Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Natural Semantics

- Aka Structural Operational Semantics, aka "Big Step Semantics"
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$

Simple Imperative Programming Language

- $I \in Identifiers$
- $N \in Numerals$
- B ::= true | false | B & B | B or B | not B | $E < E$ | $E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- C::= skip | $C;C$ | I ::= E | if B then C else C fi | while B do C od

Natural Semantics of Atomic Expressions

- Identifiers: $(I,m) \Downarrow m(I)$
- Numerals are values: $(N,m) \Downarrow N$
- Booleans: $(\text{true},m) \Downarrow \text{true}$

  $(\text{false} ,m) \Downarrow \text{false}$

Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \mathbin{\&} B', m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \mathbin{\&} B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \text{ or } B', m) \Downarrow \text{true}} \qquad \frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \text{ or } B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}} \qquad \frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$

Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation ~ hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V

Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \; op \; V = N}{(E \; op \; E', m) \Downarrow N}$$

where N is the specified value for $U \; op \; V$

Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment:

$$\frac{(E,m) \Downarrow V}{(I::=E,m) \Downarrow m[I \texttt{<--} V\,]}$$

Sequencing:

$$\frac{(C,m) \Downarrow m' \quad (C',m') \Downarrow m''}{(C;C', m) \Downarrow m''}$$

If Then Else Command

$$\frac{(B,m) \Downarrow \text{true} \quad (C,m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

$$\frac{(B,m) \Downarrow \text{false} \quad (C',m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

While Command

$$\frac{(B,m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B,m)\Downarrow\text{true} \quad (C,m)\Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m')\Downarrow m''}{(\text{while } B \text{ do C od}, m) \Downarrow m''}$$

Example: If Then Else Rule

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: If Then Else Rule

$$\frac{\overline{(x > 5, \{x \to 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi}, \{x \to 7\}) \Downarrow\ ?}$$

Example: Arith Relation

$$\dfrac{\dfrac{\begin{array}{c}? > ? = ?\\ (x,\{x\text{->}7\})\Downarrow ? \quad (5,\{x\text{->}7\})\Downarrow ?\end{array}}{(x > 5,\ \{x \text{ -> } 7\})\Downarrow ?}}{\begin{array}{c}(\text{if } x > 5 \text{ then } y{:}= 2 + 3 \text{ else } y{:}=3 + 4 \text{ fi},\\ \{x \text{ -> } 7\}) \Downarrow ?\end{array}}$$

Example: Identifier(s)

7 > 5 = true

(x,{x->7})⇓7   (5,{x->7})⇓5

(x > 5, {x -> 7})⇓?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example: Arith Relation

$$\frac{\dfrac{7 > 5 = \text{true}}{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}}{\dfrac{(x > 5,\ \{x \text{->} 7\})\Downarrow \text{true}}{(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},\ \{x \text{->} 7\}) \Downarrow\ ?}}$$

Example: If Then Else Rule

$$\dfrac{\dfrac{\dfrac{7 > 5 = \text{true}}{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5}}{(x > 5, \{x \text{->} 7\})\Downarrow \text{true}} \qquad \dfrac{}{(y{:}= 2 + 3, \{x\text{->} 7\} \Downarrow ? \quad .}}{(\text{if } x > 5 \text{ then } y{:}= 2 + 3 \text{ else } y{:}=3 + 4 \text{ fi}, \{x \text{->} 7\}) \Downarrow ?}$$

Example: Assignment

$$\dfrac{\dfrac{7 > 5 = \text{true} \quad \dfrac{}{(x,\{x\text{->}7\})\Downarrow 7} \quad \dfrac{}{(5,\{x\text{->}7\})\Downarrow 5}}{(x > 5, \{x \text{->} 7\})\Downarrow \text{true}} \qquad \dfrac{\dfrac{}{(2+3, \{x\text{->}7\})\Downarrow ?}}{(y:= 2 + 3, \{x\text{->} 7\}) \Downarrow ?}}{(\text{if } x > 5 \text{ then } y:= 2 + 3 \text{ else } y:=3 + 4 \text{ fi}, \{x \text{->} 7\}) \Downarrow ?}$$

Example: Arith Op

? + ? = ?

(2,{x->7})⇓?   (3,{x->7}) ⇓?

---

7 > 5 = true   (2+3, {x->7})⇓?

(x,{x->7})⇓7   (5,{x->7})⇓5

(x > 5, {x -> 7})⇓true   (y:= 2 + 3, {x-> 7} ⇓? .

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7}) ⇓ ?

Example: Numerals

2 + 3 = 5

(2,{x->7})⇓2 (3,{x->7}) ⇓3

---

7 > 5 = true (2+3, {x->7})⇓?

