

Programming Languages and Compilers (CS 421)

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Based in part on slides by Mattox Beckman, as updated by Vikram Adve and Gul Agha

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Two Problems

- Type checking
 - Question: Does exp. e have type τ in env Γ ?
 - Answer: Yes / No
 - Method: Type **derivation**
- Typability
 - Question Does exp. e have **some type** in env. Γ ?
If so, what is it?
 - Answer: Type τ / error
 - Method: Type **inference**

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Type Inference - Outline

- Begin by assigning a type variable as the type of the whole expression
- Decompose the expression into component expressions
- Use typing rules to generate constraints on components and whole
- Recursively find substitution that solves typing judgment of first subcomponent
- Apply substitution to next subcomponent and find substitution solving it; compose with first, etc.
- Apply comp of all substitution to orig. type var. to get answer

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Type Inference - Example

- What type can we give to
(fun x -> fun f -> f (f x))
- Start with a type variable and then look at the way the term is constructed

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Type Inference - Example

- First approximate:

$$\{ \} \vdash (\text{fun } x \rightarrow \text{fun } f \rightarrow f (f x)) : \alpha$$
- Second approximate: use fun rule

$$\frac{\{x : \beta\} \vdash (\text{fun } f \rightarrow f (f x)) : \gamma}{\{ \} \vdash (\text{fun } x \rightarrow \text{fun } f \rightarrow f (f x)) : \alpha}$$
- Remember constraint $\alpha \equiv (\beta \rightarrow \gamma)$

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Type Inference - Example

- Third approximate: use fun rule

$$\frac{\frac{\{f : \delta ; x : \beta\} \vdash f (f x) : \varepsilon}{\{x : \beta\} \vdash (\text{fun } f \rightarrow f (f x)) : \gamma}}{\{ \} \vdash (\text{fun } x \rightarrow \text{fun } f \rightarrow f (f x)) : \alpha}$$
- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Fourth approximate: use app rule

$$\frac{\frac{\frac{\{f:\delta; x:\beta\}|- f : \varphi \rightarrow \varepsilon \quad \{f:\delta; x:\beta\}|- f x : \varphi}{\{f : \delta ; x : \beta \} |- (f (f x)) : \varepsilon}}{\{x : \beta \} |- (fun f -> f (f x)) : \gamma}}{\{ \} |- (fun x -> fun f -> f (f x)) : \alpha}$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Fifth approximate: use var rule, get constraint $\delta \equiv \varphi \rightarrow \varepsilon$, Solve with same
- Apply to next sub-proof

$$\frac{\frac{\frac{\frac{\{f:\delta; x:\beta\}|- f : \varphi \rightarrow \varepsilon \quad \{f:\delta; x:\beta\}|- f x : \varphi}{\{f : \delta ; x : \beta \} |- (f (f x)) : \varepsilon}}{\{x : \beta \} |- (fun f -> f (f x)) : \gamma}}{\{ \} |- (fun x -> fun f -> f (f x)) : \alpha}}$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst: $\{\delta \equiv \varphi \rightarrow \varepsilon\}$

$$\frac{\frac{\frac{\frac{\dots \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\}|- f x : \varphi}{\{f : \delta ; x : \beta \} |- (f (f x)) : \varepsilon}}{\{x : \beta \} |- (fun f -> f (f x)) : \gamma}}{\{ \} |- (fun x -> fun f -> f (f x)) : \alpha}}$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst: $\{\delta \equiv \varphi \rightarrow \varepsilon\}$

$$\frac{\frac{\frac{\frac{\frac{\{f:\varphi \rightarrow \varepsilon; x:\beta\}|- f:\zeta \rightarrow \varphi \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\}|- x:\zeta}{\dots \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\}|- f x : \varphi}}{\{f : \delta ; x : \beta \} |- (f (f x)) : \varepsilon}}{\{x : \beta \} |- (fun f -> f (f x)) : \gamma}}{\{ \} |- (fun x -> fun f -> f (f x)) : \alpha}}$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst: $\{\delta \equiv \varphi \rightarrow \varepsilon\}$
- Var rule: Solve $\zeta \rightarrow \varphi \equiv \varphi \rightarrow \varepsilon$ **Unification**

