Programming Languages and Compilers (CS 421)

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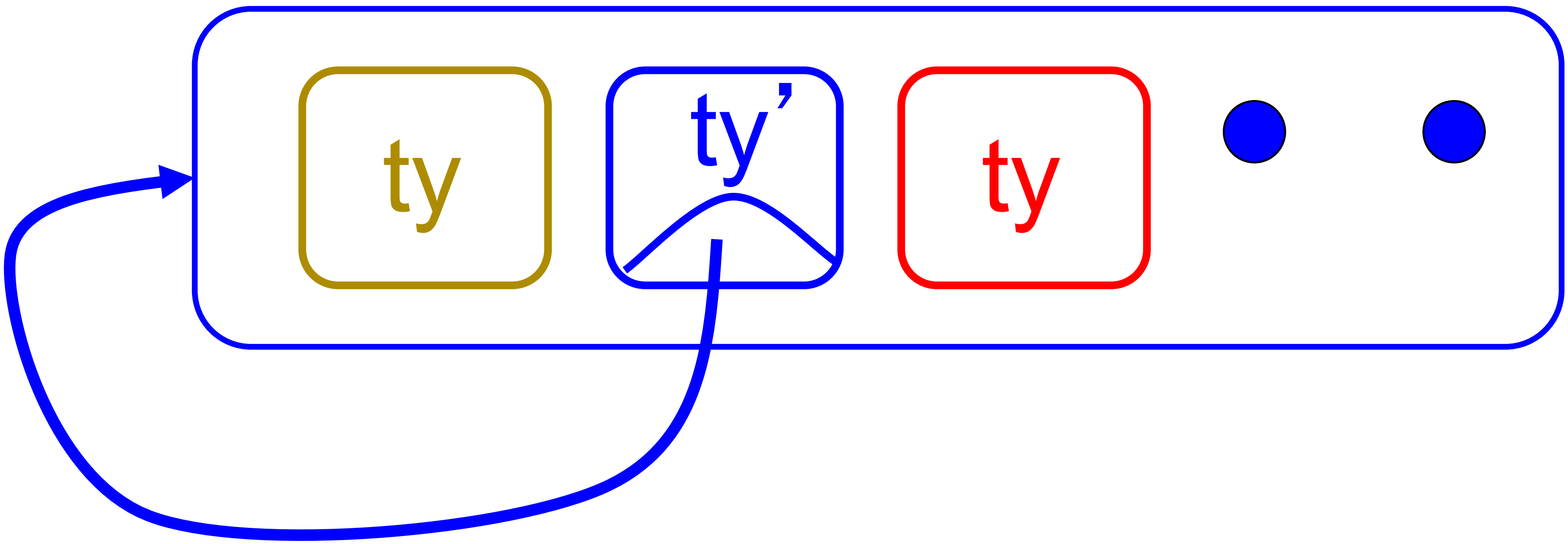
http://courses.engr.illinois.edu/cs421

Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Recursive Types

- The type being defined may be a component of itself



Recursive Data Types

```
# type int_Bin_Tree =
 Leaf of int | Node of (int_Bin_Tree *
  int_Bin_Tree);;
```

