Note Title 4/15/2010

An incorrect use of the type system T_{ocaml} from lecture 22

In this "proof," we use the following abbreviations:

Γ_0 = type environment with all built-in constants, plus $incr : int \to int$

$\Gamma_1 = \Gamma_0 [f : \forall \alpha .\ \alpha \to int * \alpha]$

$\Gamma_2 = \Gamma_0 [x : \alpha]$

$\Gamma_3 = \Gamma_2 [g : \forall \alpha, \beta .\ (\alpha \to \beta) \to \beta]$

$\Gamma_4 = \Gamma_2 [y : \alpha \to \beta]$

The judgment is given on slide 13 of lecture 22.

The proof is broken into several parts.

$$\dfrac{\overbrace{\Gamma_0 \vdash \mathit{fun}\ x \to (\ldots) : \alpha \to int * \alpha}^{A} \qquad \overbrace{\Gamma_1 \vdash f\ true : int * bool}^{B}}{\Gamma_0 \vdash \mathit{let}\ f = \mathit{fun}\ x \to (\mathit{let}\ g = \mathit{fun}\ y \to y\ x\ \mathit{in}\ g\ incr,\ x)\ \mathit{in}\ f\ true : int * bool}\ \text{Let}$$

(B) (We do B first because its simpler)

$$
\dfrac{
\dfrac{\Gamma_1(f) = \forall\alpha.\,\alpha \to \mathrm{int} * \alpha,\quad \mathrm{bool} \to \mathrm{int} * \mathrm{bool} \le \forall\alpha.\,\alpha \to \mathrm{int} * \alpha}{\Gamma_1 \vdash f : \mathrm{bool} \to \mathrm{int} * \mathrm{bool}}\,\text{Var}
\qquad
\dfrac{\Gamma_1(\mathrm{true}) = \mathrm{bool}}{\Gamma_1 \vdash \mathrm{true} : \mathrm{bool}}\,\text{Var}
}{\Gamma_1 \vdash f\ \mathrm{true} : \mathrm{int} * \mathrm{bool}}\,\text{App}
$$

A

$$\dfrac{}{\Gamma_4 : y : \alpha \to \beta} \qquad \dfrac{}{\Gamma_4 : x : \alpha} \qquad \dfrac{(int \to int) \to int \le \forall \alpha, \beta.\ (\alpha \to \beta) \to \beta}{\Gamma_3 \vdash g : (int \to int) \to int} \qquad \dfrac{}{\Gamma_3 \vdash incr : int \to int}\ \text{Va}$$

$$\Gamma_4 \vdash y\ x : \beta$$

$$\Gamma_2 \vdash fun\ y \to y\ x : (\alpha \to \beta) \to \beta \qquad \Gamma_3 \vdash g\ incr : int \qquad \dfrac{}{\Gamma_2 \vdash x : \alpha}\ \text{Var}$$

$$\Gamma_2 \vdash let\ g = fun\ y \to y\ x\ in\ g\ incr : int$$

$$\Gamma_2 \vdash (let\ g = \ldots,\ x) : int * \alpha$$

Abs

$$\Gamma_0 \vdash fun\ x \to (let\ g = \cdots,\ x) : \alpha \to int * \alpha$$

All steps are correct except:
Even though $fun\ y \to y\ x : (\alpha \to \beta) \to \beta$,
we cannot generalize both α and β
to get Γ_3. Without generalizing α —
giving g type scheme $\forall \beta. (\alpha \to \beta) \to \beta$ — the
proof cannot go through.