Lecture 23 addendum - proof trees from slides 6 and 11

Note: Abbreviations are used in these proof trees to keep the trees from getting too wide to print.
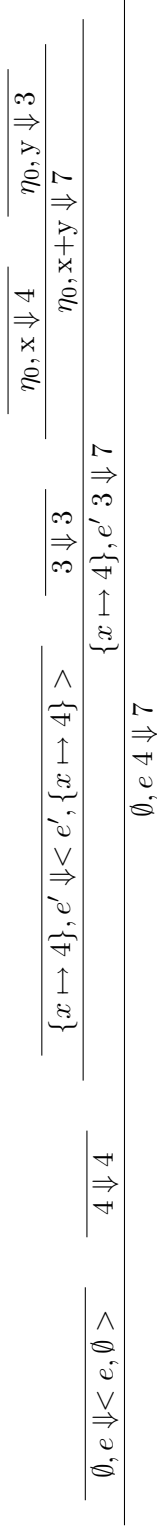
$$
\cfrac{
  \cfrac{}{\emptyset, e \Downarrow <e, \emptyset>} \quad \cfrac{}{4 \Downarrow 4} \quad
  \cfrac{
    \cfrac{}{\{x \mapsto 4\}, e' \Downarrow <e', \{x \mapsto 4\}>} \quad \cfrac{}{3 \Downarrow 3} \quad
    \cfrac{\cfrac{}{\eta_0, \text{x} \Downarrow 4} \quad \cfrac{}{\eta_0, \text{y} \Downarrow 3}}{\eta_0, \text{x+y} \Downarrow 7}
  }{\{x \mapsto 4\}, e'\ 3 \Downarrow 7}
}{\emptyset, e\ 4 \Downarrow 7}
$$

Figure 1: Proof from slide 6; e = fun x -> (fun y -> x+y)3, e' = fun y -> x+y, and $\eta_0 = \{x \mapsto 4, y \mapsto 3\}$

A:

$$\frac{\dfrac{\sigma_0, \{x \mapsto \ell\} \vdash \text{fun y -> } ee' \Downarrow <\text{fun y -> } ee', \{x \mapsto \ell\}>, \sigma_0 \qquad \dfrac{\sigma_0, \{x \mapsto \ell\} \vdash x \Downarrow \ell, \sigma_0}{}}{\sigma_0, \{x \mapsto \ell\} \vdash (\text{fun y -> } ee')x \Downarrow 1, \{\ell \mapsto 1\}}}{\sigma_0, \eta_1 \vdash e \Downarrow <e, \eta_1>, \sigma_0}$$

A

$$\frac{\sigma_1, \eta_1[z \mapsto ()] \vdash x \Downarrow \ell, \sigma_1}{\sigma_1, \eta_1[z \mapsto ()] \vdash !x \Downarrow 1, \sigma_1}$$

B

$$\frac{\dfrac{\sigma_0, \eta_1 \vdash y \Downarrow \ell, \sigma_0}{} \qquad \dfrac{\sigma_0, \eta_1 \vdash !y+1 \Downarrow 1, \sigma_0 \qquad \sigma_1 = \sigma_0[\ell \mapsto 1]}{\sigma_0, \eta_1 \vdash e' \Downarrow (), \sigma_1}}{\sigma_0, \eta_1 \vdash ee' \Downarrow 1, \sigma_1}$$

B:

$$\frac{\dfrac{\sigma_0, \eta_1 \vdash y \Downarrow \ell, \sigma_0 \qquad \sigma_0(\ell) = 0}{\sigma_0, \eta_1 \vdash !y \Downarrow 0, \sigma_0} \qquad \dfrac{}{\sigma_0, \eta_1 \vdash 1 \Downarrow 1, \sigma_0}}{\sigma_0, \eta_1 \vdash !y+1 \Downarrow 1, \sigma_0}$$

Figure 2: Proof from slide 11; $e = \text{fun z -> } !x$, $e' = y := !y+1$, $\sigma_0 = \{\ell \mapsto 0\}$, $\sigma_1 = \{\ell \mapsto 1\}$, and $\eta_1 = \{x \mapsto \ell, y \mapsto \ell\}$