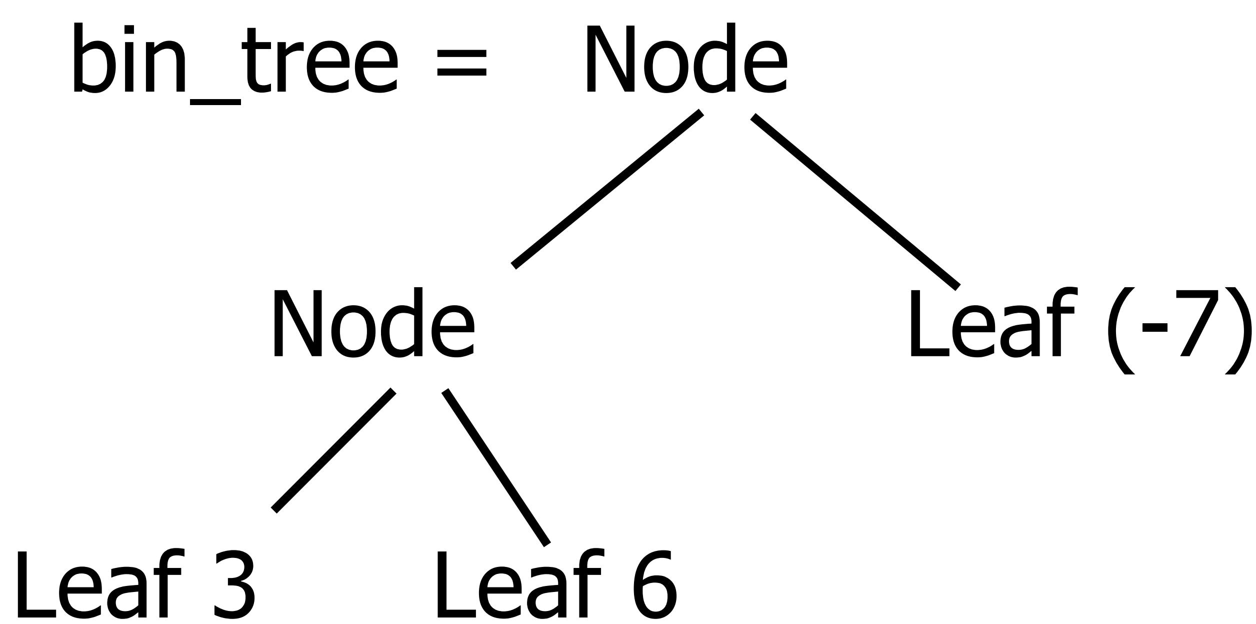
```
type int_Bin_Tree = Leaf of int | Node of
  (int_Bin_Tree * int_Bin_Tree)
```

Recursive Data Type Values

```
# let bin_tree =
 Node(Node(Leaf 3, Leaf 6),Leaf (-7));;

val bin_tree : int_Bin_Tree = Node (Node
  (Leaf 3, Leaf 6), Leaf (-7))
```

Recursive Data Type Values



Recursive Data Types

```
# type exp =
    VarExp of string
  | ConstExp of const
  | MonOpAppExp of mon_op * exp
  | BinOpAppExp of bin_op * exp * exp
  | IfExp of exp* exp * exp
  | AppExp of exp * exp
  | FunExp of string * exp
```

Recursive Data Types

```
# type bin_op = IntPlusOp |  IntMinusOp
        |  EqOp | CommaOp | ConsOp | …
# type const = BoolConst of bool | IntConst of int |
…
# type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp  | …
```

- How to represent 6 as an exp?

Recursive Data Types

```
# type bin_op = IntPlusOp |  IntMinusOp
        |  EqOp | CommaOp | ConsOp | …
# type const = BoolConst of bool | IntConst of int |
…
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp  | …
```

- How to represent 6 as an exp?
- Answer: ConstExp (IntConst 6)

Recursive Data Types

```
# type bin_op = IntPlusOp |  IntMinusOp
        |  EqOp | CommaOp | ConsOp | …
# type const = BoolConst of bool | IntConst of int |
…
# type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp  | …
```

- How to represent (6, 3) as an exp?

Recursive Data Types

type bin_op = IntPlusOp | IntMinusOp
 | EqOp | CommaOp | ConsOp | …
type const = BoolConst of bool | IntConst of int | …
type exp = VarExp of string | ConstExp of const
 | BinOpAppExp of bin_op * exp * exp | …

- How to represent (6, 3) as an exp?
- BinOpAppExp (CommaOp, ConstExp (IntConst 6),
 ConstExp (IntConst 3))

Recursive Data Types

```
# type bin_op = IntPlusOp |  IntMinusOp
        |  EqOp | CommaOp | ConsOp | …
# type const = BoolConst of bool | IntConst of int |
…
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp  | …
```

- How to represent [(6, 3)] as an exp?
- BinOpAppExp (ConsOp, BinOpAppExp (CommaOp, ConstExp (IntConst 6), ConstExp (IntConst 3)), ConstExp NilConst))));;

Problem

```
type int_Bin_Tree =Leaf of int
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write sum_tree : int_Bin_Tree -> int
- Adds all ints in tree

```
let rec sum_tree t =
```

Problem

```
type int_Bin_Tree =Leaf of int
| Node of (int_Bin_Tree * int_Bin_Tree);;
```

- Write sum_tree : int_Bin_Tree -> int
- Adds all ints in tree

```
let rec sum_tree t =
    match t with Leaf n -> n
    | Node(t1,t2) -> sum_tree t1 + sum_tree t2
```

Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp
   | FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp
  | FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =
  match exp with VarExp x ->
   | ConstExp c ->
   | BinOpAppExp (b, e1, e2) ->
   | FunExp (x,e) ->
   | AppExp (e1, e2) ->
```

Recursion over Recursive Data Types

```
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp
   | FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?

