Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha
Data type in Ocaml: lists

- Frequently used lists in recursive program
- Matched over two structural cases
  - `[ ]` - the empty list
  - `(x :: xs)` a non-empty list
- Covers all possible lists
- type `a list` = `[ ] | (::) of `a * `a list
  - Not quite legitimate declaration because of special syntax
Variants - Syntax (slightly simplified)

- type $name = C_1 [\text{of } ty_1] | \ldots | C_n [\text{of } ty_n]$
- Introduce a type called $name$
- $(\text{fun } x \to C_i x) : ty_1 \to name$
- $C_i$ is called a constructor; if the optional type argument is omitted, it is called a constant
- Constructors are the basis of almost all pattern matching
An enumeration type is a collection of distinct values.

In C and Ocaml they have an order structure; order by order of input.
Enumeration Types as Variants

# type weekday = Monday | Tuesday | Wednesday
| Thursday | Friday | Saturday | Sunday;;

type weekday =
    Monday
| Tuesday
| Wednesday
| Thursday
| Friday
| Saturday
| Sunday
Functions over Enumerations

# let day_after day = match day with
    Monday -> Tuesday
    | Tuesday -> Wednesday
    | Wednesday -> Thursday
    | Thursday -> Friday
    | Friday -> Saturday
    | Saturday -> Sunday
    | Sunday -> Monday;;
val day_after : weekday -> weekday = <fun>
# let rec days_later n day =
   match n with 0 -> day
   | _  -> if n > 0
       then day_after (days_later (n - 1) day)
       else days_later (n + 7) day;;
val days_later : int -> weekday -> weekday
  = <fun>
Functions over Enumerations

# days_later 2 Tuesday;;
- : weekday = Thursday

# days_later (-1) Wednesday;;
- : weekday = Tuesday

# days_later (-4) Monday;;
- : weekday = Thursday
Problem:

```ocaml
# type weekday = Monday | Tuesday | Wednesday
   | Thursday | Friday | Saturday | Sunday

Write function is_weekend : weekday -> bool
let is_weekend day =
```
Problem:

# type weekday = Monday | Tuesday | Wednesday
  | Thursday | Friday | Saturday | Sunday;;

Write function is_weekend : weekday -> bool

let is_weekend day =
  match day with Saturday -> true
  | Sunday -> true
  | _ -> false
Example Enumeration Types

# type bin_op = IntPlusOp | IntMinusOp | EqOp | CommaOp | ConsOp

# type mon_op = HdOp | TlOp | FstOp | SndOp
Disjoint Union Types

- Disjoint union of types, with some possibly occurring more than once

- We can also add in some new singleton elements
Disjoint Union Types

```ocaml
# type id = DriversLicense of int
    | SocialSecurity of int | Name of string;;

type id = DriversLicense of int | SocialSecurity of int | Name of string

# let check_id id = match id with
    DriversLicense num ->
        not (List.mem num [13570; 99999])
    | SocialSecurity num -> num < 900000000
    | Name str -> not (str = "John Doe");;

val check_id : id -> bool = <fun>
```
Problem

Create a type to represent the currencies for US, UK, Europe and Japan
Problem

- Create a type to represent the currencies for US, UK, Europe and Japan

```plaintext
type currency =
    Dollar of int
| Pound of int
| Euro of int
| Yen of int
```
Example Disjoint Union Type

# type const =
  BoolConst of bool
| IntConst of int
| FloatConst of float
| StringConst of string
| NilConst
| UnitConst
Example Disjoint Union Type

```haskell
# type const = BoolConst of bool
    | IntConst of int | FloatConst of float
    | StringConst of string | NilConst
    | UnitConst
```

- How to represent 7 as a const?
- Answer: `IntConst 7`
Polymorphism in Variants

- The type 'a option is gives us something to represent non-existence or failure

```ocaml
# type 'a option = Some of 'a | None;;
type 'a option = Some of 'a | None
```

