Programming Languages and Compilers (CS 421)

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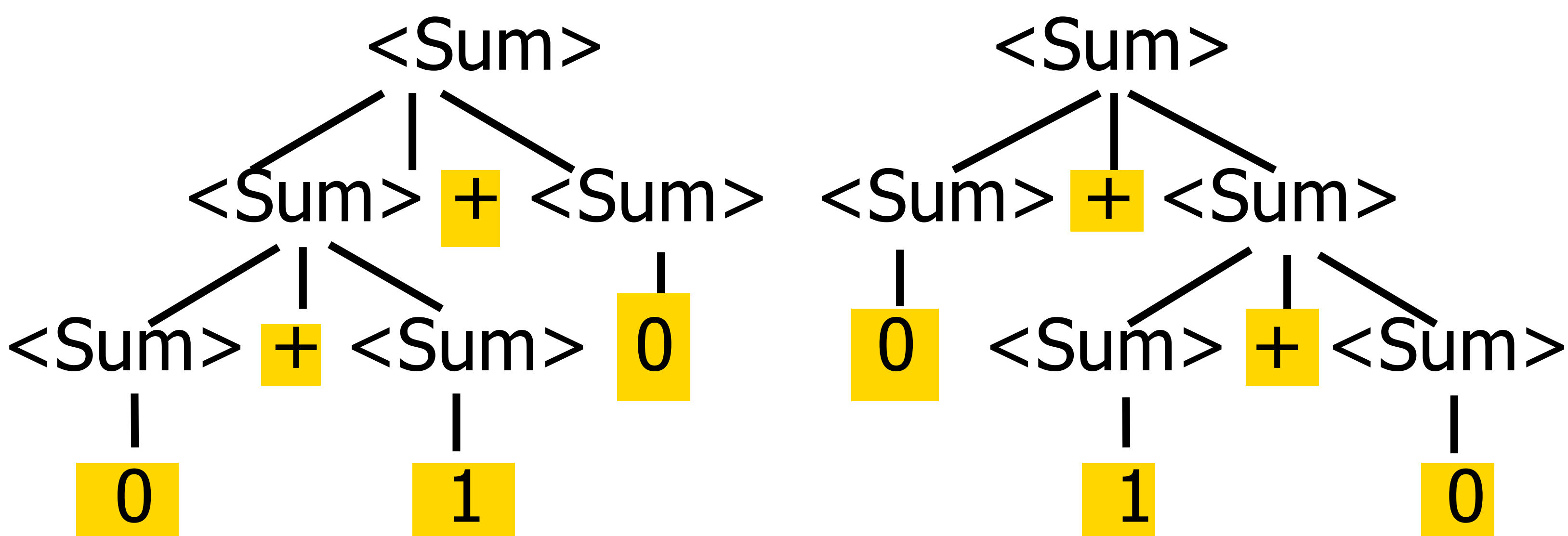
Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

Ambiguous Grammars and Languages

- A BNF grammar is *ambiguous* if its language contains strings for which there is more than one parse tree
- If all BNF's for a language are ambiguous then the language is *inherently ambiguous*

Example: Ambiguous Grammar

- 0 + 1 + 0



Example

- What is the result for:

$$3 + 4 * 5 + 6$$

Example

- What is the result for:

$$3 + 4 * 5 + 6$$

- Possible answers:
  - 41 = ((3 + 4) * 5) + 6
  - 47 = 3 + (4 * (5 + 6))
  - 29 = (3 + (4 * 5)) + 6 = 3 + ((4 * 5) + 6)
  - 77 = (3 + 4) * (5 + 6)

Example

- What is the value of:

$$7 - 5 - 2$$

Example

- What is the value of:

$$7 - 5 - 2$$

- Possible answers:
  - In Pascal, C++, SML assoc. left

  $7 - 5 - 2 = (7 - 5) - 2 = 0$
  - In APL, associate to right

  $7 - 5 - 2 = 7 - (5 - 2) = 4$

Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator assoicativity

- Not the only sources of ambiguity

Disambiguating a Grammar

- Given ambiguous grammar G, with start symbol S, find a grammar G′ with same start symbol, such that

  language of G = language of G′

- Not always possible
- No algorithm in general

Disambiguating a Grammar

- Idea: Each non-terminal represents all strings having some property
- Identify these properties (often in terms of things that can’t happen)
- Use these properties to inductively guarantee every string in language has a unique parse

