Programming Languages and Compilers (CS 421)

Sasa Misailovic
4110 SC, UIUC

https://courses.engr.illinois.edu/cs421/fa2017/CS421A

Based in part on slides by Mattox Beckman, as updated by Vikram Adve, Gul Agha, and Elsa L Gunter
BNF Grammars

- Start with a set of characters, $a, b, c, \ldots$
  - We call these *terminals*
- Add a set of different characters, $X, Y, Z, \ldots$
  - We call these *nonterminals*
- One special nonterminal $S$ called *start symbol*
BNF Grammars

- BNF rules (aka *productions*) have form 
  \[ X ::= y \]
  where \( X \) is any nonterminal and \( y \) is a string of terminals and nonterminals

- BNF *grammar* is a set of BNF rules such that every nonterminal appears on the left of some rule
Sample Grammar

- Terminals: 0 1 + ( )
- Nonterminals: <Sum>
- Start symbol = <Sum>

- <Sum> ::= 0
- <Sum> ::= 1
- <Sum> ::= <Sum> + <Sum>
- <Sum> ::= ( <Sum> )
- Can be abbreviated as
  <Sum> ::= 0 | 1
  | <Sum> + <Sum> | ( )
BNF Derivations

- Given rules

  \[ X ::= yZw \text{ and } Z ::= \nu \]

  we may replace \( Z \) by \( \nu \) to say

  \[ X \Rightarrow yZw \Rightarrow y\nu w \]

- Sequence of such replacements called \textit{derivation}

- Derivation called \textit{right-most} if always replace the right-most non-terminal
The meaning of a BNF grammar is the set of all strings consisting only of terminals that can be derived from the Start symbol.
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

Start with the start symbol:

<Sum> =>
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a non-terminal

<Sum> =>
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum>
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a non-terminal:

<Sum> => <Sum> + <Sum>
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute:
  - <Sum> ::= ( <Sum> )

<Sum> => <Sum> + <Sum>

=> ( <Sum> ) + <Sum>
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a non-terminal:

<Sum> => <Sum> + <Sum>
=> ( <Sum> ) + <Sum>
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute:
  - <Sum> ::= <Sum> + <Sum>

<Sum> => <Sum> + <Sum>

=> ( <Sum> ) + <Sum>

=> ( <Sum> + <Sum> ) + <Sum>
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

Pick a non-terminal:

<Sum> => <Sum> + <Sum> 
=> ( <Sum> ) + <Sum>
=> ( <Sum> + <Sum> ) + <Sum>
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute:
  - <Sum> ::= 1

<Sum> => <Sum> + <Sum>

  => ( <Sum> ) + <Sum>

  => ( <Sum> + <Sum> ) + <Sum>

  => ( <Sum> + 1 ) + <Sum>
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a non-terminal:

<Sum> => <Sum> + <Sum>
=> ( <Sum> ) + <Sum>
=> ( <Sum> + <Sum> ) + <Sum>
=> ( <Sum> + 1 ) + <Sum>
BNF Derivations

\(<\text{Sum}\> ::= 0 \mid 1 \mid <\text{Sum}\> + <\text{Sum}\> \mid (<\text{Sum}\>)\)

- Pick a rule and substitute:
  - \(<\text{Sum}\> ::= 0\)

\(<\text{Sum}\> \Rightarrow <\text{Sum}\> + <\text{Sum}\>\)

\[\Rightarrow ( <\text{Sum}\> ) + <\text{Sum}\>\]

\[\Rightarrow ( <\text{Sum}\> + <\text{Sum}\> ) + <\text{Sum}\>\]

\[\Rightarrow ( <\text{Sum}\> + 1 ) + <\text{Sum}\>\]

\[\Rightarrow ( <\text{Sum}\> + 1 ) + 0\]
BNF Derivations

\[ \text{<Sum>} ::= 0 | 1 | \text{<Sum>} + \text{<Sum>} | (\text{<Sum>}) \]

- Pick a non-terminal:

\[
\begin{align*}
\text{<Sum>} & \Rightarrow \text{<Sum>} + \text{<Sum>} \\
& \Rightarrow (\text{<Sum>}) + \text{<Sum>} \\
& \Rightarrow (\text{<Sum>} + \text{<Sum>}) + \text{<Sum>} \\
& \Rightarrow (\text{<Sum>} + 1) + \text{<Sum>} \\
& \Rightarrow (\text{<Sum>} + 1) + 0
\end{align*}
\]
BNF Derivations

<Sum> ::= 0 | 1 | <Sum> + <Sum> | (<Sum>)

- Pick a rule and substitute
  - <Sum> ::= 0

<Sum> => <Sum> + <Sum>
  => ( <Sum> ) + <Sum>
  => ( <Sum> + <Sum> ) + <Sum>
  => ( <Sum> + 1 ) + <Sum>
  => ( <Sum> + 1 ) 0
  => ( 0 + 1 ) + 0
BNF Derivations

\[ <\text{Sum}> ::= 0 | 1 | <\text{Sum}> + <\text{Sum}> | (<\text{Sum}>) \]