(x,{x->7})⇓7 (5,{x->7})⇓5

(y:= 2 + 3, {x-> 7}

(x > 5, {x -> 7})⇓true ⇓ ?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓ ?

Example: Arith Op

$$2 + 3 = 5$$

$$\underline{(2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\})\Downarrow 3}$$

$$7 > 5 = \text{true} \qquad \underline{(2+3,\ \{x\text{->}7\})\Downarrow 5}$$

$$\underline{(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5} \qquad (y{:=}\ 2 + 3,\ \{x\text{->}\ 7\}$$

$$\underline{(x > 5,\ \{x \text{->}\ 7\})\Downarrow \text{true} \qquad \Downarrow ?}$$

$$(\text{if } x > 5 \text{ then } y{:=}\ 2 + 3 \text{ else } y{:=}3 + 4 \text{ fi},$$

$$\{x \text{->}\ 7\})\ \Downarrow\ ?$$

Example: Assignment

2 + 3 = 5

(2,{x->7})⇓2 (3,{x->7}) ⇓3

---

7 > 5 = true (2+3, {x->7})⇓5

(x,{x->7})⇓7 (5,{x->7})⇓5 (y:= 2 + 3, {x-> 7}

(x > 5, {x -> 7})⇓true ⇓ {x->7, y->5}

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7}) ⇓?

Example: If Then Else Rule

$$2 + 3 = 5$$

$$(2,\{x\text{->}7\})\Downarrow 2 \quad (3,\{x\text{->}7\})\Downarrow 3$$

$$7 > 5 = \text{true} \qquad (2+3, \{x\text{->}7\})\Downarrow 5$$

$$(x,\{x\text{->}7\})\Downarrow 7 \quad (5,\{x\text{->}7\})\Downarrow 5 \qquad (y{:}= 2 + 3, \{x\text{->} 7\}$$

$$(x > 5, \{x \text{->} 7\})\Downarrow \text{true} \qquad \Downarrow \{x\text{->}7, y\text{->}5\}$$

$$(\text{if } x > 5 \text{ then } y{:}= 2 + 3 \text{ else } y{:}=3 + 4 \text{ fi},$$

$$\{x \text{->} 7\}) \Downarrow \{x\text{->}7, y\text{->}5\}$$

Let in Command

$$\frac{(E,m) \Downarrow v \quad (C,m[I\text{<-}v]) \Downarrow m'}{(\text{let } I = E \text{ in } C,\ m) \Downarrow m''}$$

Where $m''(y) = m'(y)$ for $y \neq I$ and $m''(I) = m(I)$ if $m(I)$ is defined, and $m''(I)$ is undefined otherwise

Example

$$\dfrac{(5,\{x\text{->}17\}) \Downarrow 5 \qquad \dfrac{\dfrac{(x,\{x\text{->}5\}) \Downarrow 5 \quad (3,\{x\text{->}5\}) \Downarrow 3}{(x+3,\{x\text{->}5\}) \Downarrow 8}}{(x{:=}x+3,\{x\text{->}5\}) \Downarrow \{x\text{->}8\}}}{(\text{let } x = 5 \text{ in } (x{:=}x+3),\ \{x \text{ -> } 17\}) \Downarrow\ ?}$$

Example

$$\dfrac{(5,\{x\text{->}17\}) \Downarrow 5 \qquad \dfrac{\dfrac{(x,\{x\text{->}5\}) \Downarrow 5 \quad (3,\{x\text{->}5\}) \Downarrow 3}{(x+3,\{x\text{->}5\}) \Downarrow 8}}{(x:=x+3,\{x\text{->}5\}) \Downarrow \{x\text{->}8\}}}{(\text{let } x = 5 \text{ in } (x:=x+3),\ \{x \text{ -> } 17\}) \Downarrow \{x\text{->}17\}}$$

Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explictly*
- Clash of constructs apparent in awkward semantics

Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed

Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
  - Start with literals
  - Variables
  - Primitive operations
  - Evaluation of expressions
  - Evaluation of commands/declarations

Interpreter

- Takes abstract syntax trees as input
  - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
  - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
  - To get final value, put in a loop

Natural Semantics Example

- compute_exp (Var(v), m) = look_up v m
- compute_exp (Int(n), _) = Num (n)
- …
- compute_com(IfExp(b,c1,c2),m) =

  if compute_exp (b,m) = Bool(true)

  then compute_com (c1,m)

  else compute_com (c2,m)

Natural Semantics Example

- compute_com(While(b,c), m) =

  if compute_exp (b,m) = Bool(false)

  then m

  else compute_com

  (While(b,c), compute_com(c,m))

```
compute_com(While(b,c), m) =
    if compute_exp (b,m) = Bool(false)
    then m
    else compute_com
            (While(b,c), compute_com(c,m))
```

- May fail to terminate - exceed stack limits
- Returns no useful information then