- Used to encode partial functions
- Often can replace the raising of an exception
Functions producing option

```ocaml
# let rec first p list =  
  match list with [ ] -> None 
  | (x::xs) -> if p x then Some x else first p xs;;
val first : ('a -> bool) -> 'a list -> 'a option = <fun>
# first (fun x -> x > 3) [1;3;4;2;5];;
- : int option = Some 4
# first (fun x -> x > 5) [1;3;4;2;5];;
- : int option = None
```
Functions over option

```ocaml
# let result_ok r =
    match r with None -> false
    | Some _ -> true;;
val result_ok : 'a option -> bool = <fun>
# result_ok (first (fun x -> x > 3) [1;3;4;2;5]);;
- : bool = true
# result_ok (first (fun x -> x > 5) [1;3;4;2;5]);;
- : bool = false
```
Problem

- Write a hd and tl on lists that doesn’t raise an exception and works at all types of lists.
Problem

- Write a hd and tl on lists that doesn’t raise an exception and works at all types of lists.

- let hd list =
  
  match list with [] -> None
  | (x::xs) -> Some x

- let tl list =
  
  match list with [] -> None
  | (x::xs) -> Some xs
Mapping over Variants

```ocaml
# let optionMap f opt =
  match opt with
    | None -> None
    | Some x -> Some (f x);
val optionMap : ('a -> 'b) -> 'a option -> 'b option

# optionMap
  (fun x -> x - 2)
  (first (fun x -> x > 3) [1;3;4;2;5]);;
- : int option = Some 2
```
Folding over Variants

# let optionFold someFun noneVal opt = 
  match opt with None -> noneVal 
  | Some x -> someFun x;;
val optionFold : ('a -> 'b) -> 'b -> 'a option -> 'b = <fun>

# let optionMap f opt = 
  optionFold (fun x -> Some (f x)) None opt;;
val optionMap : ('a -> 'b) -> 'a option -> 'b option = <fun>
Recursive Types

- The type being defined may be a component of itself

\[ \text{ty} \rightarrow \text{ty}' \rightarrow \text{ty} \]
Recursive Data Types

# type int_Bin_Tree =
Leaf of int | Node of (int_Bin_Tree * int_Bin_Tree);

type int_Bin_Tree = Leaf of int | Node of (int_Bin_Tree * int_Bin_Tree)
Recursive Data Type Values

```plaintext
# let bin_tree = Node(Node(Leaf 3, Leaf 6), Leaf (-7));

val bin_tree : int_Bin_Tree = Node (Node (Leaf 3, Leaf 6), Leaf (-7))
```
Recursive Data Type Values

```
bin_tree = Node
         /   \
        /     /
Node   Leaf (-7)
       /   /  \\
Leaf 3 Leaf 6
```
# type exp =
  VarExp of string
| ConstExp of const
| MonOpAppExp of mon_op * exp
| BinOpAppExp of bin_op * exp * exp
| IfExp of exp * exp * exp
| AppExp of exp * exp
| FunExp of string * exp
Recursive Data Types

# type bin_op = IntPlusOp | IntMinusOp
   | EqOp | CommaOp | ConsOp | ...
# type const = BoolConst of bool | IntConst of int |
...
# type exp = VarExp of string | ConstExp of const
   | BinOpAppExp of bin_op * exp * exp | ...

- How to represent 6 as an exp?
Recursive Data Types

#  type bin_op = IntPlusOp | IntMinusOp
    | EqOp | CommaOp | ConsOp | ...
#  type const = BoolConst of bool | IntConst of int |
    ...
#  type exp = VarExp of string | ConstExp of const
    | BinOpAppExp of bin_op * exp * exp | ...

- How to represent 6 as an exp?
- Answer: ConstExp (IntConst 6)
Recursive Data Types

# type bin_op = IntPlusOp | IntMinusOp | EqOp | CommaOp | ConsOp | ...
# type const = BoolConst of bool | IntConst of int | ...
# type exp = VarExp of string | ConstExp of const | BinOpAppExp of bin_op * exp * exp | ...

