

Programming Languages and Compilers (CS 421)

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<https://courses.engr.illinois.edu/cs421/fa2017/CS421A>

Based on slides by [Elsa Gunter](#), which were inspired by earlier slides by Mattox Beckman, Vikram Adve, and Gul Agha

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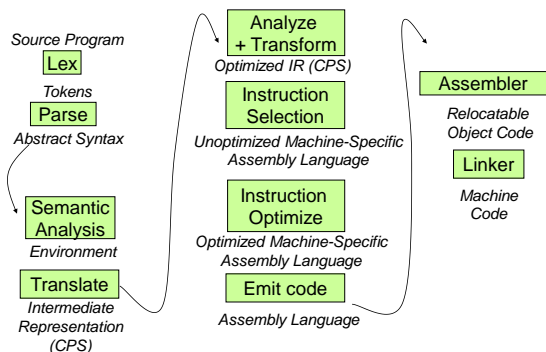
Course Objectives

- New programming paradigm
 - Functional programming
 - Environments and Closures
 - Patterns of Recursion
 - Continuation Passing Style
- Phases of an interpreter / compiler
 - Lexing and parsing
 - Type systems
 - Interpretation
- Programming Language Semantics
 - Lambda Calculus
 - Operational Semantics
 - Axiomatic Semantics

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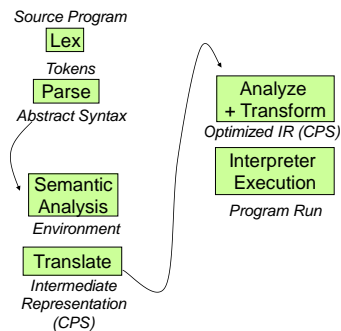
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Major Phases of a Compiler



Modified from "Modern Compiler Implementation in ML", by Andrew Appel

Major Phases of a PicoML Interpreter



Meta-discourse

Language Syntax and Semantics

- Syntax
 - Regular Expressions, DFSAs and NDFSAs
 - Grammars
- Semantics
 - Natural Semantics
 - Transition Semantics

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Where We Are Going Next?

- We want to turn strings (code) into computer instructions
- Done in phases
- Break the big strings into tokens (lex)
- Turn tokens into abstract syntax trees (parse)
- Translate abstract syntax trees into executable instructions (interpret or compile)

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Syntax of English Language

- Pattern 1

Subject	Verb
David	sings
The dog	barked
Susan	yawned
- Pattern 2

Subject	Verb	Direct Object
David	sings	ballads
The professor	wants	to retire
The jury	found	the defendant guilty

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Elements of Syntax

- Character set – previously always ASCII, now often 64 character sets
- Keywords – usually reserved
- Special constants – cannot be assigned to
- Identifiers – can be assigned to
- Operator symbols
- Delimiters (parenthesis, braces, brackets)
- Blanks (aka white space)

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Elements of Syntax

- Expressions
 - if ... then begin ... ; ... end else begin ... ; ... end
- Type expressions
 - $type_{expr_1} \rightarrow type_{expr_2}$
- Declarations (in functional languages)
 - let *pattern* = *expr*
- Statements (in imperative languages)
 - $a = b + c$
- Subprograms
 - let $pattern_1 = expr_1$ in *expr*

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Elements of Syntax

- Modules
- Interfaces
- Classes (for object-oriented languages)

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Lexing and Parsing

- Converting strings to abstract syntax trees done in two phases
 - Lexing:** Converting string (or streams of characters) into lists (or streams) of tokens (the “words” of the language)
 - Specification Technique: Regular Expressions
 - Parsing:** Convert a list of tokens into an abstract syntax tree
 - Specification Technique: BNF Grammars

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Formal Language Descriptions

- Regular expressions, regular grammars, finite state automata
- Context-free grammars, BNF grammars, syntax diagrams
- Whole family more of grammars and automata – covered in automata theory

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Grammars

- Grammars are formal descriptions of which strings over a given character set are in a particular language
- Language designers write grammar
- Language implementers use grammar to know what programs to accept
- Language users use grammar to know how to write legitimate programs

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Regular Expressions - Review

- Start with a given character set – **a, b, c...**
- Each character is a **regular expression**
 - It represents the set of one string containing just that character

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Regular Expressions

- If **x** and **y** are regular expressions, then **xy** is a regular expression
 - It represents the set of all strings made from first a string described by **x** then a string described by **y**
If $x = \{a, ab\}$ and $y = \{c, d\}$ then $xy = \{ac, ad, abc, abd\}$.
- If **x** and **y** are regular expressions, then **xvy** is a regular expression
 - It represents the set of strings described by either **x** or **y**
If $x = \{a, ab\}$ and $y = \{c, d\}$ then $x \vee y = \{a, ab, c, d\}$

