ((\text{bool} \to \text{bool})\text{ref} \le \forall\beta.(\beta \to \beta)\text{ref})$

$\Gamma_1 \vdash \text{not} : \text{bool} \to \text{bool}$

$\Gamma_1 \vdash (!\ fp)\ 5 : \text{int}$

$\Gamma_1 \vdash !\ fp[(\text{int} \to \text{int})\text{ref}] : \text{int} \to \text{int} \quad (\le \forall\alpha.\alpha\text{ref} \to \alpha)$

$\Gamma_1 \vdash\ ! : (\text{int} \to \text{int})\text{ref} \to (\text{int} \to \text{int})$

$\Gamma_1 \vdash fp[(\text{int} \to \text{int})\text{ref}] : (\text{int} \to \text{int})\text{ref} \quad (\le \forall\beta.(\beta \to \beta)\text{ref})$