Steps to Grammar Disambiguation

- Identify the rules and a smallest use that display ambiguity
- Decide which parse to keep; why should others be thrown out?
- What syntactic restrictions on subexpressions are needed to throw out the bad (while keeping the good)?
- Add a new non-terminal and rules to describe this set of restricted subexpressions (called stratifying, or refactoring)
- Replace old rules to use new non-terminals
- Rinse and repeat

Example

- Ambiguous grammar:
  <exp> ::= 0 | 1 | <exp> + <exp>
  | <exp> * <exp>
- String with more then one parse:
  0 + 1 + 0
  1 * 1 + 1
- Sourceof ambiuity: associativity and precedence

Two Major Sources of Ambiguity

- Lack of determination of operator precedence
- Lack of determination of operator assoicativity

- Not the only sources of ambiguity

How to Enforce Associativity

- Have at most one recursive call per production

- When two or more recursive calls would be natural, leave right-most one for right associativity, left-most one for left associativity

Example

- <Sum> ::= 0 | 1 | <Sum> + <Sum>
  | (<Sum>)
- Becomes
  - <Sum> ::= <Num> | <Num> + <Sum>
  - <Num> ::= 0 | 1 | (<Sum>)

Operator Precedence

- Operators of highest precedence evaluated first (bind more tightly).
- Precedence for infix binary operators given in following table
- Needs to be reflected in grammar

Precedence Table - Sample

	Fortan	Pascal	C/C++	Ada	SML
highest	**	*, /, div, mod	++, --	**	div, mod, / , *
	*, /	+, -	*, /, %	*, /, mod	+, -, ^
	+, -		+, -	+, -	::

First Example Again

- In any above language, 3 + 4 * 5 + 6 = 29
- In APL, all infix operators have same precedence
  - Thus we still don't know what the value is (handled by associativity)
- How do we handle precedence in grammar?

Predence in Grammar

- Higher precedence translates to longer derivation chain
- Example:

<exp> ::= 0 | 1 | <exp> + <exp>
| <exp> * <exp>

- Becomes

<exp> ::= <mult_exp>
| <exp> + <mult_exp>
<mult_exp> ::= <id> | <mult_exp> * <id>
<id> ::= 0 | 1

Recursive Descent Parsing

- Recursive descent parsers are a class of parsers derived fairly directly from BNF grammars

- A recursive descent parser traces out a parse tree in top-down order, corresponding to a left-most derivation (LL - left-to-right scanning, leftmost derivation)

Recursive Descent Parsing

- Each nonterminal in the grammar has a subprogram associated with it; the subprogram parses all phrases that the nonterminal can generate

- Each nonterminal in right-hand side of a rule corresponds to a recursive call to the associated subprogram

Recursive Descent Parsing

- Each subprogram must be able to decide how to begin parsing by looking at the left-most character in the string to be parsed
  - May do so directly, or indirectly by calling another parsing subprogram
- Recursive descent parsers, like other top-down parsers, cannot be built from left-recursive grammars
  - Sometimes can modify grammar to suit

Sample Grammar

<expr> ::= <term> | <term> + <expr>
| <term> - <expr>

<term> ::= <factor> | <factor> * <term>
| <factor> / <term>

<factor> ::= <id> | (<expr>)

Tokens as OCaml Types

- \+ - * / ( ) <id>
- Becomes an OCaml datatype

```
type token =
    Id_token of string
  | Left_parenthesis | Right_parenthesis
  | Times_token | Divide_token
  | Plus_token | Minus_token
```

Parse Trees as Datatypes

<expr> ::= <term> | <term> + <expr>
| <term> - <expr>

```
type expr =
    Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
```

