

A Natural Semantics

Simple Imperative Programming Language:

$$\begin{aligned}
 & I \in \text{Identifiers} && N \in \text{Integers} \\
 & B ::= \text{true} \mid \text{false} \mid B \& B \mid B \text{ or } B \mid \text{not } E \mid E < E \mid E = E \\
 & E ::= N \mid I \mid E + E \mid E * E \mid E - E \\
 & C ::= \text{skip} \mid I := E \mid C ; C \mid \text{if } B \text{ then } C \text{ else } C \text{ fi} \mid \text{while } B \text{ od } C \text{ od } C
 \end{aligned}$$

where I ranges over program identifiers, E ranges over program arithmetic expressions, B ranges over boolean-valued expressions and C ranges over commands.

Natural Semantics Rules:

Identifiers: $\frac{}{(I, \{m\}) \Downarrow m(I)}$ **Numerals:** $\frac{}{(N, \{m\}) \Downarrow N}$

Booleans: $\frac{}{(\text{true}, \{m\}) \Downarrow \text{true}}$ $\frac{}{(\text{false}, \{m\}) \Downarrow \text{false}}$

$$\frac{(\text{B}, \{m\}) \Downarrow \text{false}}{(\text{B} \& \text{B}', \{m\}) \Downarrow \text{false}} \qquad \frac{(\text{B}, \{m\}) \Downarrow \text{true} \quad (\text{B}', \{m\}) \Downarrow b}{(\text{B} \& \text{B}', \{m\}) \Downarrow b}$$

$$\frac{(\text{B}, \{m\}) \Downarrow \text{true}}{(\text{B} \text{ or } \text{B}', \{m\}) \Downarrow \text{true}} \qquad \frac{(\text{B}, \{m\}) \Downarrow \text{false} \quad (\text{B}', \{m\}) \Downarrow b}{(\text{B} \text{ or } \text{B}', \{m\}) \Downarrow b}$$

$$\frac{(\text{B}, \{m\}) \Downarrow \text{true}}{(\text{not}(\text{B}), \{m\}) \Downarrow \text{false}} \qquad \frac{(\text{B}, \{m\}) \Downarrow \text{false}}{(\text{not}(\text{B}), \{m\}) \Downarrow \text{true}}$$

$$\frac{(\text{E}, \{m\}) \Downarrow U \quad (\text{E}', \{m\}) \Downarrow V \quad U \sim V = b}{(E \sim E', \{m\}) \Downarrow b} \quad \sim \text{ a relation}$$

Arithmetic Operators: $\frac{(\text{E}, \{m\}) \Downarrow U \quad (\text{E}', \{m\}) \Downarrow V \quad U \text{ op } V = b}{(E \sim E', \{m\}) \Downarrow b}$
 (op and arith binary operator)

Commands:

Skip: $\frac{}{(\text{skip}, \{m\}) \Downarrow m}$ **Assignment:** $\frac{(\text{E}, \{m\}) \Downarrow V}{(I ::= \text{E}, \{m\}) \Downarrow m[I \leftarrow V]}$

Commands (cont):

$$\text{Sequencing: } \frac{(C, \{m\}) \Downarrow m' \quad (C', \{m'\}) \Downarrow m''}{(C ; C', \{m\}) \Downarrow m''}$$

If The Else Command:

$$\frac{(B, \{m\}) \Downarrow \text{true} \quad (C, \{m\}) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, \{m\}) \Downarrow m'} \qquad \frac{(B, \{m\}) \Downarrow \text{false} \quad (C', \{m\}) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, \{m\}) \Downarrow m'}$$

While Command:

$$\frac{(B, \{m\}) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, \{m\}) \Downarrow m}$$

$$\frac{(B, \{m\}) \Downarrow \text{true} \quad (C, \{m\}) \Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, \{m'\}) \Downarrow m''}{(\text{while } B \text{ do } C \text{ od}, \{m\}) \Downarrow m''}$$