\( (0 + 1) + 0 \) is generated by grammar

\[ <\text{Sum}> \Rightarrow <\text{Sum}> + <\text{Sum}> \]
\[ \Rightarrow ( <\text{Sum}> ) + <\text{Sum}> \]
\[ \Rightarrow ( <\text{Sum}> + <\text{Sum}> ) + <\text{Sum}> \]
\[ \Rightarrow ( <\text{Sum}> + 1 ) + <\text{Sum}> \]
\[ \Rightarrow ( <\text{Sum}> + 1 ) + 0 \]
\[ \Rightarrow (0 + 1) + 0 \]
Regular Grammars

- Subclass of BNF
- Only rules of form
  \[ <\text{nonterminal}> ::= <\text{terminal}><\text{nonterminal}> \text{ or } <\text{nonterminal}> ::= <\text{terminal}> \text{ or } <\text{nonterminal}> ::= \varepsilon \]
- Defines same class of languages as regular expressions
- Important for writing lexers (programs that convert strings of characters into strings of tokens)
Example

- Regular grammar:
  \[
  \langle \text{Balanced} \rangle ::= \varepsilon \\
  \langle \text{Balanced} \rangle ::= 0\langle \text{OneAndMore} \rangle \\
  \langle \text{Balanced} \rangle ::= 1\langle \text{ZeroAndMore} \rangle \\
  \langle \text{OneAndMore} \rangle ::= 1\langle \text{Balanced} \rangle \\
  \langle \text{ZeroAndMore} \rangle ::= 0\langle \text{Balanced} \rangle
  \]

- Generates even length strings where every initial substring of even length has same number of 0’s as 1’s
Extended BNF Grammars

- Alternatives: allow rules of from $X ::= y/z$
  - Abbreviates $X ::= y$, $X ::= z$
- Options: $X ::= y[n]z$
  - Abbreviates $X ::= ynz$, $X ::= yz$
- Repetition: $X ::= y\{n\}*z$
  - Can be eliminated by adding new nonterminal $V$ and rules $X ::= yz$, $X ::= yVz$, $V ::= n$, $V ::= nV$
Parse Trees

- Graphical representation of derivation
- Each node labeled with either non-terminal or terminal
- If node is labeled with a terminal, then it is a leaf (no sub-trees)
- If node is labeled with a non-terminal, then it has one branch for each character in the right-hand side of rule used to substitute for it
Example

- Consider grammar:
  ```
  <exp> ::= <factor>
  |   <factor> + <factor>
  <factor> ::= <bin>
  |   <bin> * <exp>
  <bin> ::= 0 | 1
  ```

- Problem: Build parse tree for $1 * 1 + 0$ as an `<exp>`
Example cont.

1 * 1 + 0:  <exp>

<exp> is the start symbol for this parse tree
Example cont.

1 * 1 + 0:  \( <exp> \)

\[ \begin{array}{c}
\text{Use rule: } <exp> ::= <factor>
\end{array} \]
Example cont.

- 1 * 1 + 0:  
  \[
  \begin{array}{c}
  \text{<exp>}
  \\
  \text{<factor>}
  \\
  \text{<bin>} \ast \text{<exp>}
  \end{array}
  \]

Use rule:  \text{<factor> ::= <bin> * <exp>}
Example cont.

- $1 \times 1 + 0$: 
  
  $\text{use rules: } \langle \text{bin} \rangle ::= 1 \text{ and } \\
  \langle \text{exp} \rangle ::= \langle \text{factor} \rangle + \langle \text{factor} \rangle$
Example cont.

1 * 1 + 0:  \( \text{<exp>\text{<factor>\text{<bin>\text{*\text{<exp>\text{<factor>\text{<bin>\text{+\text{<factor>\text{<bin>}}}}}}}}}} \)

Use rule:  \( \text{<factor> ::= <bin>} \)
Example cont.

- $1 * 1 + 0$: 
  
  Use rules: $\text{<bin>} ::= 1 \mid 0$
Example cont.

1 * 1 + 0:  

\[
\text{Fringe of tree is string generated by grammar}
\]
Your Turn: $1 \times 0 + 0 \times 1$

```
<exp> ::= <factor>
    | <factor> + <factor>

<factor> ::= <bin>
    | <bin> * <exp>

<bin> ::= 0 | 1
```
Parse Tree Data Structures

- Parse trees may be represented by OCaml datatypes
- One datatype for each nonterminal
- One constructor for each rule
- Defined as mutually recursive collection of datatype declarations
Example

- Recall grammar:

\[
\text{<exp>} ::= \text{<factor>} \mid \text{<factor>} + \text{<factor>}
\]
\[
\text{<factor>} ::= \text{<bin>} \mid \text{<bin>} * \text{<exp>}
\]
\[
\text{<bin>} ::= 0 \mid 1
\]

- type exp = Factor2Exp of factor
  | Plus of factor * factor
  and factor = Bin2Factor of bin
  | Mult of bin * exp
  and bin = Zero | One
Example cont.