```
# let rec varCnt exp =
   match exp with VarExp x -> 1
    | ConstExp c -> 0
    | BinOpAppExp (b, e1, e2) -> varCnt e1 + varCnt e2
    | FunExp (x,e) -> 1 + varCnt e
    | AppExp (e1, e2) -> varCnt e1 + varCnt e2
```

Mapping over Recursive Types

```
# let rec ibtreeMap f tree =
   match tree with (Leaf n) -> Leaf (f n)
   | Node (left_tree, right_tree) ->
    Node (ibtreeMap f left_tree,
          ibtreeMap f right_tree);;
val ibtreeMap : (int -> int) -> int_Bin_Tree ->
  int_Bin_Tree = <fun>
```

Mapping over Recursive Types

```
# ibtreeMap ((+) 2) bin_tree;;

- : int_Bin_Tree = Node (Node (Leaf 5, Leaf 8), Leaf (-5))
```

Folding over Recursive Types

```
# let rec ibtreeFoldRight leafFun nodeFun tree =
    match tree with Leaf n -> leafFun n
    | Node (left_tree, right_tree) ->
      nodeFun
      (ibtreeFoldRight leafFun nodeFun left_tree)
      (ibtreeFoldRight leafFun nodeFun right_tree);;
val ibtreeFoldRight : (int -> 'a) -> ('a -> 'a -> 'a) ->
   int_Bin_Tree -> 'a = <fun>
```

Folding over Recursive Types

```
# let tree_sum =
    ibtreeFoldRight (fun x -> x) (+);;
val tree_sum : int_Bin_Tree -> int = <fun>
# tree_sum bin_tree;;
- : int = 2
```

Mutually Recursive Types

```
# type 'a tree = TreeLeaf of 'a
  | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree
  | More of ('a tree * 'a treeList);;
```

type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList

and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList)

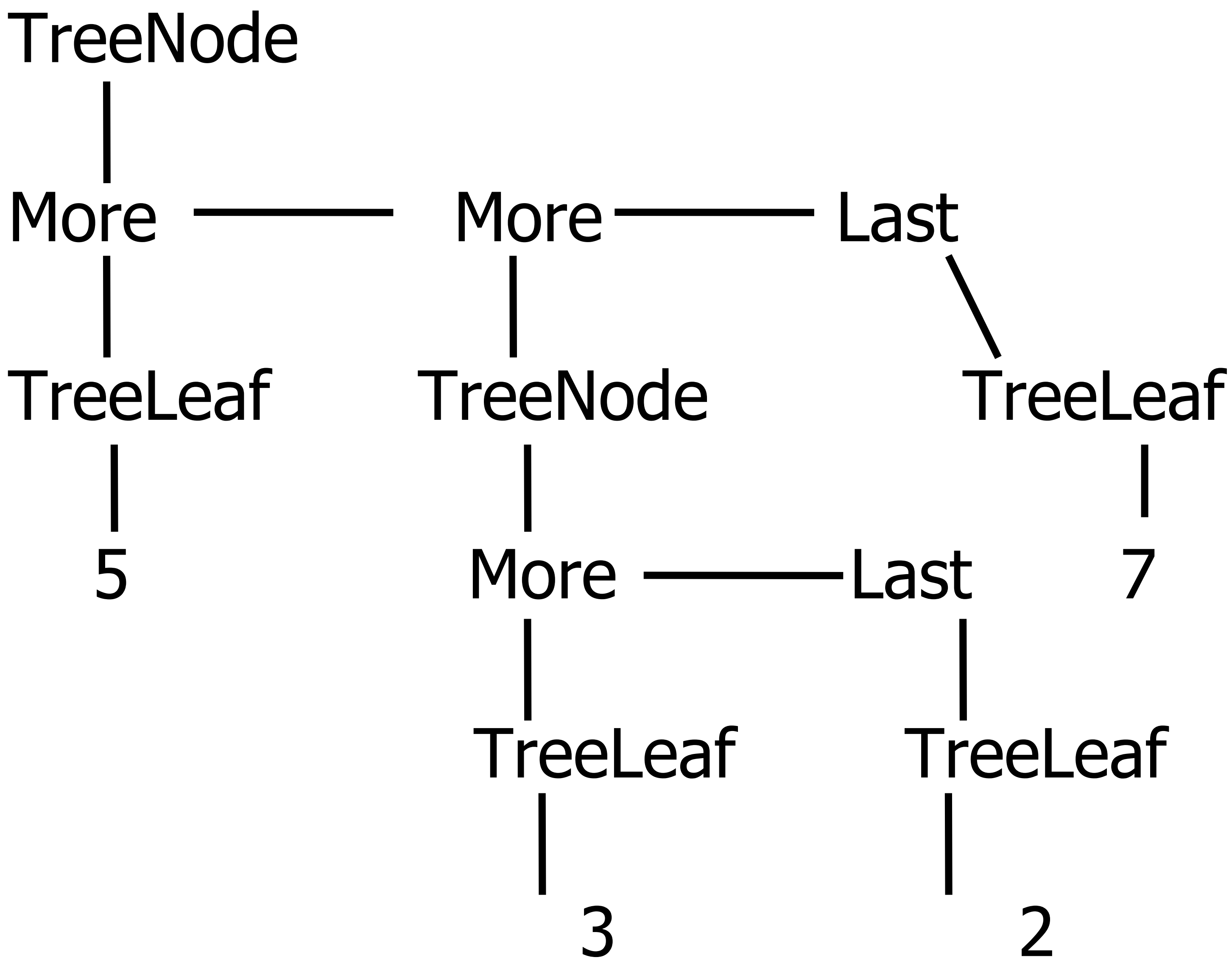
Mutually Recursive Types - Values

