Below is a program that computes the n^{th} Fibonacci number. Recall that $fib_0 = 1$, $fib_1 = 1$, and $fib_{n+2} = fib_n + fib_{n+1}$.

```
while(i!=n) {
    b = a+b;
    a = b-a;
    i = i+1;
}
```

Prove that, after the loop terminates, $a = fib_n$, $b = fib_{n+1}$.

Initial values are: $b=1$, $a=1$, $i=0$ and $n \geq 0$.

That is, show the proof tree for the following Hoare judgment:

$$\left\{ \begin{array}{l} a=1 \ \& \\ b=1 \ \& \\ i=0 \ \& \\ n \geq 0 \end{array} \right\} \texttt{while(i!=n)\{b=a+b; a=b-a; i=i+1;\}} \left\{ \begin{array}{l} b = fib_{n+1} \ \& \\ a = fib_n \end{array} \right\}$$

You can use the following shorthand notation for the consequence rule:

$$\frac{P \Rightarrow P' \{S\} Q' \Rightarrow Q}{P\{S\}Q}$$

which stands for

$$\frac{P \Rightarrow P' \qquad P'\{S\}Q' \qquad Q' \Rightarrow Q}{P\{S\}Q}$$

ASSIGN

$a=fib_{i+1}$ & $b=fib_{i+2}$ { $i=i+1;$ } $a=fib_i$ & $b=fib_{i+1}$

ASSIGN.

$b-a=fib_{i+1}$ & $b=fib_{i+2}$ { $a=b-a;$ } $a=fib_{i+1}$ & $b=fib_{i+2}$

ASSIGN.

& ... ;& $\Rightarrow$ $a=fib_i$ & $a+b=fib_{i+2}$ { $b=a+b;$ } $a=fib_i$ & $b=fib_{i+2}$ $\Rightarrow$ $b-a=fib_{i+1}$ & $b=fib_{i+2}$

$i+1$

SEQ

$b-a=fib_{i+1}$ & $b=fib_{i+2}$ { $a=b-a;$ $i=i+1;$ } $a=fib_i$ $b=fib_{i+1}$

CONSEQ.

$i\neq n$ & $a=fib_i$ & $b=fib_{i+1}$ { $b=a+b;$ } $b-a=fib_{i+1}$ & $b=fib_{i+2}$

SEQ

$i\neq n$ & $a=fib_i$ & $b=fib_{i+1}$ { $b=a+b;$ $a=b-a;$ $i=i+1;$ } $a=fib_i$ & $b=fib_{i+1}$

WHILE

$a=1$ & $b=1$ & $i=0$ & $n\geq 0$ $\Rightarrow$ $a=fib_i$ & $b=fib_{i+1}$ { while (i!=n){ b=a+b; a=b-a; i=i+1;} } $i=n$ & $a=fib_i$ & $b=fib_{i+1}$ $\Rightarrow$ $a=fib_n$ $b=fib_{n+1}$

Conseq.

$a=1$ & $b=1$ & $i=0$ & $n\geq 0$ { while (i!=n){ b=a+b; a=b-a; i=i+1;} } $b=fib_n$ ... $a=fib$ ...