Parse Trees as Datatypes

```
<term> ::= <factor> | <factor> * <term>
             | <factor> / <term>

and term =
    Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
```

Parse Trees as Datatypes

<factor> ::= <id> | (<expr>)

```
and factor =
    Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```

Parsing Lists of Tokens

- Will create three mutually recursive functions:
  - expr : token list -> (expr * token list)
  - term : token list -> (term * token list)
  - factor : token list -> (factor * token list)
- Each parses what it can and gives back parse and remaining tokens

Parsing an Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
   with ( term_parse , tokens_after_term) ->
     (match tokens_after_term
      with( Plus_token  :: tokens_after_plus) ->
```

Parsing an Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
    with ( term_parse , tokens_after_term) ->
      (match tokens_after_term
       with ( Plus_token  :: tokens_after_plus) ->
```

Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
    with ( term_parse , tokens_after_term) ->
      (match tokens_after_term
        with ( Plus_token  :: tokens_after_plus) ->
```

Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
    with ( term_parse , tokens_after_term) ->
      (match tokens_after_term
        with ( Plus_token  :: tokens_after_plus) ->
```

Parsing a Plus Expression

<expr> ::= <term> [(+ | -) <expr>]

```
let rec expr tokens =
  (match term tokens
   with ( term_parse , tokens_after_term) ->
     (match tokens_after_term
      with ( Plus_token  :: tokens_after_plus) ->
```

Parsing a Plus Expression

<expr> ::= <term> + <expr>

```
(match expr tokens_after_plus
 with ( expr_parse  , tokens_after_expr) ->
( Plus_Expr  ( term_parse  ,  expr_parse ),
 tokens_after_expr))
```

Parsing a Plus Expression

<expr> ::= <term> + <expr>

(match expr tokens_after_plus

with (expr_parse , tokens_after_expr) ->

(Plus_Expr (term_parse , expr_parse),

tokens_after_expr))

Building Plus Expression Parse Tree

<expr> ::= <term> + <expr>

(match expr tokens_after_plus

with (expr_parse , tokens_after_expr) ->

(Plus_Expr (term_parse , expr_parse),

tokens_after_expr))

Parsing a Minus Expression

<expr> ::= <term> - <expr>

```
| ( Minus_token :: tokens_after_minus) ->
   (match expr tokens_after_minus
   with ( expr_parse , tokens_after_expr) ->
  ( Minus_Expr ( term_parse , expr_parse ),
  tokens_after_expr))
```

Parsing a Minus Expression

```
<expr> ::= <term> - <expr>

| ( Minus_token :: tokens_after_minus) ->
  (match expr tokens_after_minus
  with ( expr_parse , tokens_after_expr) ->
 ( Minus_Expr ( term_parse , expr_parse ),
  tokens_after_expr))
```

Parsing an Expression as a Term

<expr> ::= <term>

| _ -> (Term_as_Expr term_parse , tokens_after_term)))

- Code for **term** is same except for replacing addition with multiplication and subtraction with division

Parsing Factor as Id

<factor> ::= <id>