Transition Semantics

- Form of operational semantics
- Describes how each program construct transforms machine state by *transitions*
- Rules look like

  (C, *m*) --> (C', *m'*)   or   (C,*m*) --> *m'*
- *C, C'* is code remaining to be executed
- m, m' represent the state/store/memory/ environment
  - Partial mapping from identifiers to values
  - Sometimes *m* (or *C*) not needed
- Indicates exactly one step of computation

Expressions and Values

- *C, C'* used for commands; *E, E' for* expressions; *U,V* for values
- Special class of expressions designated as *values*
  - Eg 2, 3 are values, but 2+3 is only an expression
- Memory only holds values
  - Other possibilities exist

Evaluation Semantics

- Transitions successfully stops when E/*C* is a value/memory
- Evaluation fails if no transition possible, but not at value/memory
- Value/memory is the final *meaning* of original expression/command (in the given state)
- Coarse semantics: final value / memory
- More fine grained: whole transition sequence

Simple Imperative Programming Language

- $I \in Identifiers$
- $N \in Numerals$
- B ::= true | false | B & B | B or B | not B | $E < E$ | $E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- C::= skip | $C;C$ | I ::= E
  | if B then C else C fi | while B do C od

Transitions for Expressions

- Numerals are values

- Boolean values = {true, false}

- Identifiers: (I,m) --> $(m(I), m)$

Boolean Operations:

- Operators: (short-circuit)

(false & *B*, *m*) --> (false,*m*)

(true & *B*, *m*) --> (*B*,*m*)

$$\frac{(B, m) \text{ --> } (B'', m)}{(B \,\&\, B', m) \text{ --> } (B'' \,\&\, B', m)}$$

(true or *B*, *m*) --> (true,*m*)

(false or *B*, *m*) --> (*B*,*m*)

$$\frac{(B, m) \text{ --> } (B'', m)}{(B \text{ or } B', m) \text{ --> } (B'' \text{ or } B', m)}$$

(not true, m) --> (false,*m*)

(not false, m) --> (true,*m*)

$$\frac{(B, m) \text{ --> } (B', m)}{(\text{not } B, m) \text{ --> } (\text{not } B', m)}$$

Relations

$$\frac{(E,\ m) \dashrightarrow (E'',m)}{(E \sim E',\ m) \dashrightarrow (E''\sim E',m)}$$

$$\frac{(E,\ m) \dashrightarrow (E',m)}{(V \sim E,\ m) \dashrightarrow (V\sim E',m)}$$

$(U \sim V,\ m)$ --> (true,m) or (false,m) depending on whether $U \sim V$ holds or not

Arithmetic Expressions

$$\frac{(E, m) \dashrightarrow (E'', m)}{(E \; op \; E', m) \dashrightarrow (E'' \; op \; E', m)}$$

$$\frac{(E, m) \dashrightarrow (E', m)}{(V \; op \; E, m) \dashrightarrow (V \; op \; E', m)}$$

$(U \; op \; V, m) \dashrightarrow (N, m)$ where N is the specified value for $U \; op \; V$

Commands - in English

- skip means done evaluating
- When evaluating an assignment, evaluate the expression first
- If the expression being assigned is already a value, update the memory with the new value for the identifier
- When evaluating a sequence, work on the first command in the sequence first
- If the first command evaluates to a new memory (ie completes), evaluate remainder with new memory

Commands

$$(\text{skip}, m) \dashrightarrow m$$

$$\frac{(E,m) \dashrightarrow (E',m)}{(I::=E,m) \dashrightarrow (I::=E',m)}$$

$$(I::=V,m) \dashrightarrow m[I \leftarrow V]$$

$$\frac{(C,m) \dashrightarrow (C'',m')}{(C;C', m) \dashrightarrow (C'';C',m')} \qquad \frac{(C,m) \dashrightarrow m'}{(C;C', m) \dashrightarrow (C',m')}$$

If Then Else Command - in English

- If the boolean guard in an if_then_else is true, then evaluate the first branch
- If it is false, evaluate the second branch
- If the boolean guard is not a value, then start by evaluating it first.

If Then Else Command

(if true then C else C' fi, m) --> (C, m)

(if false then C else C' fi, m) --> (C', m)

$$\frac{(B, m) \dashrightarrow (B', m)}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m)}$$

--> (if B' then C else C' fi, m)

While Command

(while *B* do *C* od, *m*) -->

(if *B* then *C*; while *B* do *C* od else skip fi, m)

In English: Expand a While into a test of the boolean guard, with the true case being to do the body and then try the while loop again, and the false case being to stop.

Example Evaluation

- First step:

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7})

--> ?

Example Evaluation

- First step:

(x > 5, {x -> 7}) --> ?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7})

--> ?