$$\frac{\frac{\frac{\frac{\frac{\{f:\varphi \rightarrow \varepsilon; x:\beta\}|- f:\zeta \rightarrow \varphi \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\}|- x:\zeta}{\dots \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\}|- f x : \varphi}}{\{f : \delta ; x : \beta \} |- (f (f x)) : \varepsilon}}{\{x : \beta \} |- (fun f -> f (f x)) : \gamma}}{\{ \} |- (fun x -> fun f -> f (f x)) : \alpha}}$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst: $\{\zeta \equiv \varepsilon, \varphi \equiv \varepsilon\} \circ \{\delta \equiv \varphi \rightarrow \varepsilon\}$
- Var rule: Solve $\zeta \rightarrow \varphi \equiv \varphi \rightarrow \varepsilon$ **Unification**

$$\frac{\frac{\frac{\frac{\frac{\{f:\varphi \rightarrow \varepsilon; x:\beta\}|- f:\zeta \rightarrow \varphi \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\}|- x:\zeta}{\dots \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\}|- f x : \varphi}}{\{f : \delta ; x : \beta \} |- (f (f x)) : \varepsilon}}{\{x : \beta \} |- (fun f -> f (f x)) : \gamma}}{\{ \} |- (fun x -> fun f -> f (f x)) : \alpha}}$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst: $\{\zeta \equiv \varepsilon, \varphi \equiv \varepsilon, \delta \equiv \varepsilon \rightarrow \varepsilon\}$
- Apply to next sub-proof

$$\frac{\dots \quad \{f:\varepsilon \rightarrow \varepsilon; x:\beta\} \mid - x:\varepsilon}{\dots \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\} \mid - f x : \varphi}$$

$$\frac{\{f:\delta; x:\beta\} \mid - (f(f x)) : \varepsilon}{\{x:\beta\} \mid - (\text{fun } f \rightarrow f(f x)) : \gamma}$$

$$\{ \} \mid - (\text{fun } x \rightarrow \text{fun } f \rightarrow f(f x)) : \alpha$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst: $\{\zeta \equiv \varepsilon, \varphi \equiv \varepsilon, \delta \equiv \varepsilon \rightarrow \varepsilon\}$
- Var rule: $\varepsilon \equiv \beta$

$$\frac{\dots \quad \{f:\varepsilon \rightarrow \varepsilon; x:\beta\} \mid - x:\varepsilon}{\dots \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\} \mid - f x : \varphi}$$

$$\frac{\{f:\delta; x:\beta\} \mid - (f(f x)) : \varepsilon}{\{x:\beta\} \mid - (\text{fun } f \rightarrow f(f x)) : \gamma}$$

$$\{ \} \mid - (\text{fun } x \rightarrow \text{fun } f \rightarrow f(f x)) : \alpha$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst: $\{\varepsilon \equiv \beta\} \circ \{\zeta \equiv \varepsilon, \varphi \equiv \varepsilon, \delta \equiv \varepsilon \rightarrow \varepsilon\}$
- Solves subproof; return one layer

$$\frac{\dots \quad \{f:\varepsilon \rightarrow \varepsilon; x:\beta\} \mid - x:\varepsilon}{\dots \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\} \mid - f x : \varphi}$$

$$\frac{\{f:\delta; x:\beta\} \mid - (f(f x)) : \varepsilon}{\{x:\beta\} \mid - (\text{fun } f \rightarrow f(f x)) : \gamma}$$

$$\{ \} \mid - (\text{fun } x \rightarrow \text{fun } f \rightarrow f(f x)) : \alpha$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst: $\{\varepsilon \equiv \beta, \zeta \equiv \beta, \varphi \equiv \beta, \delta \equiv \beta \rightarrow \beta\}$
- Solves this subproof; return one layer

$$\frac{\dots \quad \{f:\varphi \rightarrow \varepsilon; x:\beta\} \mid - f x : \varphi}{\dots \quad \{f:\delta; x:\beta\} \mid - (f(f x)) : \varepsilon}$$

$$\frac{\{x:\beta\} \mid - (\text{fun } f \rightarrow f(f x)) : \gamma}{\{ \} \mid - (\text{fun } x \rightarrow \text{fun } f \rightarrow f(f x)) : \alpha}$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst: $\{\varepsilon \equiv \beta, \zeta \equiv \beta, \varphi \equiv \beta, \delta \equiv \beta \rightarrow \beta\}$
- Need to satisfy constraint $\gamma \equiv (\delta \rightarrow \varepsilon)$, given subst: $\gamma \equiv ((\beta \rightarrow \beta) \rightarrow \beta)$

$$\frac{\{f:\delta; x:\beta\} \mid - (f(f x)) : \varepsilon}{\{x:\beta\} \mid - (\text{fun } f \rightarrow f(f x)) : \gamma}$$