```
and factor  tokens =
 (match tokens
  with (Id_token id_name :: tokens_after_id) =
   ( Id_as_Factor  id_name, tokens_after_id)
```

Parsing Factor as Parenthesized Expression

<factor> ::= (<expr>)

| factor (Left_parenthesis :: tokens) =

(match expr tokens

with (expr_parse , tokens_after_expr) ->

Parsing Factor as Parenthesized Expression

<factor> ::= (<expr>)

(match tokens_after_expr

with Right_parenthesis :: tokens_after_rparen ->

(Parenthesized_Expr_as_Factor expr_parse ,
tokens_after_rparen)

Error Cases

- What if no matching right parenthesis?

  | _ -> raise (Failure "No matching rparen")))

- What if no leading id or left parenthesis?

| _ -> raise (Failure "No id or lparen"));;

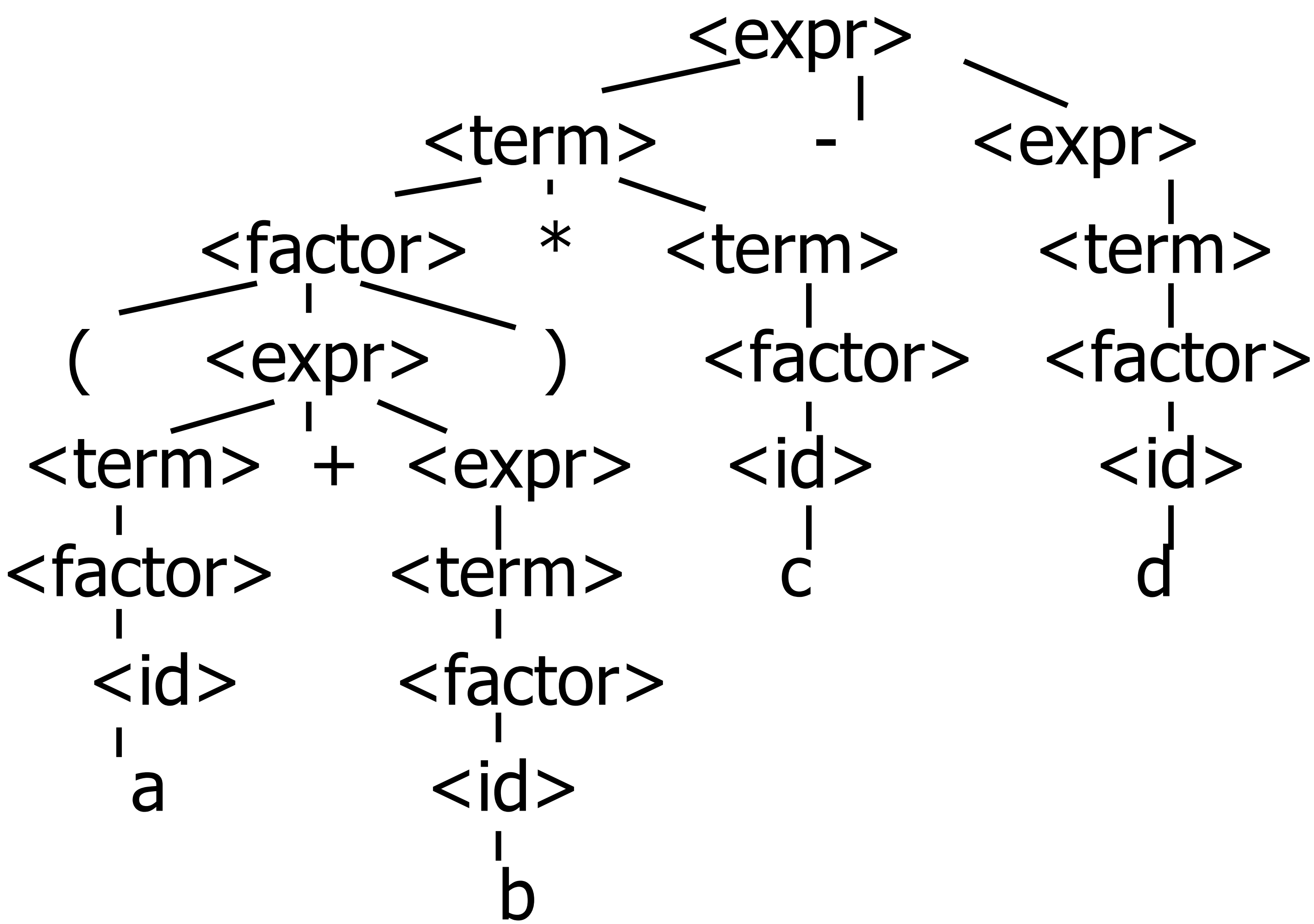
(a + b) * c - d

expr [Left_parenthesis; Id_token "a”; Plus_token; Id_token "b”; Right_parenthesis; Times_token; Id_token "c”; Minus_token; Id_token "d"];;

(a + b) * c - d