1 * 1 + 0:

\[
\begin{array}{c}
\text{type } \text{exp} = \text{Factor2Exp of factor} \\
\text{Plus of factor * factor} \\
\text{and factor = Bin2Factor of bin} \\
\text{Mult of bin * exp} \\
\text{and bin = Zero | One}
\end{array}
\]

\[
<\text{exp}> \\
| <\text{factor}> \\
| <\text{bin}> * <\text{exp}> \\
| <\text{factor}> + <\text{factor}> \\
| <\text{bin}> \\
| 1 \\
| <\text{bin}> \\
| 1 \\
| 0
\]
Example cont.

- Can be represented as

\[
\text{Factor2Exp} = (\text{Mult}(\text{One}, \\
\quad \text{Plus}(<\text{Bin2Factor}\ One, \\
\quad \quad \text{Bin2Factor}\ Zero>)))
\]
Ambiguous Grammars and Languages

- A BNF grammar is *ambiguous* if its language contains strings for which there is more than one parse tree

- If all BNF’s for a language are ambiguous then the language is *inherently ambiguous*
Example: Ambiguous Grammar

0 + 1 + 0

```
<Sum> + <Sum>
|    |    |
|    |    |
<Sum> + <Sum> 0
|   |
|    |
0    1
```

```
<Sum> + <Sum>
|    |    |
|    |    |
0    <Sum> + <Sum>
|    |    |
|    |    |
1    0
```
Example

- What is the result for:
  \[3 + 4 \times 5 + 6\]
Example

What is the result for:

\[ 3 + 4 \times 5 + 6 \]

Possible answers:

- \[ 41 = ((3 + 4) \times 5) + 6 \]
- \[ 47 = 3 + (4 \times (5 + 6)) \]
- \[ 29 = (3 + (4 \times 5)) + 6 = 3 + ((4 \times 5) + 6) \]
- \[ 77 = (3 + 4) \times (5 + 6) \]
Example

What is the value of:

\[ 7 - 5 - 2 \]
Example

- What is the value of:
  \[ 7 - 5 - 2 \]

- Possible answers:
  - In Pascal, C++, SML assoc. left
    \[ 7 - 5 - 2 = (7 - 5) - 2 = 0 \]
  - In APL, associate to right
    \[ 7 - 5 - 2 = 7 - (5 - 2) = 4 \]
Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator associativity
- Not the only sources of ambiguity
Disambiguating a Grammar

- Given ambiguous grammar $G$, with start symbol $S$, find a grammar $G'$ with same start symbol, such that
  
  $\text{language of } G = \text{language of } G'$

- Not always possible

- No algorithm in general
Disambiguating a Grammar

- Idea: Each non-terminal represents all strings having some property
- Identify these properties (often in terms of things that can’t happen)
- Use these properties to inductively guarantee every string in language has a unique parse
Steps to Grammar Disambiguation

- Identify the rules and a smallest use that display ambiguity
- Decide which parse to keep; why should others be thrown out?
- What syntactic restrictions on subexpressions are needed to throw out the bad (while keeping the good)?
- Add a new non-terminal and rules to describe this set of restricted subexpressions (called stratifying, or refactoring)
- Replace old rules to use new non-terminals
- Rinse and repeat
Example

- **Ambiguous grammar:**
  
  \[
  \text{exp} ::= 0 \mid 1 \mid \text{exp} + \text{exp} \\
  \mid \text{exp} \ast \text{exp}
  \]

- **String with more than one parse:**
  
  \[
  0 + 1 + 0 \\
  1 \ast 1 + 1
  \]

- **Source of ambiguity:** associativity and precedence
Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator associativity
- Not the only sources of ambiguity
How to Enforce Associativity

- Have at most one recursive call per production

- When two or more recursive calls would be natural leave right-most one for right associativity, left-most one for left associativity
Example

- \(<\text{Sum}> ::= 0 \mid 1 \mid <\text{Sum}> + <\text{Sum}> \mid (<\text{Sum}>)

- Becomes
  - \(<\text{Sum}> ::= <\text{Num}> \mid <\text{Num}> + <\text{Sum}>
  - \(<\text{Num}> ::= 0 \mid 1 \mid (<\text{Sum}>)


Operator Precedence

- Operators of highest precedence evaluated first (bind more tightly).