- How to represent (6, 3) as an exp?
Recursive Data Types

# type bin_op = IntPlusOp | IntMinusOp | EqOp | CommaOp | ConsOp | ...
# type const = BoolConst of bool | IntConst of int | ...
# type exp = VarExp of string | ConstExp of const | BinOpAppExp of bin_op * exp * exp | ...

- How to represent (6, 3) as an exp?
- BinOpAppExp (CommaOp, ConstExp (IntConst 6), ConstExp (IntConst 3))
Recursive Data Types

# type bin_op = IntPlusOp | IntMinusOp | EqOp | CommaOp | ConsOp | ...

# type const = BoolConst of bool | IntConst of int | ...

# type exp = VarExp of string | ConstExp of const | BinOpAppExp of bin_op * exp * exp | ...

- How to represent \([(6, 3)]\) as an exp?
Recursive Functions

# let rec first_leaf_value tree =
match tree with (Leaf n) -> n
| Node (left_tree, right_tree) ->
  first_leaf_value left_tree;;

val first_leaf_value : int_Bin_Tree -> int = <fun>

# let left = first_leaf_value bin_tree;;
val left : int = 3
type int_Bin_Tree = Leaf of int | Node of (int_Bin_Tree * int_Bin_Tree);; 
- Write sum_tree : int_Bin_Tree -> int
- Adds all ints in tree

let rec sum_tree t =
type int_Bin_Tree = Leaf of int
| Node of (int_Bin_Tree * int_Bin_Tree);

- Write sum_tree : int_Bin_Tree -> int
- Adds all ints in tree

let rec sum_tree t =
  match t with Leaf n -> n
  | Node(t1,t2) -> sum_tree t1 + sum_tree t2
Recursion over Recursive Data Types

```ocaml
# type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp
  | FunExp of string * exp | AppExp of exp * exp
```

- How to count the number of variables in an exp?
Recursion over Recursive Data Types

```ocaml
# type exp = VarExp of string | ConstExp of const
  | BinOpAppExp of bin_op * exp * exp
  | FunExp of string * exp | AppExp of exp * exp

# How to count the number of variables in an exp?
# let rec varCnt exp =
  match exp with VarExp x ->
      | ConstExp c ->
      | BinOpAppExp (b, e1, e2) ->
      | FunExp (x, e) ->
      | AppExp (e1, e2) ->
```
Recursion over Recursive Data Types

# type exp = VarExp of string | ConstExp of const | BinOpAppExp of bin_op * exp * exp | FunExp of string * exp | AppExp of exp * exp

- How to count the number of variables in an exp?

# let rec varCnt exp =

match exp with VarExp x -> 1
| ConstExp c -> 0
| BinOpAppExp (b, e1, e2) -> varCnt e1 + varCnt e2
| FunExp (x,e) -> 1 + varCnt e
| AppExp (e1, e2) -> varCnt e1 + varCnt e2
Your turn now

Try Problem 3 on MP3
Mapping over Recursive Types

```ocaml
# let rec ibtreeMap f tree =
  match tree with (Leaf n) -> Leaf (f n)
| Node (left_tree, right_tree) ->
  Node (ibtreeMap f left_tree, ibtreeMap f right_tree);

val ibtreeMap : (int -> int) -> int_Bin_Tree -> int_Bin_Tree = <fun>
```
Mapping over Recursive Types

# ibtreeMap ((+) 2) bin_tree;;

- : int_Bin_TreeNode = Node (Node (Leaf 5, Leaf 8), Leaf (-5))
Folding over Recursive Types

```ocaml
# let rec ibtreeFoldRight leafFun nodeFun tree =
  match tree with Leaf n -> leafFun n
| Node (left_tree, right_tree) ->
  nodeFun
  (ibtreeFoldRight leafFun nodeFun left_tree)
  (ibtreeFoldRight leafFun nodeFun right_tree);;

val ibtreeFoldRight : (int -> 'a) -> ('a -> 'a -> 'a) ->
  int_Bin_Tree -> 'a = <fun>
```
Folding over Recursive Types