```
- : expr * token list =
(Minus_Expr
  (Mult_Term
    (Parenthesized_Expr_as_Factor
      (Plus_Expr
        (Factor_as_Term (Id_as_Factor "a"),
         Term_as_Expr (Factor_as_Term
   (Id_as_Factor "b")))),
     Factor_as_Term (Id_as_Factor "c")),
   Term_as_Expr (Factor_as_Term (Id_as_Factor
   "d"))),
 [])
```

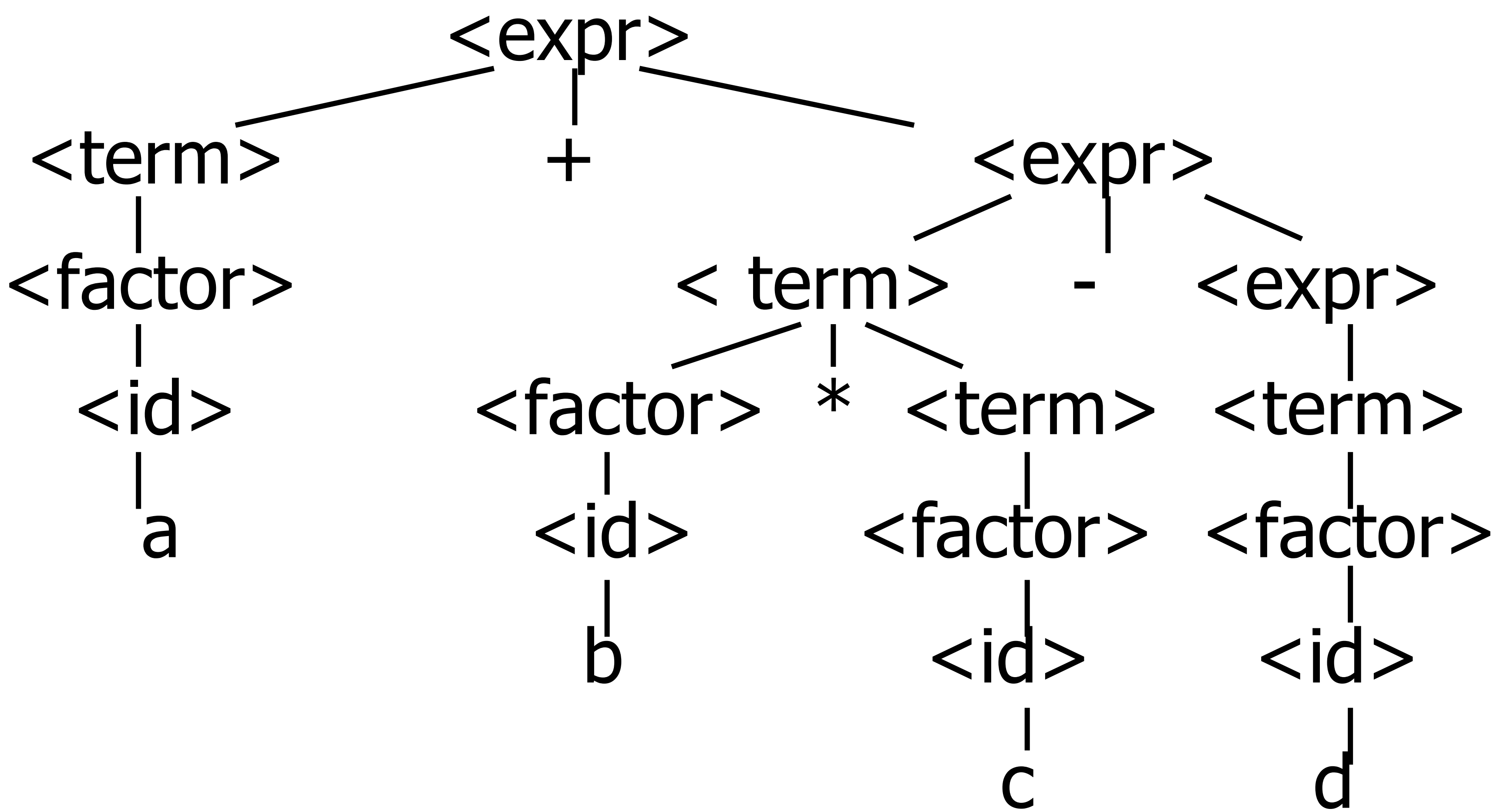
(a + b) * c – d


<expr>
<term>
-
<expr>
<factor>
*
<term>
<term>
(
<expr>
)
<factor>
<factor>
<term>
+
<expr>
<id>
<id>
<factor>
<term>
c
d
<id>
<factor>
a
<id>
b

a + b * c – d