```
# let tree =
  TreeNode
   (More (TreeLeaf 5,
         (More (TreeNode
               (More (TreeLeaf 3,
                     Last (TreeLeaf 2))),
               Last (TreeLeaf 7)))));;
```

Mutually Recursive Types - Values

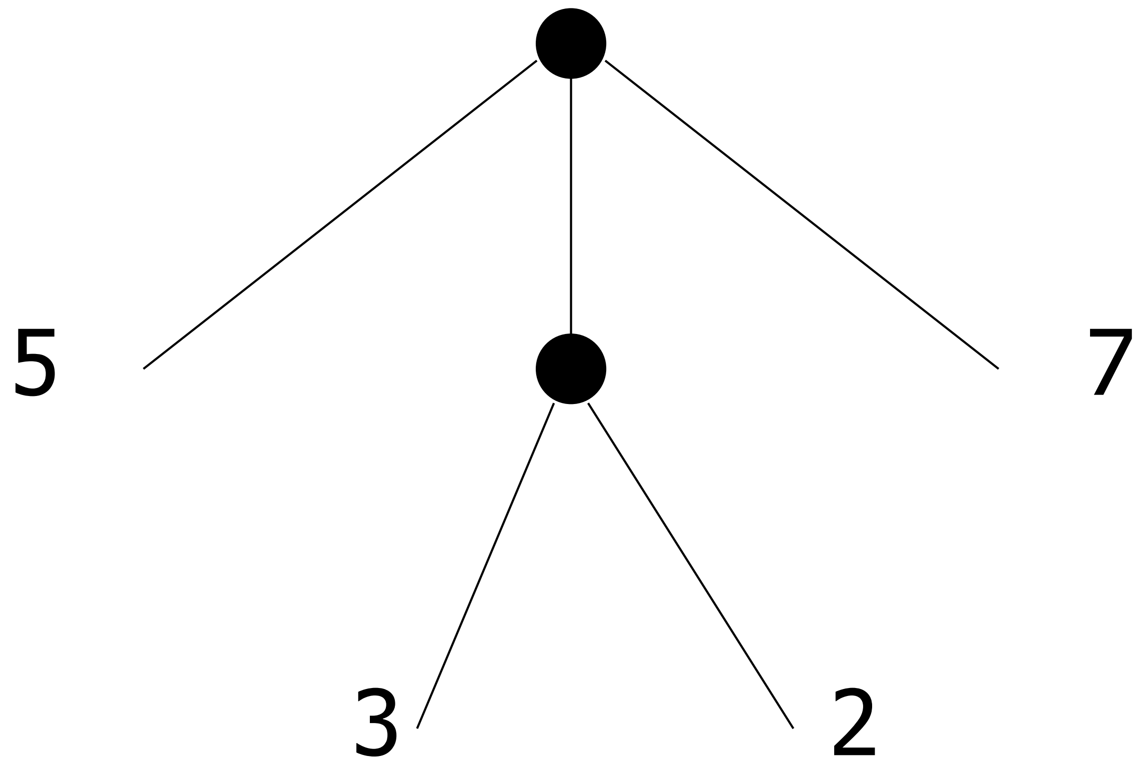
```
val tree : int tree =
 TreeNode
  (More
   (TreeLeaf 5,
    More
     (TreeNode (More (TreeLeaf 3, Last
 (TreeLeaf 2))), Last (TreeLeaf 7))))
```

Mutually Recursive Types - Values



Mutually Recursive Types - Values

A more conventional picture



Mutually Recursive Functions