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Regular Expressions

- If **x** is a regular expression, then so is **(x)**
 - It represents the same thing as **x**
- If **x** is a regular expression, then so is **x***
 - It represents strings made from concatenating zero or more strings from **x**
If $x = \{a, ab\}$ then $x^* = \{\epsilon, a, ab, aa, aab, abab, \dots\}$
- ϵ
 - It represents $\{\epsilon\}$, set containing the empty string
- \emptyset
 - It represents $\{\}$, the empty set

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Example Regular Expressions

- **(0 \vee 1)*1**
 - The set of all strings of 0's and 1's ending in 1,
■ $\{1, 01, 11, \dots\}$
- **a*b(a*)**
 - The set of all strings of a's and b's with exactly one b
- **((01) \vee (10))***
 - You tell me
- Regular expressions (equivalently, regular grammars) important for lexing, breaking strings into recognized words

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Example: Lexing

- Regular expressions good for describing lexemes (words) in a programming language
 - Identifier = $(a \vee b \vee \dots \vee z \vee A \vee B \vee \dots \vee Z) (a \vee b \vee \dots \vee z \vee A \vee B \vee \dots \vee Z \vee 0 \vee 1 \vee \dots \vee 9)^*$
 - Digit = $(0 \vee 1 \vee \dots \vee 9)$
 - Number = $0 \vee (1 \vee \dots \vee 9)(0 \vee \dots \vee 9)^* \vee -(1 \vee \dots \vee 9)(0 \vee \dots \vee 9)^*$
 - Keywords: if = if, while = while,...

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Implementing Regular Expressions

- Regular expressions reasonable way to generate strings in language
- Not so good for recognizing when a string is in language
- Problems with Regular Expressions
 - which option to choose,
 - how many repetitions to make
- Answer: finite state automata
- Should have seen in CS374

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Lexing

- Different syntactic categories of “words”: tokens

Example:

- Convert sequence of characters into sequence of strings, integers, and floating point numbers.
- "asd 123 jkl 3.14" will become:
[String "asd"; Int 123; String "jkl"; Float 3.14]

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Lex, ocamllex

- Could write the reg exp, then translate to DFA by hand
 - A lot of work
- Better: Write program to take reg exp as input and automatically generates automata
- Lex is such a program
- ocamllex version for ocaml

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How to do it

- To use regular expressions to parse our input we need:
 - Some way to identify the input string — call it a lexing buffer
 - Set of regular expressions,
 - Corresponding set of actions to take when they are matched.

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How to do it

- The lexer will take the regular expressions and generate a state machine.
- The state machine will take our lexing buffer and apply the transitions...
- If we reach an accepting state from which we can go no further, the machine will perform the appropriate action.

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Mechanics

- Put table of reg exp and corresponding actions (written in ocaml) into a file *<filename>.mll*
- Call

```
ocamllex <filename>.mll
```
- Produces Ocaml code for a lexical analyzer in file *<filename>.ml*

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Sample Input

```
rule main = parse
  ['0'-'9']+ { print_string "Int\n"}
| ['0'-'9']+ '.' ['0'-'9']+ { print_string "Float\n"}
| ['a'-'z']+ { print_string "String\n"}
| _ { main lexbuf }
{
  let newlexbuf = (Lexing.from_channel stdin) in
    print_string "Ready to lex.\n";
  main newlexbuf
}
```

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General Input

```
{ header }
let ident = regexp ...
rule entrypoint [arg1... argn] = parse
  regexp { action }
  | ...
  | regexp { action }
and entrypoint [arg1... argn] = parse
...and ...
{ trailer }
```

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Ocamllex Input

- *header* and *trailer* contain arbitrary ocaml code put at top and bottom of *<filename>.ml*
- `let ident = regexp ...` Introduces *ident* for use in later regular expressions

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Ocamllex Input

- *<filename>.ml* contains one lexing function per *entrypoint*
 - Name of function is name given for *entrypoint*
 - Each entry point becomes an Ocaml function that takes $n+1$ arguments, the extra implicit last argument being of type `Lexing.lexbuf`
- *arg1... argn* are for use in *action*

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Ocamllex Regular Expression

- Single quoted characters for letters: `'a'`
- `_`: (underscore) matches any letter
- `Eof`: special "end_of_file" marker
- Concatenation same as usual
- `"string"`: concatenation of sequence of characters
- `e1/e2`: choice - what was $e_1 \vee e_2$