```
# expr [Id_token "a”; Plus_token; Id_token "b”;
   Times_token; Id_token "c”; Minus_token;
      Id_token "d"];;
- : expr * token list =
(Plus_Expr
  (Factor_as_Term (Id_as_Factor "a"),
   Minus_Expr
    (Mult_Term (Id_as_Factor "b", Factor_as_Term
   (Id_as_Factor "c")),
     Term_as_Expr (Factor_as_Term (Id_as_Factor
   "d")))),
 [])
```

a + b * c – d


<expr>
<term>
+
<expr>
<factor>
< term>
-
<expr>
<id>
<factor>
*
<term>
<term>
a
<id>
<factor>
<factor>
b
<id>
<id>
c
d

(a + b * c - d

expr [Left_parenthesis; Id_token "a”; Plus_token; Id_token "b”; Times_token; Id_token "c”; Minus_token; Id_token "d"];;

Exception: Failure "No matching rparen".

Can’t parse because it was expecting a right parenthesis but it got to the end without finding one

a + b) * c - d *)

expr [Id_token "a”; Plus_token; Id_token "b”; Right_parenthesis; Times_token; Id_token "c”; Minus_token; Id_token "d"];;

- : expr * token list =

(Plus_Expr

(Factor_as_Term (Id_as_Factor "a"),

Term_as_Expr (Factor_as_Term (Id_as_Factor "b"))),

[Right_parenthesis; Times_token; Id_token "c"; Minus_token; Id_token "d"])

Parsing Whole String

- Q: How to guarantee whole string parses?
- A: Check returned tokens empty

```
let parse tokens  =
    match expr tokens
     with (expr_parse, []) -> expr_parse
     | _ -> raise (Failure “No parse");;
```

- Fixes <expr> as start symbol

Streams in Place of Lists

- More realistically, we don't want to create the entire list of tokens before we can start parsing
- We want to generate one token at a time and use it to make one step in parsing
- Will use (token * (unit -> token)) or (token * (unit -> token option))

  in place of token list

Problems for Recursive-Descent Parsing

- Left Recursion:

  A ::= Aw

  translates to a subroutine that loops forever
- Indirect Left Recursion:

  A ::= Bw

  B ::= Av

  causes the same problem

Problems for Recursive-Descent Parsing

- Parser must always be able to choose the next action based only only the very next token

- Pairwise Disjointedness Test: Can we always determine which rule (in the non-extended BNF) to choose based on just the first token

Pairwise Disjointedness Test

- For each rule

  A ::= y

Calculate

FIRST (y) =

$\{a \mid y \Rightarrow^* aw\} \cup \{\varepsilon \mid \text{if } y \Rightarrow^* \varepsilon\}$

- For each pair of rules A ::= y and A ::= z, require FIRST(y) $\cap$ FIRST(z) = { }

Example

Grammar:

<S> ::= <A> a <B> b

<A> ::= <A> b | b

<B> ::= a <B> | a

FIRST (<A> b) = {b}

FIRST (b) = {b}

Rules for <A> not pairwise disjoint

Eliminating Left Recursion

- Rewrite grammar to shift left recursion to right recursion
  - Changes associativity
- Given

<expr> ::= <expr> + <term> and

<expr> ::= <term>

- Add new non-terminal <e> and replace above rules with

<expr> ::= <term><e>

<e> ::= + <term><e> | ε

Factoring Grammar

- Test too strong: Can't handle

  <expr> ::= <term> [(+ | -) <expr>]

- Answer: Add new non-terminal and replace above rules by

  <expr> ::= <term><e>

  <e> ::= + <term><e>

  <e> ::= - <term><e>

  <e> ::= ε

- You are delaying the decision point

Example

Both <A> and <B> have problems:

<S> ::= <A> a <B> b

<A> ::= <A> b | b

<B> ::= a <B> | a

Transform grammar to:

<S> ::= <A> a <B> b

<A> ::-= b<A1>

<A1> :: b<A1> | ε

<B> ::= a<B1>

<B1> ::= a<B1> | ε

Ocamlyacc Input

- File format:

%{
  <*header*>
%}
  <*declarations*>
%%
  <*rules*>
%%
  <*trailer*>

Ocamlyacc <*header*>

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- <*footer*> similar. Possibly used to call parser

Ocamlyacc <declarations>

- %token *symbol … symbol*
- Declare given symbols as tokens
- %token <*type*> *symbol … symbol*
- Declare given symbols as token constructors, taking an argument of type <*type*>
- %start *symbol … symbol*
- Declare given symbols as entry points; functions of same names in <*grammar*>.ml

Ocamlyacc <*declarations*>

- %type <*type*> *symbol … symbol*

  Specify type of attributes for given symbols. Mandatory for start symbols
- %left *symbol … symbol*
- %right *symbol … symbol*
- %nonassoc *symbol … symbol*

  Associate precedence and associativity to given symbols. Same line,same precedence; earlier line, lower precedence (broadest scope)

Ocamlyacc <*rules*>

- *nonterminal* :

  symbol ... *symbol* { *semantic_action* }

  | ...

  | *symbol* ... *symbol* { *semantic_action* }

  ;
- Semantic actions are arbitrary Ocaml expressions
- Must be of same type as declared (or inferred) for *nonterminal*
- Access semantic attributes (values) of symbols by position: $1 for first symbol, $2 to second …

Example - Base types