```
# let rec fringe tree =
    match tree with (TreeLeaf x) -> [x]
  | (TreeNode list) -> list_fringe list
and list_fringe tree_list =
    match tree_list with (Last tree) -> fringe tree
  | (More (tree,list)) ->
    (fringe tree) @ (list_fringe list);;

val fringe : 'a tree -> 'a list = <fun>
val list_fringe : 'a treeList -> 'a list = <fun>
```

Mutually Recursive Functions

```
# fringe tree;;
- : int list = [5; 3; 2; 7]
```

Not covered after here

Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

Problem

```
# type 'a tree = TreeLeaf of ’a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

```
let rec tree_size t =
      match t with TreeLeaf _ ->
      | TreeNode ts ->
```

Problem

```
# type 'a tree = TreeLeaf of ’a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size

```
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size  ts
```

Problem

```
# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
      match t with TreeLeaf _ -> 1
      | TreeNode ts -> treeList_size  ts
and treeList_size ts =
```

Problem

```
# type 'a tree = TreeLeaf of ’a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size  ts
and treeList_size ts =
    match ts with Last t ->
    | More t ts’ ->
```

Problem

```
# type 'a tree = TreeLeaf of ’a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
      match t with TreeLeaf _ -> 1
      | TreeNode ts -> treeList_size  ts
and treeList_size ts =
      match ts with Last t -> tree_size t
      | More t ts’ -> tree_size t + treeList_size ts’
```

Problem

```
# type 'a tree = TreeLeaf of ’a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;
```

Define tree_size and treeList_size

```
let rec tree_size t =
      match t with TreeLeaf _ -> 1
      | TreeNode ts -> treeList_size  ts
and treeList_size ts =
      match ts with Last t -> tree_size t
      | More t ts’ -> tree_size t + treeList_size ts’
```

Nested Recursive Types