- Precedence for infix binary operators given in following table

- Needs to be reflected in grammar
<table>
<thead>
<tr>
<th></th>
<th>Fortan</th>
<th>Pascal</th>
<th>C/C++</th>
<th>Ada</th>
<th>SML</th>
</tr>
</thead>
<tbody>
<tr>
<td>highest</td>
<td>**</td>
<td>*, /, div, mod</td>
<td>++, --</td>
<td>**</td>
<td>div, mod, /, *</td>
</tr>
<tr>
<td></td>
<td>*,, /</td>
<td>+, -</td>
<td>*,, /, %</td>
<td>*,, /, mod</td>
<td>+, -, ^</td>
</tr>
<tr>
<td></td>
<td>+, -</td>
<td>+, -</td>
<td>+, -</td>
<td>+, -</td>
<td>::</td>
</tr>
</tbody>
</table>
First Example Again

- In any above language, $3 + 4 \times 5 + 6 = 29$
- In APL, all infix operators have same precedence
  - Thus we still don’t know what the value is (handled by associativity)
- How do we handle precedence in grammar?
Predence in Grammar

- Higher precedence translates to longer derivation chain
- Example:
  \[
  <\text{exp}> ::= 0 \mid 1 \mid <\text{exp}> + <\text{exp}>
  \mid <\text{exp}> * <\text{exp}>
  \]
- Becomes
  \[
  <\text{exp}> ::= <\text{mult}_{\text{exp}}>
  \mid <\text{exp}> + <\text{mult}_{\text{exp}}>
  <\text{mult}_{\text{exp}}> ::= <\text{id}> \mid <\text{mult}_{\text{exp}}>*<\text{id}>
  <\text{id}> ::= 0 \mid 1
  \]
Parser Code

- `<grammar>.ml` defines one parsing function per entry point
- Parsing function takes a lexing function (lexer buffer to token) and a lexer buffer as arguments
- Returns semantic attribute of corresponding entry point
Ocamlyacc Input

- File format:

```ocaml
%{
    <header>
%
%
%<
declarations>
%
%
%<
rules>
%
%
%<
trailer>
```
Ocamlyacc <header>

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- <footer> similar. Possibly used to call parser
Ocamlyacc <declarations>

- `%token symbol ... symbol`
  - Declare given symbols as tokens
- `%token <type> symbol ... symbol`
  - Declare given symbols as token constructors, taking an argument of type `<type>`
- `%start symbol ... symbol`
  - Declare given symbols as entry points; functions of same names in `<grammar>.ml`
Ocamlyacc <declarations>

- **%type** `<type> symbol ... symbol`
  Specify type of attributes for given symbols. Mandatory for start symbols

- **%left** `symbol ... symbol`
- **%right** `symbol ... symbol`
- **%nonassoc** `symbol ... symbol`
  Associate precedence and associativity to given symbols. Same line, same precedence; earlier line, lower precedence (broadest scope)
Ocamlyacc <rules>

- `nonterminal`:
  
  ```
  symbol ... symbol { semantic_action }
  | ...
  | symbol ... symbol { semantic_action }
  
  ;
  ```

- Semantic actions are arbitrary Ocaml expressions
- Must be of same type as declared (or inferred) for `nonterminal`
- Access semantic attributes (values) of symbols by position: $1$ for first symbol, $2$ to second ...
Example - Base types

(* File: expr.ml *)

type expr =
  Term_as_expr of term
| Plus.Expr of (term * expr)
| Minus.Expr of (term * expr)

and term =
  Factor_as_Term of factor
| Mul_Term of (factor * term)
| Div_Term of (factor * term)

and factor =
  Id_as_Factor of string
| Parenthesized.Expr_as_Factor of expr
Example - Lexer (exprlex.mll)

```mll
{ (*open Exprrparse*) }
let numeric = ['0' - '9']
let letter = ['a' - 'z' 'A' - 'Z']
rule token = parse
  | "\+" {Plus_token}
  | ",\-" {Minus_token}
  | ",\*" {Times_token}
  | ",/" {Divide_token}
  | "\(\" {Left_parenthesis}
  | ",\)\" {Right_parenthesis}
  | letter (letter|numeric|"\_")* as id {Id_token id}
  | [' ' \t ' \n'] {token lexbuf}
  | eof {EOL}
```
Example - Parser (exprparse.mly)

{% open Expr
%
%token <string> Id_token
%token Left_parenthesis Right_parenthesis
%token Times_token Divide_token
%token Plus_token Minus_token
%token EOL
%start main
%type <expr> main
%%
Example - Parser (exprparse.mly)

expr:
    term
      { Term_as_Expr $1 }
    | term Plus_token expr
      { Plus_Expr ($1, $3) }
    | term Minus_token expr
      { Minus_Expr ($1, $3) }