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Ocamllex Regular Expression

- `[c1 - c2]`: choice of any character between first and second inclusive, as determined by character codes
- `[^c1 - c2]`: choice of any character NOT in set
- `e*`: same as before
- `e+`: same as $e e^*$
- `e?`: option - was $e_1 \vee \varepsilon$

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Ocamllex Regular Expression

- $e_1 \# e_2$: the characters in e_1 but not in e_2 ; e_1 and e_2 must describe just sets of characters
- *ident*: abbreviation for earlier reg exp in let $ident = regexp$
- e_1 as *id*: binds the result of e_1 to *id* to be used in the associated *action*

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Example : test.mll

```
{
  type result = Int of int | Float of float |
                String of string
}
let digit = ['0'-'9']
let digits = digit+
let lower_case = ['a'-'z']
let upper_case = ['A'-'Z']
let letter = upper_case | lower_case
let letters = letter+
```

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Example

```
# #use "test.mll";;
...
val main : Lexing.lexbuf -> result = <fun>
val __ocaml_lex_main_rec : Lexing.lexbuf ->
  int -> result = <fun>
Ready to lex.
hi there 234 5.2
- : result = String "hi"
```

What happened to the rest?!?

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Ocamllex Manual

- More details can be found at

<http://caml.inria.fr/pub/docs/manual-ocaml/lex yacc.html>

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Example : test.mll

```
rule main = parse
  (digits)'.digits as f
    { Float (float_of_string f) }
  | digits as n   { Int (int_of_string n) }
  | letters as s  { String s }
  | _             { main lexbuf }
{
  let newlexbuf = (Lexing.from_channel stdin) in
  print_string "Ready to lex.";
  print_newline ();
  main newlexbuf
}
```

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Example

```
# let b = Lexing.from_channel stdin;;
# main b;;
hi 673 there
- : result = String "hi"
# main b;;
- : result = Int 673
# main b;;
- : result = String "there"
```

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Problem

- How to get lexer to look at more than the first token at one time?
- Answer: *action* has to tell it to -- recursive calls
- Side Benefit: can add “state” into lexing
- Note: already used this with the `_` case

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Example

```
rule main = parse
  (digits) '.' digits as f
  { Float (float_of_string f) :: main lexbuf }
| digits as n
  { Int (int_of_string n) :: main lexbuf }
| letters as s
  { String s :: main lexbuf }
| eof { [] }
| _ { main lexbuf }
```

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Example Results

```
Ready to lex.
hi there 234 5.2
- : result list = [String "hi"; String "there";
                  Int 234; Float 5.2]
#
```

Used Ctrl-d to send the end-of-file signal

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Dealing with comments

First Attempt

```
let open_comment = "("
let close_comment = ")"

rule main = parse
  (digits) '.' digits as f
  { Float (float_of_string f) :: main lexbuf }
| digits as n
  { Int (int_of_string n) :: main lexbuf }
| letters as s
  { String s :: main lexbuf }
```

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Dealing with comments

```
(* Continued from rule main *)
| open_comment { comment lexbuf }
| eof { [] }
| _ { main lexbuf }

and comment = parse
  close_comment { main lexbuf }
| _ { comment lexbuf }
```

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Dealing with nested comments

```
rule main = parse ...
| open_comment { comment 1 lexbuf }
| eof { [] }
| _ { main lexbuf }

and comment depth = parse
  open_comment { comment (depth+1) lexbuf }
| close_comment { if depth = 1
                  then main lexbuf
                  else comment (depth - 1)
                  lexbuf }
| _ { comment depth lexbuf }
```

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Types of Formal Language Descriptions

- Regular expressions, regular grammars
- Context-free grammars, BNF grammars, syntax diagrams
- Finite state automata
- Pushdown automata

- Whole family more of grammars and automata – covered in automata theory

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BNF Grammars

- Start with a set of characters, **a,b,c,...**
 - We call these *terminals*
- Add a set of different characters, **X,Y,Z,...**
 - We call these *nonterminals*
- One special nonterminal **S** called *start symbol*

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BNF Grammars

- BNF rules (aka *productions*) have form
$$\mathbf{X} ::= y$$
where **X** is any nonterminal and *y* is a string of terminals and nonterminals
- BNF *grammar* is a set of BNF rules such that every nonterminal appears on the left of some rule

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Example: Regular Grammars

- Regular grammar:
$$\langle \text{Balanced} \rangle ::= \epsilon$$
$$\langle \text{Balanced} \rangle ::= 0 \langle \text{OneAndMore} \rangle$$
$$\langle \text{Balanced} \rangle ::= 1 \langle \text{ZeroAndMore} \rangle$$
$$\langle \text{OneAndMore} \rangle ::= 1 \langle \text{Balanced} \rangle$$
$$\langle \text{ZeroAndMore} \rangle ::= 0 \langle \text{Balanced} \rangle$$
- Generates even length strings where every initial substring of even length has same number of 0's as 1's

