Solutions for Sample Questions for Midterm 1 (CS 421 Fall 2017)

On the actual midterm, you will have plenty of space to put your answers. The actual midterm will likely have no more than 9 questions plus one extra credit question. In addition to questions of the kind asked below, you should expect to see questions from your MPs and WA1 on the exams. Some of these questions may be reused for the exam.

1. Given the following OCAML code:
   ```ocaml
   let x = 3;;
   let f y = x + y;;
   let x = 5;;
   let z = f 2;;
   let x = "hi";;
   ```
   What value will \( z \) have? Will the last declaration (let \( x = \text{"hi"}; \)) cause a type error? What is the value of \( x \) after this code has been executed?

   **Solution:**
   - \( z \) is bound to 5
   - \( \text{let } x = \text{"hi"}; \) will not cause a type error
   - \( x \) is bound to "hi"

2. What environment is in effect after each declaration in the code in Problem 1? What is the step-by-step formal evaluation of \( f \, 2 \) in the fourth declaration, starting from the environment after the third declaration?

   **Solution:**
   ```ocaml
   let x = 3;;
   \{x \rightarrow 3\}
   
   let f y = x + y;;
   \{f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle, \ x \rightarrow 3\}
   
   let x = 5;;
   \{x \rightarrow 5\} + \{f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle, \ x \rightarrow 3\} =
   \{x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle\}
   
   let z = f 2;;
   \text{Eval}(f, \{x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle\}) =
   \text{App}(\text{Eval}(f, \{x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle\}),
   \text{Eval}(2, \{x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle\})) =
   \text{App}(\langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle, 2) =
   \text{Eval}(x+y, \{y \rightarrow 2\} + \{x \rightarrow 3\}) =
   \text{Eval}(x, \{y \rightarrow 2, x \rightarrow 3\}) + \text{Eval}(y, \{y \rightarrow 2, x \rightarrow 3\}) = 3 + 2 = 5
   
   \{z \rightarrow 5, x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle\}
   ```
let x = "hi";;
{x → "hi"} + {z → 5, x → 5, f → <y → x+y, {x → 3}>} =
{x → "hi", z → 5, f → <y → x+y, {x → 3}>}

3. What is the effect of each of the following pieces of code?
   a. (fun x -> (print_string "a"; x + 2)) (print_string "b"; 4);;
      It prints ba and returns 6
   b. let f = (print_string "a"; fun x -> x + 2) in f (print_string "b"; 4);;
      It prints ab and returns 6
   c. let f = fun g -> (print_string "a"; g 2) in f (fun x -> print_string "b"; 4 + x);;
      It prints ab and returns 6

4. Consider the following two OCaml functions, loop1 and loop2:
   let rec loop1 () = loop1(); ()
   let rec loop2 () = loop2();;
   val loop1 : unit -> unit = <fun>
   val loop2 : unit -> 'a = <fun>

   Suppose you were to run loop1();; and loop2();; in OCaml, (pressing CTRL + C after at
   least a minute to terminate infinite loops when necessary).
   a. For each program, what behavior would you expect to see?
   b. What is the difference between loop1 and loop2?
   c. For each program state if it is:
      i. recursive,
      ii. forward recursive,
      iii. tail-recursive.

   Solution:
   a. The first program generates a stack overflow, while the second program runs
      indefinitely.
   b. Because loop1 is not tail-recursive, each new recursive call must push a new
      activation record onto the stack, hence the stack overflow, but since loop2 is tail-
      recursive, each new activation record may overwrite the previous call, and thus the
      stack does not grow. It should also be observed that loop2 has a more general type
      (:unit -> 'a) than that of loop1:unit -> unit, and hence may be used in places
      where other return types beside unit are required. (Of course, it had better never
      actually be applied.)
   c. Both programs are recursive, and in fact forward recursive, but loop2 is tail-
      recursive while loop1 is not

