OS_{subst}

(Const) $$\frac{}{k \Downarrow k}$$

(Abstr) $$\frac{}{fun\ x \to e \Downarrow fun\ x \to e}$$

(Rec) $$\frac{}{rec\ f = e \Downarrow e\,[rec\ f = e\ /\ f]}$$

(δ) $$\frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2 \quad v = v_1 \oplus v_2}{e_1 \oplus e_2 \Downarrow v}$$

(If-true) $$\frac{e_1 \Downarrow true \qquad e_2 \Downarrow v}{if\ e_1\ then\ e_2\ else\ e_3 \Downarrow v}$$

(If-false) $$\frac{e_1 \Downarrow false \qquad e_3 \Downarrow v}{if\ e_1\ then\ e_2\ else\ e_3 \Downarrow v}$$

(App) $$\frac{e_1 \Downarrow fun\ x \to e \quad e_2 \Downarrow v \quad e[v/x] \Downarrow v'}{e_1 e_2 \Downarrow v'}$$

OS_{clo}

(Const) $\dfrac{}{\eta, k \Downarrow k}$ (Var) $\dfrac{}{\eta, x \Downarrow \eta(x)}$

(Abstr) $\dfrac{}{\eta, fun\ x \to e \Downarrow \langle fun\ x \to e, \eta \rangle}$

(Rec) $\dfrac{}{\eta, rec\ f = e \Downarrow \langle e, \eta' \rangle}$
where $\eta' = \eta[f \to \langle e, \eta' \rangle]$

(δ) $\dfrac{\eta, e_1 \Downarrow v_1 \quad \eta, e_2 \Downarrow v_2 \quad v = v_1 \oplus v_2}{\eta, e_1 \oplus e_2 \Downarrow v}$

(App) $\dfrac{\eta, e_1 \Downarrow \langle fun\ x \to e, \eta' \rangle \quad \eta, e_2 \Downarrow v \quad \eta'[x \to v], e \Downarrow v'}{\eta, e_1 e_2 \Downarrow v'}$

OS_{state}

(Const) $\dfrac{}{\sigma,\eta \vdash k \Downarrow k,\sigma}$ (Var) $\dfrac{}{\sigma,\eta \vdash x \Downarrow \eta(x),\sigma}$

(Abstr) $\dfrac{}{\sigma,\eta \vdash \mathit{fun}\ x \to e \Downarrow \langle \mathit{fun}\ x \to e,\eta\rangle,\sigma}$

(δ) $\dfrac{\sigma,\eta \vdash e_1 \Downarrow v_1,\sigma_1 \quad \sigma_1,\eta \vdash e_2 \Downarrow v_2,\sigma_2 \quad v{=}v_1 \oplus v_2}{\sigma,\eta \vdash e_1 \oplus e_2 \Downarrow v,\sigma_2}$

(App) $\dfrac{\sigma,\eta \vdash e_1 \Downarrow \langle \mathit{fun}\ x \to e,\eta'\rangle,\sigma_1 \quad \sigma_1,\eta \vdash e_2 \Downarrow v,\sigma_2 \quad \sigma_2,\eta'[x \to v] \vdash e \Downarrow v',\sigma_3}{\sigma,\eta \vdash e_1 e_2 \Downarrow v',\sigma_3}$

(Deref) $\dfrac{\sigma,\eta \vdash e \Downarrow \ell,\sigma' \quad \ell\ \mathit{a\ location} \quad \sigma'(\ell) = v}{\sigma,\eta \vdash !e \Downarrow v,\sigma'}$

(Assign) $\dfrac{\sigma,\eta \vdash e_1 \Downarrow \ell,\sigma' \quad \sigma',\eta \vdash e_2 \Downarrow v,\sigma''}{\sigma,\eta \vdash e_1 := e_2 \Downarrow (),\sigma''[\ell \to v]}$

(Ref) $\dfrac{\sigma,\eta \vdash e \Downarrow v,\sigma' \quad \ell\ \mathit{a\ fresh\ location}}{\sigma,\eta \vdash \mathit{ref}\ e \Downarrow \ell,\sigma'[\ell \to v]}$