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Example of BNF: Regular Grammars

- Subclass of BNF -- has only rules of the form:
$$\langle \text{nonterminal} \rangle ::= \langle \text{terminal} \rangle \langle \text{nonterminal} \rangle$$
 or
$$\langle \text{nonterminal} \rangle ::= \langle \text{terminal} \rangle$$
 or
$$\langle \text{nonterminal} \rangle ::= \epsilon$$
- Defines same class of languages as regular expressions
- Important for writing lexers (programs that convert strings of characters into strings of tokens)
- Close connection to nondeterministic finite state automata
 - nonterminals = states;
 - rule = edge

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BNF Grammars

- BNF rules (aka *productions*) have form
$$\mathbf{X} ::= y$$
where **X** is any nonterminal and *y* is a string of terminals and nonterminals
- BNF *grammar* is a set of BNF rules such that every nonterminal appears on the left of some rule

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Sample BNF Grammar

- Language: Parenthesized sums of 0's and 1's
- $\langle \text{Sum} \rangle ::= 0$
- $\langle \text{Sum} \rangle ::= 1$
- $\langle \text{Sum} \rangle ::= \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$
- $\langle \text{Sum} \rangle ::= (\langle \text{Sum} \rangle)$

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Sample Grammar

- Terminals: 0 | + | (|)
- Nonterminals: $\langle \text{Sum} \rangle$
- Start symbol = $\langle \text{Sum} \rangle$
- $\langle \text{Sum} \rangle ::= 0$
- $\langle \text{Sum} \rangle ::= 1$
- $\langle \text{Sum} \rangle ::= \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$
- $\langle \text{Sum} \rangle ::= (\langle \text{Sum} \rangle)$
- Can be abbreviated as
 $\langle \text{Sum} \rangle ::= 0 \mid 1 \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \mid (\langle \text{Sum} \rangle)$

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BNF Derivations

- Given rules
 $X ::= yZw$ and $Z ::= v$
we may replace Z by v to say
 $X \Rightarrow yZw \Rightarrow yvw$
- Sequence of such replacements called *derivation*
- Derivation called *right-most* if always replace the right-most non-terminal

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BNF Derivations

- Start with the start symbol:

$\langle \text{Sum} \rangle \Rightarrow$

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BNF Derivations

- Pick a non-terminal

$\langle \text{Sum} \rangle \Rightarrow$

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BNF Derivations

- Pick a rule and substitute:
 $\langle \text{Sum} \rangle ::= \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

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BNF Derivations

- Pick a non-terminal:

$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$

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BNF Derivations

- Pick a rule and substitute:

■ $\langle \text{Sum} \rangle ::= (\langle \text{Sum} \rangle)$
 $\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$
 $\Rightarrow (\langle \text{Sum} \rangle) + \langle \text{Sum} \rangle$

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BNF Derivations

- Pick a non-terminal:

$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$
 $\Rightarrow (\langle \text{Sum} \rangle) + \langle \text{Sum} \rangle$

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BNF Derivations

- Pick a rule and substitute:

■ $\langle \text{Sum} \rangle ::= \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$
 $\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$
 $\Rightarrow (\langle \text{Sum} \rangle) + \langle \text{Sum} \rangle$
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle) + \langle \text{Sum} \rangle$

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BNF Derivations

- Pick a non-terminal:

$\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$
 $\Rightarrow (\langle \text{Sum} \rangle) + \langle \text{Sum} \rangle$
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle) + \langle \text{Sum} \rangle$

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BNF Derivations

- Pick a rule and substitute:

■ $\langle \text{Sum} \rangle ::= I$
 $\langle \text{Sum} \rangle \Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle$
 $\Rightarrow (\langle \text{Sum} \rangle) + \langle \text{Sum} \rangle$
 $\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle) + \langle \text{Sum} \rangle$
 $\Rightarrow (\langle \text{Sum} \rangle + I) + \langle \text{Sum} \rangle$

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BNF Derivations

- Pick a non-terminal:

$$\begin{aligned} \langle \text{Sum} \rangle &\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + 1) + \langle \text{Sum} \rangle \end{aligned}$$

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BNF Derivations

- Pick a rule and substitute:

- $\langle \text{Sum} \rangle ::= 0$

$$\begin{aligned} \langle \text{Sum} \rangle &\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + 1) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + 1) + 0 \end{aligned}$$