```
(* File: expr.ml *)
type expr =
    Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
    Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
and factor =
    Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```

Example - Lexer (exprlex.mll)

```
{ (*open Exprparse*) }
let numeric = ['0' - '9']
let letter =['a' - 'z' 'A' - 'Z']
rule token = parse
  | "+" {Plus_token}
  | "-"  {Minus_token}
  | "*"  {Times_token}
  | "/"  {Divide_token}
  | "("  {Left_parenthesis}
  | ")"  {Right_parenthesis}
  | letter (letter|numeric|"_")* as id  {Id_token id}
  | [' ' '\t' '\n'] {token lexbuf}
  | eof {EOL}
```

Example - Parser (exprparse.mly)

```
%{ open Expr
%}
%token <string> Id_token
%token Left_parenthesis Right_parenthesis
%token Times_token Divide_token
%token Plus_token Minus_token
%token EOL
%start main
%type <expr> main
%%
```

Example - Parser (exprparse.mly)

```
expr:
   term
       { Term_as_Expr $1 }
 | term Plus_token expr
       { Plus_Expr ($1, $3) }
 | term Minus_token expr
       { Minus_Expr ($1, $3) }
```

Example - Parser (exprparse.mly)

```
term:
  factor
      { Factor_as_Term $1 }
 | factor Times_token term
      { Mult_Term ($1, $3) }
 | factor Divide_token term
      { Div_Term ($1, $3) }
```

Example - Parser (exprparse.mly)

```
factor:
   Id_token
       { Id_as_Factor $1 }
 | Left_parenthesis expr Right_parenthesis
       {Parenthesized_Expr_as_Factor $2 }
main:
 | expr EOL
       { $1 }
```

Example - Using Parser

```
# #use "expr.ml";;
…
# #use "exprparse.ml";;
…
# #use "exprlex.ml";;
…
# let test s =
  let lexbuf = Lexing.from_string (s^"\n") in
      main token lexbuf;;
```

Example - Using Parser

```
# test "a + b";;
- : expr =
Plus_Expr
 (Factor_as_Term (Id_as_Factor "a"),
  Term_as_Expr (Factor_as_Term
   (Id_as_Factor "b")))
```