Example Evaluation

- First step:

(x,{x -> 7}) --> (7, {x -> 7})

(x > 5, {x -> 7}) --> ?

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7})

--> ?

Example Evaluation

- First step:

$$\frac{\dfrac{(x,\{x \to 7\}) \dashrightarrow (7, \{x \to 7\})}{(x > 5, \{x \to 7\}) \dashrightarrow (7 > 5, \{x \to 7\})}}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi}, \{x \to 7\}) \dashrightarrow\ ?}$$

(x,{x -> 7}) --> (7, {x -> 7})

(x > 5, {x -> 7}) --> (7 > 5, {x -> 7})

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,

{x -> 7})

--> ?

Example Evaluation

- First step:

(x,{x -> 7}) --> (7, {x -> 7})

(x > 5, {x -> 7}) --> (7 > 5, {x -> 7})

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7})
--> (if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi,
{x -> 7})

Example Evaluation

- Second Step:

  (7 > 5, {x -> 7}) --> (true, {x -> 7})

  (if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi,
  {x -> 7})
  --> (if true then y:=2 + 3 else y:=3 + 4 fi,
  {x -> 7})

- Third Step:

  (if true then y:=2 + 3 else y:=3 + 4 fi, {x -> 7})
  -->(y:=2+3, {x->7})

Example Evaluation

- Fourth Step:

$$\frac{(2+3, \{x \text{->} 7\}) \text{ --> } (5, \{x \text{ ->} 7\})}{(y:=2+3, \{x\text{->}7\}) \text{ --> } (y:=5, \{x\text{->}7\})}$$

- Fifth Step:

$$(y:=5, \{x\text{->}7\}) \text{ --> } \{y \text{ ->} 5, x \text{ ->} 7\}$$

Example Evaluation

- Bottom Line:

(if x > 5 then y:= 2 + 3 else y:=3 + 4 fi,
{x -> 7})

--> (if 7 > 5 then y:=2 + 3 else y:=3 + 4 fi,
{x -> 7})

-->(if true then y:=2 + 3 else y:=3 + 4 fi,
{x -> 7})

-->(y:=2+3, {x->7})

--> (y:=5, {x->7}) --> {y -> 5, x -> 7}

Transition Semantics Evaluation

- A sequence of steps with trees of justification for each step

(C_1,m_1) --> (C_2,m_2) --> (C_3,m_3) --> … --> m

- Let -->* be the transitive closure of -->
- Ie, the smallest transitive relation containing -->

Adding Local Declarations

- Add to expressions:
- E ::= … | let I = E in E' | fun I -> E | E E'
- fun I -> E is a value
- Could handle local binding using state, but have assumption that evaluating expressions doesn't alter the environment
- We will use substitution here instead
- **Notation**: $E\,[E' / I\,]$ means replace all free occurrence of I by E' in E

Call-by-value (Eager Evaluation)

$$(\text{let } I = V \text{ in } E, m) \text{ --> } (E[V/I], m)$$

$$\frac{(E, m) \text{ --> } (E'', m)}{(\text{let } I = E \text{ in } E', m) \text{ --> } (\text{let } I = E'' \text{ in } E')}$$

$$((\text{fun } I \text{ -> } E)\ V, m) \text{ --> } (E[V / I], m)$$

$$\frac{(E', m) \text{ --> } (E'', m)}{((\text{fun } I \text{ -> } E)\ E', m) \text{ --> } ((\text{fun } I \text{ -> } E)\ \text{E}'', m)}$$

Call-by-name (Lazy Evaluation)

- (let I = E in E', m) --> (E' [E / I],m)

- ((fun I -> E') E, m) --> (E' [E / I],m)

- Question: Does it make a difference?
- It can depending on the language

Church-Rosser Property

- Church-Rosser Property: If E-->* E_1 and E-->* E_2, if there exists a value V such that E_1 -->* V, then E_2 -->* V
- Also called **confluence** or **diamond property**
- Example: E= 2 + 3 + 4

E_1 = 5 + 4 E_2= 2 + 7

V =9

Does It always Hold?

- No. Languages with side-effects tend not be Church-Rosser with the combination of call-by-name and call-by-value
- Alonzo Church and Barkley Rosser proved in 1936 the λ-calculus does have it
- Benefit of Church-Rosser: can check equality of terms by evaluating them (Given evaluation strategy might not terminate, though)

Transition Semantics for λ-Calculus

- Application (version 1)

$$(\lambda\ x\ .\ E)\ E' \dashrightarrow E[E'/x]$$

- Application (version 2)

$$(\lambda\ x\ .\ E)\ V \dashrightarrow E[V/x]$$

$$\frac{E' \dashrightarrow E''}{(\lambda\ x\ .\ E)\ E' \dashrightarrow (\lambda\ x\ .\ E)\ E''}$$