```
# type 'a labeled_tree =
 TreeNode of ('a * 'a labeled_tree
  list);;
type 'a labeled_tree = TreeNode of ('a
  * 'a labeled_tree list)
```

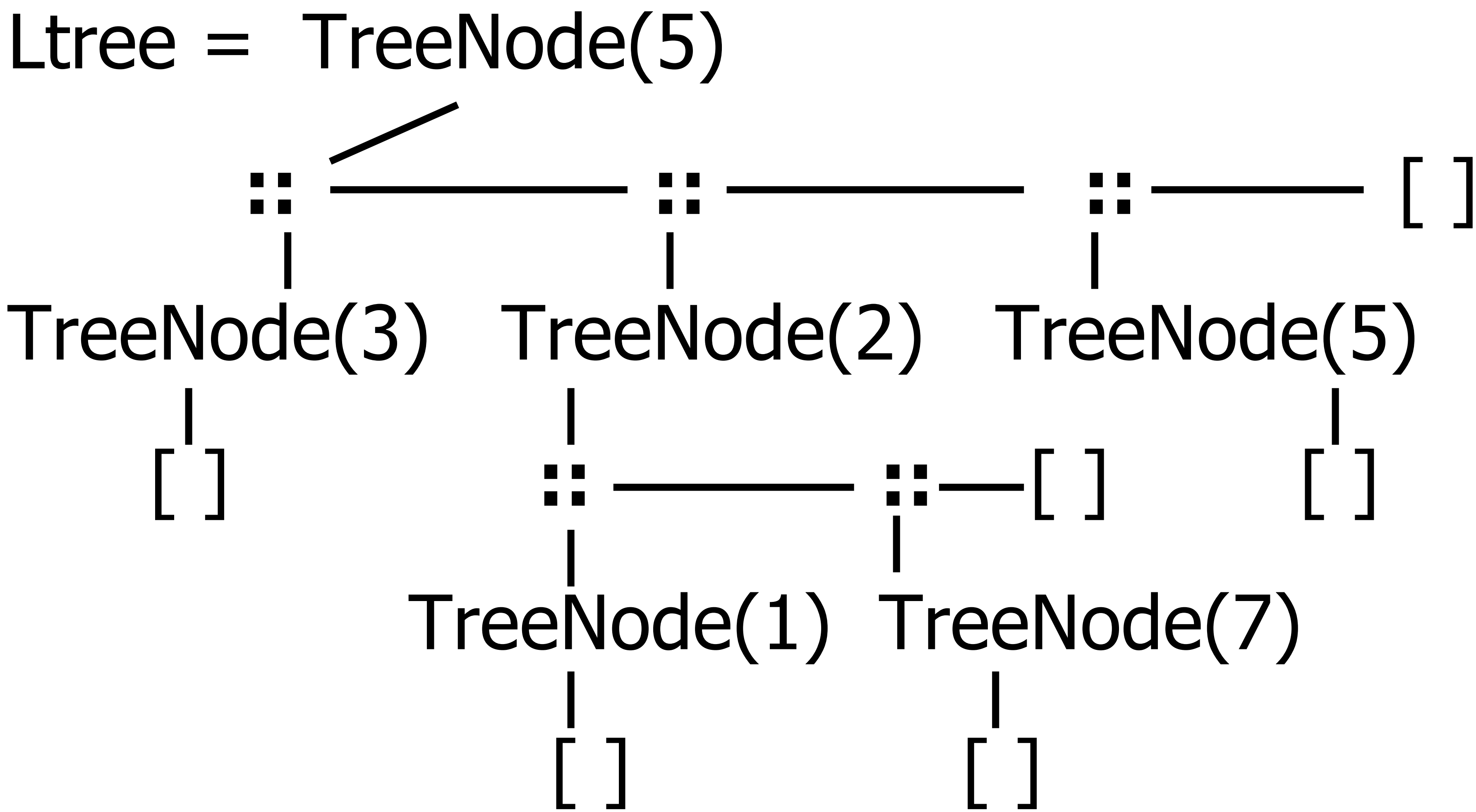
Nested Recursive Type Values

```
# let ltree =
  TreeNode(5,
    [TreeNode (3, []);
     TreeNode (2, [TreeNode (1, []);
                   TreeNode (7, [])]);
     TreeNode (5, [])]);;
```

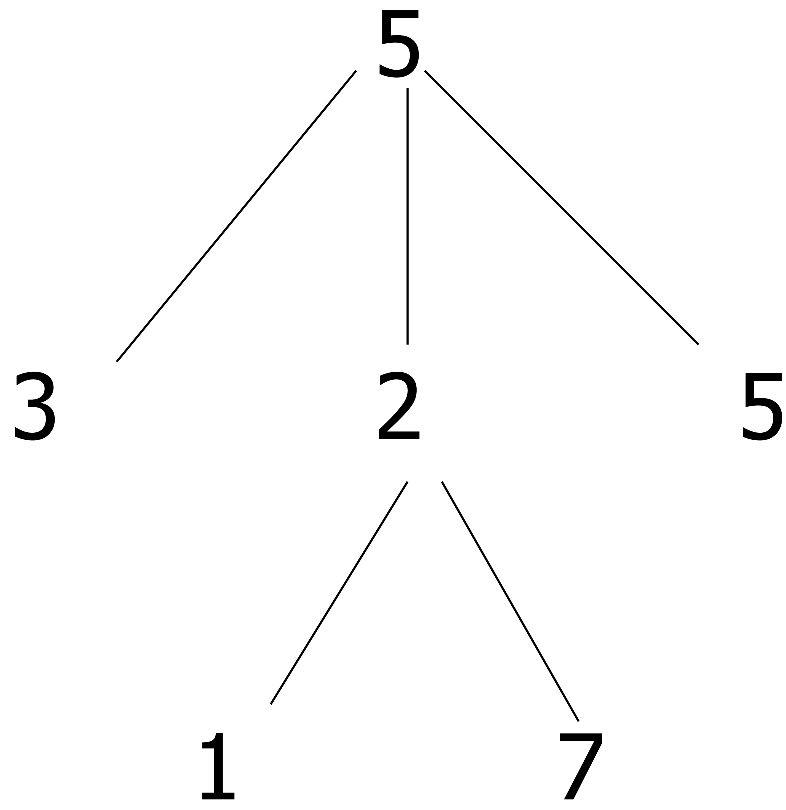
Nested Recursive Type Values