$$\{ \} \mid - (\text{fun } x \rightarrow \text{fun } f \rightarrow f(f x)) : \alpha$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst: $\{\gamma \equiv ((\beta \rightarrow \beta) \rightarrow \beta), \varepsilon \equiv \beta, \zeta \equiv \beta, \varphi \equiv \beta, \delta \equiv \beta \rightarrow \beta\}$
- Solves subproof; return one layer

$$\frac{\{f:\delta; x:\beta\} \mid - (f(f x)) : \varepsilon}{\{x:\beta\} \mid - (\text{fun } f \rightarrow f(f x)) : \gamma}$$

$$\{ \} \mid - (\text{fun } x \rightarrow \text{fun } f \rightarrow f(f x)) : \alpha$$

- $\alpha \equiv (\beta \rightarrow \gamma); \gamma \equiv (\delta \rightarrow \varepsilon)$

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Type Inference - Example

- Current subst:

$\{\gamma \equiv ((\beta \rightarrow \beta) \rightarrow \beta), \varepsilon \equiv \beta, \zeta \equiv \beta, \varphi \equiv \beta, \delta \equiv \beta \rightarrow \beta\}$

- Need to satisfy constraint $\alpha \equiv (\beta \rightarrow \gamma)$
given subst: $\alpha \equiv (\beta \rightarrow ((\beta \rightarrow \beta) \rightarrow \beta))$

$$\frac{\{x : \beta\} \vdash (\text{fun } f \rightarrow f (f x)) : \gamma}{\{ \} \vdash (\text{fun } x \rightarrow \text{fun } f \rightarrow f (f x)) : \alpha}$$

- $\alpha \equiv (\beta \rightarrow \gamma)$;

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Type Inference - Example

- Current subst:

$\{\alpha \equiv (\beta \rightarrow ((\beta \rightarrow \beta) \rightarrow \beta)),$

$\gamma \equiv ((\beta \rightarrow \beta) \rightarrow \beta), \varepsilon \equiv \beta, \zeta \equiv \beta, \varphi \equiv \beta, \delta \equiv \beta \rightarrow \beta\}$

- Solves subproof; return on layer

$$\frac{\{x : \beta\} \vdash (\text{fun } f \rightarrow f (f x)) : \gamma}{\{ \} \vdash (\text{fun } x \rightarrow \text{fun } f \rightarrow f (f x)) : \alpha}$$

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Type Inference - Example

- Current subst:

$\{\alpha \equiv (\beta \rightarrow ((\beta \rightarrow \beta) \rightarrow \beta)),$

$\gamma \equiv ((\beta \rightarrow \beta) \rightarrow \beta), \varepsilon \equiv \beta, \zeta \equiv \beta, \varphi \equiv \beta, \delta \equiv \beta \rightarrow \beta\}$

- Done: $\alpha \equiv (\beta \rightarrow ((\beta \rightarrow \beta) \rightarrow \beta))$

$$\{ \} \vdash (\text{fun } x \rightarrow \text{fun } f \rightarrow f (f x)) : \alpha$$

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Type Inference Algorithm

Let $\text{infer}(\Gamma, e, \tau) = \sigma$

- Γ is a typing environment (giving polymorphic types to expression variables)
- e is an expression
- τ is a type (with type variables),
- σ is a substitution of types for type variables
- Idea: σ is the constraints on type variables necessary for $\Gamma \vdash e : \tau$
- Should have $\sigma(\Gamma) \vdash e : \sigma(\tau)$

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Type Inference Algorithm

$\text{has_type}(\Gamma, \text{exp}, \tau) =$

- Case exp of
 - Var $v \rightarrow$ return $\text{Unify}\{\tau \equiv \text{freshInstance}(\Gamma(v))\}$
 - Replace all quantified type vars by fresh ones
 - Const $c \rightarrow$ return $\text{Unify}\{\tau \equiv \text{freshInstance } \varphi\}$
where $\Gamma \vdash c : \varphi$ by the constant rules
 - fun $x \rightarrow e \rightarrow$
 - Let α, β be fresh variables
 - Let $\sigma = \text{infer}(\{x: \alpha\} + \Gamma, e, \beta)$
 - Return $\text{Unify}\{\{\sigma(\tau) \equiv \sigma(\alpha \rightarrow \beta)\}\} \circ \sigma$

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Type Inference Algorithm (cont)