Example - Base types

(* File: expr.ml *)

type expr =
  Term_as_Expr of term
| Plus_Expr of (term * expr)
| Minus_Expr of (term * expr)
|
Example - Parser (exprparse.mly)

term:
  factor
  { Factor_as_Term $1 }  
| factor Times_token term
  { Mult_Term ($1, $3) }  
| factor Divide_token term
  { Div_Term ($1, $3) }

Example - Base types

(* File: expr.ml *)
type expr =
  Term_as.Expr of term
| Plus.Expr of (term * expr)
| Minus.Expr of (term * expr)
and term =
  Factor_as.Term of factor
| Mult.Term of (factor * term)
| Div.Term of (factor * term)
Example - Parser (exprparse.mly)

factor:
   Id_token
       { Id_as_Factor $1 }
   | Left_parenthesis expr Right_parenthesis
       { Parenthesized_Expr_as_Factor $2 }

main:
   | expr EOL
     { $1 }

Example - Base types

(* File: expr.ml *)
type expr =
    Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
    Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
and factor =
    Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
Example - Using Parser

```plaintext
# #use "expr.ml";;
...
# #use "exprparse.ml";;
...
# #use "exprlex.ml";;
...
# let test s =
    let lexbuf = Lexing.from_string (s^"\n") in
    main token lexbuf;;
```
Example - Using Parser

# test "a + b";;
- : expr =
  Plus_Expr
  (Factor_as_Term (Id_as_Factor "a"),
   Term_as_Expr
    (Factor_as_Term (Id_as_Factor "b")))
LR Parsing

- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced
Example

\[(0 + 1) + 0\]
Example

\[(0 + 1) + 0\]
Example

\[
\langle \text{Sum} \rangle = \left( 0 + 1 \right) + 0
\]
Example

\[
\langle \text{Sum} \rangle \quad 0 + 1 \quad ) + 0
\]
Example

\[ \langle \text{Sum} \rangle (0 + 1) + 0 \]
Example

\[ \langle \text{Sum} \rangle \quad 0 \quad + \quad \langle \text{Sum} \rangle \quad 1 \quad + \quad 0 \]

10/31/2017 78
Example

\[
( \langle \text{Sum} \rangle 0 + \langle \text{Sum} \rangle 1 ) + 0
\]
Example

\[(0 + 1) + 0\]
Example

\[ \text{<Sum>} \]

\[ (0 + 1) + 0 \]
Example

\[
\langle \text{Sum} \rangle \quad \langle \text{Sum} \rangle \\
\langle \text{Sum} \rangle \\
(0 + 1) + 0
\]
Example

\[
\begin{align*}
\langle \text{Sum} \rangle & + 0 \\
\langle \text{Sum} \rangle & + 1 \\
\langle \text{Sum} \rangle & + 0
\end{align*}
\]
Example

\[
\left( 0 + 1 \right) + 0
\]
Example

( \langle \text{Sum} \rangle \langle \text{Sum} \rangle \langle \text{Sum} \rangle \langle \text{Sum} \rangle \langle \text{Sum} \rangle \langle \text{Sum} \rangle ) + 0 + 1 + 0
Example

\[
\left( \sum_0 \quad + \quad \sum_1 \quad \right) \quad + \quad \sum_0
\]
LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
  - This is the hardest part, we omit here
  - Rows labeled by states
  - For Action, columns labeled by terminals and “end-of-tokens” marker
    - (more generally strings of terminals of fixed length)
  - For Goto, columns labeled by non-terminals
Action and Goto Tables

- Given a state and the next input, Action table says either
  - **shift** and go to state \( n \), or
  - **reduce** by production \( k \) (explained in a bit)
  - **accept** or **error**

- Given a state and a non-terminal, Goto table says
  - go to state \( m \)
Example: $<\text{Sum}> = 0 \mid 1 \mid (<\text{Sum}>)$

$<\text{Sum}> \Rightarrow$

$= \bullet (0 + 1) + 0$  

$\text{shift}$
Example: $<\text{Sum}> = 0 \mid 1 \mid ( <\text{Sum}> ) \mid <\text{Sum}> + <\text{Sum}>$

$<\text{Sum}> \implies$

\[
= ( \textcolor{red}{0 + 1} ) + 0 \quad \text{shift}
\]
\[
= \textcolor{red}{0 + 1} + 0 \quad \text{shift}
\]
Example: \(<\text{Sum}> = 0 \mid 1 \mid (<\text{Sum}>)) \mid <\text{Sum}> + <\text{Sum}>\)

\(<\text{Sum}> => \)

\[
\begin{align*}
=> & \ (0 \bullet + 1) + 0 & \text{reduce} \\
= & \ (\bullet 0 + 1) + 0 & \text{shift} \\
= & \ (\bullet (0 + 1) + 0) & \text{shift}
\end{align*}
\]
Example: \(<\text{Sum}> = 0 \mid 1 \mid (\langle\text{Sum}\rangle)\)
\mid \langle\text{Sum}\rangle + \langle\text{Sum}\rangle\)