# let tree_sum = ibtreeFoldRight (fun x -> x) (+);; 
val tree_sum : int_Bin_Tree -> int = <fun>
# tree_sum bin_tree;; 
- : int = 2
Mutually Recursive Types

# type 'a tree = TreeLeaf of 'a
   | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree
   | More of ('a tree * 'a treeList);

type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList)
Mutually Recursive Types - Values

# let tree =
TreeNode
  (More (TreeLeaf 5, 
    (More (TreeNode 
      (More (TreeLeaf 3, 
        Last (TreeLeaf 2))), 
      Last (TreeLeaf 7))))));;
Mutually Recursive Types - Values

val tree : int tree =
TreeNode
(More
(TreeLeaf 5,
More
(TreeLeaf 3, Last
(TreeLeaf 2))), Last (TreeLeaf 7)))
Mutually Recursive Types - Values

TreeNode
  | More
  | More
  | TreeLeaf
  | 5

TreeNode
  | More
  | More
  | Last
  | TreeLeaf
  | 7

TreeLeaf
  | More
  | Last
  | TreeLeaf
  | 2

TreeLeaf
  | 3
Mutually Recursive Types - Values

A more conventional picture

```
5

3

2
```

7
Mutually Recursive Functions

```ocaml
# let rec fringe tree = 
  match tree with (TreeLeaf x) -> [x] 
| (TreeNode list) -> list_fringe list 
and list_fringe tree_list = 
  match tree_list with (Last tree) -> fringe tree 
| (More (tree,list)) -> 
  (fringe tree) @ (list_fringe list);;

val fringe : 'a tree -> 'a list = <fun>
val list_fringe : 'a treeList -> 'a list = <fun>
```
Mutually Recursive Functions

# fringe tree;;
- : int list = [5; 3; 2; 7]
Problem

# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;

Define tree_size
Problem

# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);

Define tree_size

let rec tree_size t =
    match t with TreeLeaf _ ->
     | TreeNode ts ->
Problem

# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);

Define tree_size

let rec tree_size t =
  match t with TreeLeaf _ -> 1
  | TreeNode ts -> treeList_size ts
  | TreeNode ts -> treeList_size ts
Problem

# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);;

Define tree_size and treeList_size

let rec tree_size t =
  match t with TreeLeaf _ -> 1
  | TreeNode ts -> treeList_size ts

and treeList_size ts =
Problem

# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);

Define tree_size and treeList_size

let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size ts
and treeList_size ts =
    match ts with Last t ->
    | More t ts' ->
Problem

# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);

Define tree_size and treeList_size

let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size ts
and treeList_size ts =
    match ts with Last t -> tree_size t
    | More t ts' -> tree_size t + treeList_size ts'
Problem

# type 'a tree = TreeLeaf of 'a | TreeNode of 'a treeList
and 'a treeList = Last of 'a tree | More of ('a tree * 'a treeList);

Define tree_size and treeList_size

let rec tree_size t =
    match t with TreeLeaf _ -> 1
    | TreeNode ts -> treeList_size ts
and treeList_size ts =
    match ts with Last t -> tree_size t
    | More t ts' -> tree_size t + treeList_size ts'
Nested Recursive Types

# type 'a labeled_tree =
   TreeNode of ('a * 'a labeled_tree list);

  type 'a labeled_tree = TreeNode of ('a * 'a labeled_tree list)
# let ltree =
TreeNode(5,
    [TreeNode (3, []);
     TreeNode (2, [TreeNode (1, []);
                  TreeNode (7, [])]);
     TreeNode (5, [])]);
Nested Recursive Type Values

val ltree : int labeled_tree = 
TreeNode
  (5,
   [TreeNode (3, []); TreeNode (2,
     [TreeNode (1, []); TreeNode (7, [])];
    TreeNode (5, [])])