- Case exp of

- App ($e_1 e_2$) \rightarrow

- Let α be a fresh variable
- Let $\sigma_1 = \text{infer}(\Gamma, e_1, \alpha \rightarrow \tau)$
- Let $\sigma_2 = \text{infer}(\sigma(\Gamma), e_2, \sigma(\alpha))$
- Return $\sigma_2 \circ \sigma_1$

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Type Inference Algorithm (cont)

- Case *exp* of
 - If e_1 then e_2 else $e_3 \rightarrow$
 - Let $\sigma_1 = \text{infer}(\Gamma, e_1, \text{bool})$
 - Let $\sigma_2 = \text{infer}(\sigma_1\Gamma, e_2, \sigma_1(\tau))$
 - Let $\sigma_3 = \text{infer}(\sigma_2 \circ \sigma_1(\Gamma), e_3, \sigma_2 \circ \sigma_1(\tau))$
 - Return $\sigma_3 \circ \sigma_2 \circ \sigma_1$

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Type Inference Algorithm (cont)

- Case *exp* of
 - let $x = e_1$ in $e_2 \rightarrow$
 - Let α be a fresh variable
 - Let $\sigma_1 = \text{infer}(\Gamma, e_1, \alpha)$
 - Let $\sigma_2 =$
 $\text{infer}(\{x:\text{GEN}(\sigma_1(\Gamma), \sigma_1(\alpha))\} + \sigma_1(\Gamma),$
 $e_2, \sigma_1(\tau))$
 - Return $\sigma_2 \circ \sigma_1$

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Type Inference Algorithm (cont)

- Case *exp* of
 - let rec $x = e_1$ in $e_2 \rightarrow$
 - Let α be a fresh variable
 - Let $\sigma_1 = \text{infer}(\{x:\alpha\} + \Gamma, e_1, \alpha)$
 - Let $\sigma_2 = \text{infer}(\{x:\text{GEN}(\sigma_1(\Gamma), \sigma_1(\alpha))\}$
 $+ \sigma_1(\Gamma)\}, e_2, \sigma_1(\tau))$
 - Return $\sigma_2 \circ \sigma_1$

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Type Inference Algorithm (cont)

- To infer a type, introduce `type_of`
- Let α be a fresh variable
- `type_of` (Γ, e) =
 - Let $\sigma = \text{infer}(\Gamma, e, \alpha)$
 - Return $\sigma(\alpha)$
- Need an algorithm for Unif

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Background for Unification

- **Terms** made from **constructors** and **variables** (for the simple first order case)
- Constructors may be applied to arguments (other terms) to make new terms
- Variables and constructors with no arguments are base cases
- Constructors applied to different number of arguments (arity) considered different
- **Substitution** of terms for variables

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Simple Implementation Background

```
type term = Variable of string
          | Const of (string * term list)
```

```
let rec subst var_name residue term =
  match term with Variable name ->
    if var_name = name then residue else term
  | Const (c, tys) ->
    Const (c, List.map (subst var_name residue)
                     tys);;
```

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Unification Problem

Given a set of pairs of terms (“equations”)

$$\{(s_1, t_1), (s_2, t_2), \dots, (s_n, t_n)\}$$

(the *unification problem*) does there exist a substitution σ (the *unification solution*) of terms for variables such that

$$\sigma(s_i) = \sigma(t_i),$$

for all $i = 1, \dots, n$?

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Uses for Unification

- Type Inference and type checking
- Pattern matching as in OCAML
 - Can use a simplified version of algorithm
- Logic Programming - Prolog
- Simple parsing

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Unification Algorithm

- Let $S = \{(s_1, t_1), (s_2, t_2), \dots, (s_n, t_n)\}$ be a unification problem.
- Case $S = \{ \}$: $\text{Unif}(S) = \text{Identity function}$ (i.e., no substitution)
- Case $S = \{(s, t)\} \cup S'$: Four main steps

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Unification Algorithm

- **Delete**: if $s = t$ (they are the same term) then $\text{Unif}(S) = \text{Unif}(S')$
- **Decompose**: if $s = f(q_1, \dots, q_m)$ and $t = f(r_1, \dots, r_m)$ (same f , same $m!$), then $\text{Unif}(S) = \text{Unif}(\{(q_1, r_1), \dots, (q_m, r_m)\} \cup S')$
- **Orient**: if $t = x$ is a variable, and s is not a variable, $\text{Unif}(S) = \text{Unif}(\{(x, s)\} \cup S')$