\(<\text{Sum}>) =>

= (\langle\text{Sum}\rangle * 1 + 1) + 0 \quad \text{shift}
= (0 * 1 + 1) + 0 \quad \text{reduce}
= (0 + 1) + 0 \quad \text{shift}
= 0 + 0 \quad \text{shift}
Example: \(<\text{Sum}\> = 0 \mid 1 \mid (\langle\text{Sum}\rangle) \mid \langle\text{Sum}\rangle + \langle\text{Sum}\rangle\)

\(<\text{Sum}\> \Rightarrow \)

\=

\[
= (\langle\text{Sum}\rangle + \bullet 1) + 0 \quad \text{shift}
\]

\[
= (\langle\text{Sum}\rangle \bullet + 1) + 0 \quad \text{shift}
\]

\[
=> (0 \bullet + 1) + 0 \quad \text{reduce}
\]

\[
= (0 + 1) + 0 \quad \text{shift}
\]

\[
= \bullet (0 + 1) + 0 \quad \text{shift}
\]
Example: \(<\text{Sum}\> = 0 \mid 1 \mid (<\text{Sum}\>)\) 
\mid <\text{Sum}\> + <\text{Sum}\> 

\(<\text{Sum}\> \Rightarrow \)

\[\Rightarrow ( <\text{Sum}\> + 1 ) + 0 \quad \text{reduce}\]
\[= ( <\text{Sum}\> + 1 ) + 0 \quad \text{shift}\]
\[= ( <\text{Sum}\> + 1 ) + 0 \quad \text{shift}\]
\[\Rightarrow ( 0 + 1 ) + 0 \quad \text{reduce}\]
\[= ( 0 + 1 ) + 0 \quad \text{shift}\]
\[= ( 0 + 1 ) + 0 \quad \text{shift}\]
Example: $<\text{Sum}> = 0 \mid 1 \mid (<\text{Sum}>)$
\[ <\text{Sum}> + <\text{Sum}> \]

$<\text{Sum}> \Rightarrow$

\[
=> ( <\text{Sum}> + <\text{Sum}> \bullet ) + 0 \quad \text{reduce}
\]
\[
=> ( <\text{Sum}> + 1 \bullet ) + 0 \quad \text{reduce}
\]
\[
= ( <\text{Sum}> + \bullet 1 ) + 0 \quad \text{shift}
\]
\[
= ( <\text{Sum}> \bullet + 1 ) + 0 \quad \text{shift}
\]
\[
=> ( 0 \bullet + 1 ) + 0 \quad \text{reduce}
\]
\[
= ( \bullet 0 + 1 ) + 0 \quad \text{shift}
\]
\[
= \bullet ( 0 + 1 ) + 0 \quad \text{shift}
\]
Example: $<\text{Sum}> = 0 \mid 1 \mid (\langle<\text{Sum}＞\rangle)$
$\mid <\text{Sum}> + <\text{Sum}>$

$<\text{Sum}> \Rightarrow$

$$= (\langle<\text{Sum}＞\rangle \bullet) + 0 \quad \text{shift}$$
$=> (\langle<\text{Sum}＞ + <\text{Sum}＞ \bullet \rangle) + 0 \quad \text{reduce}$
$=> (\langle<\text{Sum}＞ + 1 \bullet \rangle) + 0 \quad \text{reduce}$
$= (\langle<\text{Sum}＞ + \bullet 1 \rangle) + 0 \quad \text{shift}$
$= (\langle<\text{Sum}＞ \bullet + 1 \rangle) + 0 \quad \text{shift}$
$=> (0 \bullet + 1) + 0 \quad \text{reduce}$
$= (\bullet 0 + 1) + 0 \quad \text{shift}$
$= \bullet (0 + 1) + 0 \quad \text{shift}$
Example: \(<\text{Sum}> = 0 \mid 1 \mid (\langle\text{Sum}\rangle)\) \\
\mid \langle\text{Sum}\rangle + \langle\text{Sum}\rangle\)

\[
\langle\text{Sum}\rangle \Rightarrow
\]

\[
\Rightarrow (\langle\text{Sum}\rangle) \bullet + 0 \quad \text{reduce}
\]

\[
= (\langle\text{Sum}\rangle \bullet) + 0 \quad \text{shift}
\]

\[
\Rightarrow (\langle\text{Sum}\rangle + \langle\text{Sum}\rangle \bullet) + 0 \quad \text{reduce}
\]

\[
\Rightarrow (\langle\text{Sum}\rangle + 1 \bullet) + 0 \quad \text{reduce}
\]

\[
= (\langle\text{Sum}\rangle + \bullet 1) + 0 \quad \text{shift}
\]

\[
= (\langle\text{Sum}\rangle \bullet + 1) + 0 \quad \text{shift}
\]

\[
\Rightarrow (0 \bullet + 1) + 0 \quad \text{reduce}
\]