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BNF Derivations

- Pick a non-terminal:

$$\begin{aligned} \langle \text{Sum} \rangle &\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + 1) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + 1) + 0 \end{aligned}$$

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BNF Derivations

- Pick a rule and substitute

- $\langle \text{Sum} \rangle ::= 0$

$$\begin{aligned} \langle \text{Sum} \rangle &\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + 1) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + 1) 0 \\ &\Rightarrow (0 + 1) + 0 \end{aligned}$$

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BNF Derivations

- $(0 + 1) + 0$ is generated by grammar

$$\begin{aligned} \langle \text{Sum} \rangle &\Rightarrow \langle \text{Sum} \rangle + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + \langle \text{Sum} \rangle) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + 1) + \langle \text{Sum} \rangle \\ &\Rightarrow (\langle \text{Sum} \rangle + 1) + 0 \\ &\Rightarrow (0 + 1) + 0 \end{aligned}$$

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Parse Trees

- Graphical representation of derivation
- Each node labeled with either non-terminal or terminal
- If node is labeled with a terminal, then it is a leaf (no sub-trees)
- If node is labeled with a non-terminal, then it has one branch for each character in the right-hand side of rule used to substitute for it

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Example

- Consider grammar:

$\langle \text{exp} \rangle ::= \langle \text{factor} \rangle$
 | $\langle \text{factor} \rangle + \langle \text{factor} \rangle$

$\langle \text{factor} \rangle ::= \langle \text{bin} \rangle$
 | $\langle \text{bin} \rangle * \langle \text{exp} \rangle$

$\langle \text{bin} \rangle ::= 0 \mid 1$

- Goal: Build parse tree for $1 * 1 + 0$ as an $\langle \text{exp} \rangle$

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Example cont.

- $1 * 1 + 0: \quad \langle \text{exp} \rangle$

$\langle \text{exp} \rangle$ is the start symbol for this parse tree

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Example cont.

- $1 * 1 + 0: \quad \langle \text{exp} \rangle$
 |
 $\langle \text{factor} \rangle$

Use rule: $\langle \text{exp} \rangle ::= \langle \text{factor} \rangle$

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Example cont.

- $1 * 1 + 0: \quad \langle \text{exp} \rangle$
 |
 $\langle \text{factor} \rangle$
 / | \
 $\langle \text{bin} \rangle$ * $\langle \text{exp} \rangle$

Use rule: $\langle \text{factor} \rangle ::= \langle \text{bin} \rangle * \langle \text{exp} \rangle$

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Example cont.

- $1 * 1 + 0: \quad \langle \text{exp} \rangle$
 |
 $\langle \text{factor} \rangle$
 / | \
 $\langle \text{bin} \rangle$ * $\langle \text{exp} \rangle$
 | / | \
 | $\langle \text{factor} \rangle$ + $\langle \text{factor} \rangle$
 | | |
 | | |
 | | |

Use rules: $\langle \text{bin} \rangle ::= 1$ and
 $\langle \text{exp} \rangle ::= \langle \text{factor} \rangle + \langle \text{factor} \rangle$

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Example cont.

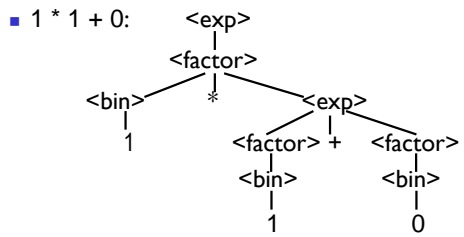
- $1 * 1 + 0: \quad \langle \text{exp} \rangle$
 |
 $\langle \text{factor} \rangle$
 / | \
 $\langle \text{bin} \rangle$ * $\langle \text{exp} \rangle$
 | / | \
 | $\langle \text{factor} \rangle$ + $\langle \text{factor} \rangle$
 | | |
 | | |
 | | |
 | | |

Use rule: $\langle \text{factor} \rangle ::= \langle \text{bin} \rangle$

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Example cont.

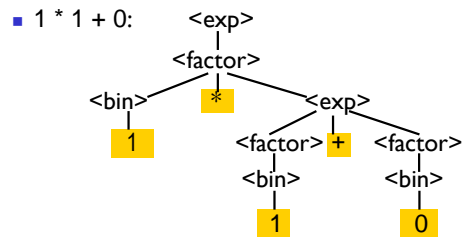


Use rules: $\langle \text{bin} \rangle ::= 1 \mid 0$

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Example cont.



Use rules: $\langle \text{bin} \rangle ::= 1 \mid 0$

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