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Unification Algorithm

- **Eliminate**: if $s = x$ is a variable, and x does not occur in t (the occurs check), then
 - Let $\varphi = x \mapsto t$
 - Let $\psi = \text{Unif}(\varphi(S'))$
 - $\text{Unif}(S) = \{x \mapsto \psi(t)\} \circ \psi$
 - Note: $\{x \mapsto a\} \circ \{y \mapsto b\} = \{y \mapsto (\{x \mapsto a\}(b))\} \circ \{x \mapsto a\}$ if y not in a

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Tricks for Efficient Unification

- Don't return substitution, rather do it incrementally
- Make substitution be constant time
 - Requires implementation of terms to use mutable structures (or possibly lazy structures)
 - We won't discuss these

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Example

- x,y,z variables, f,g constructors
- $S = \{(f(x), f(g(y,z))), (g(y,f(y)), x)\}$

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Example

- x,y,z variables, f,g constructors
- S is nonempty
- $S = \{(f(x), f(g(y,z))), (g(y,f(y)), x)\}$

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Example

- x,y,z variables, f,g constructors
- Pick a pair: $(g(y,f(y)), x)$
- $S = \{(f(x), f(g(y,z))), (g(y,f(y)), x)\}$

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Example

- x,y,z variables, f,g constructors
- Pick a pair: $(g(y,f(y)), x)$
- Orient: $(x, g(y,f(y)))$
- $S = \{(f(x), f(g(y,z))), (g(y,f(y)), x)\}$
- $\rightarrow \{(f(x), f(g(y,z))), (x, g(y,f(y)))\}$

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Example

- x,y,z variables, f,g constructors
- $S \rightarrow \{(f(x), f(g(y,z))), (x, g(y,f(y)))\}$

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Example

- x,y,z variables, f,g constructors
- Pick a pair: $(f(x), f(g(y,z)))$
- $S \rightarrow \{(f(x), f(g(y,z))), (x, g(y,f(y)))\}$

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Example

- x, y, z variables, f, g constructors
- Pick a pair: $(f(x), f(g(y, z)))$
- Decompose: $(x, g(y, z))$
- $S \rightarrow \{(f(x), f(g(y, z))), (x, g(y, f(y)))\}$
- $\rightarrow \{(x, g(y, z)), (x, g(y, f(y)))\}$

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Example

- x, y, z variables, f, g constructors
- Pick a pair: $(x, g(y, f(y)))$
- Substitute: $\{x \mapsto g(y, f(y))\}$
- $S \rightarrow \{(x, g(y, z)), (x, g(y, f(y)))\}$
- $\rightarrow \{(g(y, f(y)), g(y, z))\}$

- With $\{x \mapsto g(y, f(y))\}$

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Example

- x, y, z variables, f, g constructors
- Pick a pair: $(g(y, f(y)), g(y, z))$

- $S \rightarrow \{(g(y, f(y)), g(y, z))\}$

With $\{x \mapsto g(y, f(y))\}$

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Example

- x, y, z variables, f, g constructors
- Pick a pair: $(g(y, f(y)), g(y, z))$
- Decompose: (y, y) and $(f(y), z)$
- $S \rightarrow \{(g(y, f(y)), g(y, z))\}$
- $\rightarrow \{(y, y), (f(y), z)\}$

With $\{x \mapsto g(y, f(y))\}$

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Example

- x, y, z variables, f, g constructors
- Pick a pair: (y, y)

- $S \rightarrow \{(y, y), (f(y), z)\}$

With $\{x \mapsto g(y, f(y))\}$

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Example

- x, y, z variables, f, g constructors
- Pick a pair: (y, y)
- Delete
- $S \rightarrow \{(y, y), (f(y), z)\}$
- $\rightarrow \{(f(y), z)\}$

With $\{x \mapsto g(y, f(y))\}$

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Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(y), z)
- S -> {(f(y), z)}

With {x |→ g(y,f(y))}

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Example

- x,y,z variables, f,g constructors
- Pick a pair: (f(y), z)
- Orient: (z, f(y))
- S -> {(f(y), z)}
- -> {(z, f(y))}

With {x |→ g(y,f(y))}

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Example

- x,y,z variables, f,g constructors
- Pick a pair: (z, f(y))
- S -> {(z, f(y))}

With {x |→ g(y,f(y))}

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Example

- x,y,z variables, f,g constructors
- Pick a pair: (z, f(y))
- Eliminate: {z|→ f(y)}
- S -> {(z, f(y))}
- -> { }

With {x |→ {z |→ f(y)} (g(y,f(y))) }
o {z |→ f(y)}

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Example

- x,y,z variables, f,g constructors
- Pick a pair: (z, f(y))
- Eliminate: {z|→ f(y)}
- S -> {(z, f(y))}
- -> { }