\[
= (\bullet 0 + 1) + 0 \quad \text{shift}
\]

\[
= \bullet (0 + 1) + 0 \quad \text{shift}
\]
Example: \( <\text{Sum}> = 0 \mid 1 \mid ( <\text{Sum}> ) \mid <\text{Sum}> + <\text{Sum}> \)

\[ <\text{Sum}> \Rightarrow \]

\[ = \quad <\text{Sum}> \bullet + 0 \quad \text{shift} \]
\[ => ( \quad <\text{Sum}> \bullet \quad + 0 \quad \text{reduce} \]
\[ = \quad ( \quad <\text{Sum}> \bullet \quad ) + 0 \quad \text{shift} \]
\[ => ( \quad <\text{Sum}> + <\text{Sum}> \bullet \quad ) + 0 \quad \text{reduce} \]
\[ => ( \quad <\text{Sum}> + 1 \bullet \quad ) + 0 \quad \text{reduce} \]
\[ = \quad ( \quad <\text{Sum}> + \bullet 1 \quad ) + 0 \quad \text{shift} \]
\[ = \quad ( \quad <\text{Sum}> \bullet + 1 \quad ) + 0 \quad \text{shift} \]
\[ => ( \quad 0 \bullet + 1 \quad ) + 0 \quad \text{reduce} \]
\[ = \quad ( \bullet 0 + 1 \quad ) + 0 \quad \text{shift} \]
\[ = \quad \bullet ( \quad 0 + 1 \quad ) + 0 \quad \text{shift} \]
Example: \(<\text{Sum}> = 0 \mid 1 \mid (<\text{Sum}>))
\mid <\text{Sum}> + <\text{Sum}>\)

\begin{align*}
<\text{Sum}> & \Rightarrow \\
& = <\text{Sum}> + \bullet 0 \quad \text{shift} \\
& = <\text{Sum}> \bullet + 0 \quad \text{shift} \\
& \Rightarrow ( <\text{Sum}> ) \bullet + 0 \quad \text{reduce} \\
& = ( <\text{Sum}> \bullet ) + 0 \quad \text{shift} \\
& \Rightarrow ( <\text{Sum}> + <\text{Sum}> \bullet ) + 0 \quad \text{reduce} \\
& \Rightarrow ( <\text{Sum}> + 1 \bullet ) + 0 \quad \text{reduce} \\
& = ( <\text{Sum}> + \bullet 1 ) + 0 \quad \text{shift} \\
& = ( <\text{Sum}> \bullet + 1 ) + 0 \quad \text{shift} \\
& \Rightarrow ( 0 \bullet + 1 ) + 0 \quad \text{reduce} \\
& = ( \bullet 0 + 1 ) + 0 \quad \text{shift} \\
& = \bullet ( 0 + 1 ) + 0 \quad \text{shift}
\end{align*}
Example: $<\text{Sum}> = 0 \mid 1 \mid ( <\text{Sum}> ) \\
\mid <\text{Sum}> + <\text{Sum}>$

$<\text{Sum}> \Rightarrow$

$=> <\text{Sum}> + 0$  \hspace{1cm} \text{reduce}

$= <\text{Sum}> + 0$  \hspace{1cm} \text{shift}

$= <\text{Sum}> + 0$  \hspace{1cm} \text{shift}

$=> ( <\text{Sum}> ) + 0$  \hspace{1cm} \text{reduce}

$= ( <\text{Sum}> ) + 0$  \hspace{1cm} \text{shift}

$=> ( <\text{Sum}> + <\text{Sum}> ) + 0$  \hspace{1cm} \text{reduce}

$=> ( <\text{Sum}> + 1 ) + 0$  \hspace{1cm} \text{reduce}

$= ( <\text{Sum}> + 1 ) + 0$  \hspace{1cm} \text{shift}

$= ( <\text{Sum}> + 1 ) + 0$  \hspace{1cm} \text{shift}

$=> ( 0 + 1 ) + 0$  \hspace{1cm} \text{reduce}

$= ( 0 + 1 ) + 0$  \hspace{1cm} \text{shift}

$= ( 0 + 1 ) + 0$  \hspace{1cm} \text{shift}
Example: \( <\text{Sum}> = 0 \mid 1 \mid ( <\text{Sum}> ) \mid <\text{Sum}> + <\text{Sum}> \)