```
val ltree : int labeled_tree =
  TreeNode
   (5,
    [TreeNode (3, []); TreeNode (2,
  [TreeNode (1, []); TreeNode (7, [])]);
     TreeNode (5, [])])
```

Nested Recursive Type Values



Nested Recursive Type Values



Mutually Recursive Functions

```
# let rec flatten_tree labtree =
    match labtree with TreeNode (x,treelist)
      -> x::flatten_tree_list treelist
    and flatten_tree_list treelist =
    match treelist with [] -> []
    | labtree::labtrees
      -> flatten_tree labtree
        @ flatten_tree_list labtrees;;
```

Mutually Recursive Functions

```
val flatten_tree : 'a labeled_tree -> 'a list =
  <fun>
val flatten_tree_list : 'a labeled_tree list -> 'a
  list = <fun>
# flatten_tree ltree;;
- : int list = [5; 3; 2; 1; 7; 5]
```

- Nested recursive types lead to mutually recursive functions

Infinite Recursive Values

```
# let rec ones = 1::ones;;
val ones : int list =
  [1; 1; 1; 1; ...]
# match ones with x::_ -> x;;
Characters 0-25:
Warning: this pattern-matching is not exhaustive.
Here is an example of a value that is not matched:
[]
  match ones with x::_ -> x;;
  ^^^^^^^^^^^^^^^^^^^^^^^^^
- : int = 1
```

Infinite Recursive Values

```
# let rec lab_tree = TreeNode(2, tree_list)
   and tree_list = [lab_tree; lab_tree];;
val lab_tree : int labeled_tree =
 TreeNode (2, [TreeNode(...); TreeNode(...)])
val tree_list : int labeled_tree list =
 [TreeNode (2, [TreeNode(...);
  TreeNode(...)]);
  TreeNode (2, [TreeNode(...);
  TreeNode(...)])]
```

Infinite Recursive Values

```
# match lab_tree
  with TreeNode (x, _) -> x;;
- : int = 2
```

Records

- Records serve the same programming purpose as tuples
- Provide better documentation, more readable code
- Allow components to be accessed by label instead of position
  - Labels (aka *field names* must be unique)
  - Fields accessed by suffix dot notation

Record Types

- Record types must be declared before they can be used in OCaml

```
# type person = {name : string; ss : (int * int * int); age : int};;
type person = { name : string; ss : int * int * int; age : int; }
```

- person is the type being introduced
- name, ss and age are the labels, or fields

Record Values

- Records built with labels; order does not matter

```
# let teacher = {name = "Elsa L. Gunter";
  age = 102; ss = (119,73,6244)};;
val teacher : person =
  {name = "Elsa L. Gunter"; ss = (119, 73,
  6244); age = 102}
```

Record Pattern Matching

```
# let {name = elsa; age = age; ss =
  (_,_,s3)} = teacher;;
val elsa : string = "Elsa L. Gunter"
val age : int = 102
val s3 : int = 6244
```

Record Field Access

```
# let soc_sec = teacher.ss;;
val soc_sec : int * int * int = (119,
  73, 6244)
```

Record Values

```
# let student = {ss=(325,40,1276);
  name="Joseph Martins"; age=22};;
val student : person =
 {name = "Joseph Martins"; ss = (325, 40,
  1276); age = 22}
# student = teacher;;
- : bool = false
```

New Records from Old

```
# let birthday person = {person with age =
  person.age + 1};;
val birthday : person -> person = <fun>
# birthday teacher;;
- : person = {name = "Elsa L. Gunter"; ss =
  (119, 73, 6244); age = 103}
```

New Records from Old

```
# let new_id name soc_sec person =
 {person with name = name; ss = soc_sec};;
val new_id : string -> int * int * int -> person
  -> person = <fun>
# new_id "Guieseppe Martin" (523,04,6712)
  student;;
- : person = {name = "Guieseppe Martin"; ss
  = (523, 4, 6712); age = 22}
```