With {x |→ g(y,f(y))} o {z |→ f(y)}

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Example

S = {(f(x), f(g(y,z))), (g(y,f(y)),x)}
Solved by {x |→ g(y,f(y))} o {z |→ f(y)}
$$f(\underbrace{g(y,f(y))}_x) = f(g(y,\underbrace{f(y)}_z))$$

and

$$g(y,f(y)) = \underbrace{g(y,f(y))}_x$$

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Example of Failure: Decompose

- $S = \{(f(x,g(y)), f(h(y),x))\}$
- Decompose: $(f(x,g(y)), f(h(y),x))$
- $S \rightarrow \{(x,h(y)), (g(y),x)\}$
- Orient: $(g(y),x)$
- $S \rightarrow \{(x,h(y)), (x,g(y))\}$
- Eliminate: $(x,h(y))$
- $S \rightarrow \{(h(y), g(y))\}$ with $\{x \mapsto h(y)\}$
- No rule to apply! Decompose fails!

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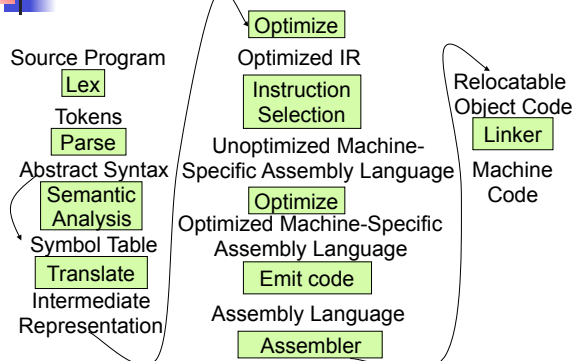
Example of Failure: Occurs Check

- $S = \{(f(x,g(x)), f(h(x),x))\}$
- Decompose: $(f(x,g(x)), f(h(x),x))$
- $S \rightarrow \{(x,h(x)), (g(x),x)\}$
- Orient: $(g(y),x)$
- $S \rightarrow \{(x,h(x)), (x,g(x))\}$
- No rules apply.

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Major Phases of a Compiler



Modified from "Modern Compiler Implementation in ML", by Andrew Appel

Meta-discourse

- Language Syntax and Semantics
- Syntax
 - Regular Expressions, DFSAs and NDFSAs
 - Grammars
- Semantics
 - Natural Semantics
 - Transition Semantics

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Language Syntax

- Syntax is the description of which strings of symbols are meaningful expressions in a language
- It takes more than syntax to understand a language; need meaning (semantics) too
- Syntax is the entry point

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Syntax of English Language

- Pattern 1

Subject	Verb
David	sings
The dog	barked
Susan	yawned
- Pattern 2

Subject	Verb	Direct Object
David	sings	ballads
The professor	wants	to retire
The jury	found	the defendant guilty

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Elements of Syntax

- Character set – previously always ASCII, now often 64 character sets
- Keywords – usually reserved
- Special constants – cannot be assigned to
- Identifiers – can be assigned to
- Operator symbols
- Delimiters (parenthesis, braces, brackets)
- Blanks (aka white space)

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Elements of Syntax

- Expressions
if ... then begin ... ; ... end else begin ... ; ... end
- Type expressions
 $type_{expr_1} \rightarrow type_{expr_2}$
- Declarations (in functional languages)
 $let\ pattern_1 = expr_1\ in\ expr$
- Statements (in imperative languages)
 $a = b + c$
- Subprograms
 $let\ pattern_1 = let\ rec\ inner = \dots\ in\ expr$

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Elements of Syntax

- Modules
- Interfaces
- Classes (for object-oriented languages)

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Lexing and Parsing

- Converting strings to abstract syntax trees done in two phases
 - **Lexing:** Converting string (or streams of characters) into lists (or streams) of tokens (the “words” of the language)
 - Specification Technique: Regular Expressions
 - **Parsing:** Convert a list of tokens into an abstract syntax tree
 - Specification Technique: BNF Grammars

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Formal Language Descriptions

- Regular expressions, regular grammars, finite state automata
- Context-free grammars, BNF grammars, syntax diagrams
- Whole family more of grammars and automata – covered in automata theory

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Grammars

- Grammars are formal descriptions of which strings over a given character set are in a particular language
- Language designers write grammar
- Language implementers use grammar to know what programs to accept
- Language users use grammar to know how to write legitimate programs

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