\[
<\text{Sum}> \Rightarrow <\text{Sum}> + <\text{Sum}> \qquad \text{reduce}
\]
\[
= <\text{Sum}> + 0 \qquad \text{reduce}
\]
\[
= <\text{Sum}> + 0 \qquad \text{shift}
\]
\[
= <\text{Sum}> + 0 \qquad \text{shift}
\]
\[
= ( <\text{Sum}> ) + 0 \qquad \text{reduce}
\]
\[
= ( <\text{Sum}> ) + 0 \qquad \text{shift}
\]
\[
= ( <\text{Sum}> + <\text{Sum}> ) + 0 \qquad \text{reduce}
\]
\[
= ( <\text{Sum}> + 1 ) + 0 \qquad \text{reduce}
\]
\[
= ( <\text{Sum}> + 1 ) + 0 \qquad \text{shift}
\]
\[
= ( <\text{Sum}> + 1 ) + 0 \qquad \text{shift}
\]
\[
= ( 0 + 1 ) + 0 \qquad \text{reduce}
\]
\[
= ( 0 + 1 ) + 0 \qquad \text{shift}
\]
\[
= ( 0 + 1 ) + 0 \qquad \text{shift}
\]
Example: \(<\text{Sum}> = 0 \mid 1 \mid (<\text{Sum}>) \mid \text{<Sum>} + \text{<Sum>}\)

\(<\text{Sum}> \bullet \Rightarrow \text{<Sum>} + \text{<Sum>} \bullet \quad \text{reduce} \)
\Rightarrow \text{<Sum>} + 0 \bullet \quad \text{reduce} \)
\text{= \<Sum>} + \bullet 0 \quad \text{shift} \)
\text{= \<Sum>} \bullet + 0 \quad \text{shift} \)
\Rightarrow ( \text{<Sum>} ) \bullet + 0 \quad \text{reduce} \)
\text{= ( \<Sum> \bullet ) + 0 \quad shift} \)
\Rightarrow ( \text{<Sum>} + \text{<Sum>} \bullet ) + 0 \quad \text{reduce} \)
\Rightarrow ( \text{<Sum>} + 1 \bullet ) + 0 \quad \text{reduce} \)
\text{= ( \<Sum> + \bullet 1 ) + 0 \quad shift} \)
\text{= ( \<Sum> \bullet + 1 ) + 0 \quad shift} \)
\Rightarrow ( 0 \bullet + 1 ) + 0 \quad \text{reduce} \)
\text{= ( \bullet 0 + 1 ) + 0 \quad shift} \)
\text{= \bullet ( 0 + 1 ) + 0 \quad shift} \)
LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals
LR(i) Parsing Algorithm

0. Insure token stream ends in special “end-of-tokens” symbol

1. Start in state 1 with an empty stack

2. Push state(1) onto stack

3. Look at next i tokens from token stream (toks) (don’t remove yet)

4. If top symbol on stack is state(n), look up action in Action table at (n, toks)
LR(i) Parsing Algorithm

5. If action = \textbf{shift} \( m \),
   a) Remove the top token from token stream and push it onto the stack
   b) Push \textbf{state}(m) onto stack
   c) Go to step 3
LR(i) Parsing Algorithm

6. If action = reduce \( k \) where production \( k \) is \( E ::= u \)

   a) Remove \( 2 \times \text{length}(u) \) symbols from stack (\( u \) and all the interleaved states)

   b) If new top symbol on stack is \( \text{state}(m) \), look up new state \( \rho \) in \( \text{Goto}(m,E) \)

   c) Push \( E \) onto the stack, then push \( \text{state}(\rho) \) onto the stack

   d) Go to step 3
LR(i) Parsing Algorithm

7. If action = accept
   ■ Stop parsing, return success

8. If action = error,
   ■ Stop parsing, return failure
Adding Synthesized Attributes

- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
  - gather the recorded attributes from each non-terminal popped from stack
  - Compute new attribute for non-terminal pushed onto stack
Shift-Reduce Conflicts

- **Problem**: can’t decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar
Example: \(<\text{Sum}> = 0 \mid 1 \mid (\langle \text{Sum} \rangle) \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle\)

- \(0 + 1 + 0\) \hspace{1cm} \text{shift}
- \(0 \circ + 1 + 0\) \hspace{1cm} \text{reduce}
- \(\langle \text{Sum} \rangle \circ + 1 + 0\) \hspace{1cm} \text{shift}
- \(\langle \text{Sum} \rangle + 1 \circ + 0\) \hspace{1cm} \text{shift}
- \(\langle \text{Sum} \rangle + 1 \circ + 0\) \hspace{1cm} \text{reduce}
- \(\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \circ + 0\)
Example - cont

- **Problem:** shift or reduce?

- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce

- Shift first - right associative
- Reduce first - left associative
Reduce - Reduce Conflicts

- **Problem:** can’t decide between two different rules to reduce by
- Again caused by ambiguity in grammar
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors
Example

- $S ::= A | aB$
- $A ::= abc$
- $B ::= bc$

- $\text{abc}$: shift
- $a \text{bc}$: shift
- $ab \text{c}$: shift
- $\text{abc}$: shift

Problem: reduce by $B ::= bc$ then by $S ::= aB$, or by $A ::= abc$ then $S ::= A$?