5. Write an OCAML function pair_up that takes first a function, then an input list and
   returns a list of pairs of an element from input list (the second argument), paired with the
   result of applying the first argument to that element. What is the OCAML type of
   pair_up? What is the result of the following expressions:
   a. pair_up (fun x -> x + 3) [6;4;1];;
   b. pair_up ((fun x -> "Hi, "^x), ["John"; "Mary";"Dana"]);;
   c. pair_up (fun x -> x *. 2.0);;
Solution:

let rec pair_up f l =
  (match l with [] -> []
   | x :: xs -> (x, f x)::pair_up f xs)
 alternately let pair_up f = List.map (fun x -> (x, f x))

pair_up : ('a -> 'b) -> 'a list -> ('a * 'b) list

a. [(6, 9); (4, 7); (1, 4)];;
b. type error
c. A function of type float list -> (float * float) list that returns a list of pairs of an
   element from the input list paired with twice itself.

6. Write an Ocaml function palindrome :string list -> unit that first prints the strings in
   the list from left to right, followed by printing them right to left, recursing over the list
   only once. (Potential extra credit problem: Do this using each of List.fold_right and
   List.fold_left but no explicit use of let rec.)

Solution:

let rec palindrome l =
  match l with [] -> ()
   | s::ss -> (print_string s; palindrome ss; print_string s);;

let rec palindrome l =
  List.fold_right (fun s -> fun print_middle -> (fun () -> (print_string s; print_middle (); print_string s)))
  l
  (fun () -> ());

let palindrome l =
  List.fold_left (fun print_middle -> fun s -> (print_string s; fun () -> (print_middle (print_string s))))
  (fun () -> ())
  l
  ();

7. Using fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b, but without using explicit
   recursion, write a function concat : ' a list list -> 'a list that appends all the lists in the
   input list of lists, preserving the order of elements. You may use the append function @.

Solution: let concat lst = List.fold_right (@) lst [];;

8. Write an Ocaml function list_print : string list -> unit that prints all the strings in a list
   from left to right:
   a. using tail recursion, but no higher order functions,
   b. using fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a but no explicit recursion.

Solution:
a. let rec list_print lst = match lst with [ ] -> ( | s::ss -> (print_string s; list_print ss));
b. let list_print lst = List.fold_left (fun ( ) -> fun s -> print_string s) () lst;;

9. Put the following function in full continuation passing style:
let rec sum_odd n = if n <= 0 then 0 else ((2 * n) – 1) + sum_odd (n – 1);;

Primitive operations (+, -, *, <=) do not have to be converted to CPS, but all procedure calls must be.

Solution:
let add_k a b k = k(a + b)
let minus_k a b k = k(a - b)
let times_k a b k = k(a * b)
let leq_k a b k = k(a <= b)

let rec sum_odd_k n k =
    leq_k n 0 (fun b -> if b then k 0
                        else minus_k n 1
                        (fun d -> sum_odd_k d
                        (fun r -> times_k 2 n
                        (fun t -> minus_k t 1
                        (fun m -> add_k m r k ))))))

10. Write the definition of an OCAML variant type reg_exp to express abstract syntax trees for regular expressions over a base character set of booleans. Thus, a boolean is a reg_exp, epsilon is a reg_exp, the concatenation of two reg_exp’s is a reg_exp, the “choice” of two reg_exp’s is a reg_exp, and the Kleene star of a reg_exp is a reg_exp.

Solution:
type reg_exp =
    Char of bool
    | Epsilon
    | Concat of (reg_exp * reg_exp)
    | Choice of (reg_exp * reg_exp)
    | Kleene_star of reg_exp
    | Paren of reg_exp (* I would accept it with this case missing *)

11. Given the following OCAML datatype:
type int_seq = Null | Snoc of (int_seq * int)
write a tail-recursive function in OCAML all_pos : int_seq -> bool that returns true if every integer in the input int_seq to which all_pos is applied is strictly greater than 0 and false otherwise. Thus all_pos (Snoc(Snoc(Snoc(Null,3),5),7)) should returns true, but all_pos (Snoc(Snoc(Null,~1))) and all_pos (Snoc(Snoc(Null,3),0)) should both return false.

Solution:
let rec all_pos s =
    (match s with Null -> true
     | Snoc(seq, x) -> if x <= 0 then false else all_pos seq);;