Nested Recursive Type Values

\[
\text{Ltree} = \text{TreeNode(5)}
\]

\[
\begin{array}{ccc}
\text{TreeNode(3)} & \text{TreeNode(2)} & \text{TreeNode(5)} \\
[ ] & [ ] & [ ]
\end{array}
\]

\[
\begin{array}{ccc}
\text{TreeNode(1)} & \text{TreeNode(7)} \\
[ ] & [ ] & [ ]
\end{array}
\]
Nested Recursive Type Values
Mutually Recursive Functions

# let rec flatten_tree labtree = match labtree with TreeNode (x, treelist) -> x::flatten_tree_list treelist
and flatten_tree_list treelist = match treelist with [] -> [] |
labtree::labtrees -> flatten_tree labtree @ flatten_tree_list labtrees;;
Mutually Recursive Functions

```ocaml
cval flatten_tree : 'a labeled_tree -> 'a list = <fun>
cval flatten_tree_list : 'a labeled_tree list -> 'a list = <fun>

# flatten_tree ltree;;
- : int list = [5; 3; 2; 1; 7; 5]
```

- Nested recursive types lead to mutually recursive functions
Infinite Recursive Values

# let rec ones = 1::ones;;
val ones : int list =  
   [1; 1; 1; 1; ...]
# match ones with x::_ -> x;;

Characters 0-25:
Warning: this pattern-matching is not exhaustive. Here is an example of a value that is not matched: []
   match ones with x::_ -> x;;
   ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
- : int = 1
Infinite Recursive Values

# let rec lab_tree = TreeNode(2, tree_list)
   and tree_list = [lab_tree; lab_tree];;

val lab_tree : int labeled_tree =
   TreeNode (2, [TreeNode(...); TreeNode(...)])

val tree_list : int labeled_tree list =
   [TreeNode (2, [TreeNode(...); TreeNode(...)]);
    TreeNode (2, [TreeNode(...); TreeNode(...)])]
Infinite Recursive Values

# match lab_tree

  with TreeNode (x, _) -> x;;

- : int = 2
Records

- Records serve the same programming purpose as tuples
- Provide better documentation, more readable code
- Allow components to be accessed by label instead of position
  - Labels (aka field names must be unique)
  - Fields accessed by suffix dot notation
Record types must be declared before they can be used in OCaml.

```ocaml
# type person = { name : string; ss : (int * int * int); age : int };;

type person = { name : string; ss : int * int * int; age : int; }
```

- person is the type being introduced
- name, ss and age are the labels, or fields
- Records built with labels; order does not matter

```ocaml
# let teacher = {name = "Elsa L. Gunter"; age = 102; ss = (119,73,6244)};;

val teacher : person =
{name = "Elsa L. Gunter"; ss = (119,73,6244); age = 102}
```
Record Pattern Matching

# let {name = elsa; age = age; ss = (_,_,s3)} = teacher;;

val elsa : string = "Elsa L. Gunter"
val age : int = 102
val s3 : int = 6244
Record Field Access

# let soc_sec = teacher.ss;;

val soc_sec : int * int * int = (119, 73, 6244)
Record Values

# let student = {ss=(325,40,1276); name="Joseph Martins"; age=22};;
val student : person =
 {name = "Joseph Martins"; ss = (325, 40, 1276); age = 22}

# student = teacher;;
- : bool = false
New Records from Old

```oca
# let birthday person = {person with age = person.age + 1};;
val birthday : person -> person = <fun>
# birthday teacher;;
- : person = {name = "Elsa L. Gunter"; ss = (119, 73, 6244); age = 103}
```
New Records from Old

# let new_id name soc_sec person = {person with name = name; ss = soc_sec};;
val new_id : string -> int * int * int -> person -> person = <fun>

# new_id "Guieseppe Martin" (523,04,6712) student;;
- : person = {name = "Guieseppe Martin"; ss = (523, 4, 6712); age = 22}