Sample Questions for Midterm 1 (CS 421 Fall 2013)

On the actual midterm, you will have plenty of space to put your answers. The actual midterm will likely have no more than 9 questions plus one extra credit question. In addition to questions of the kind asked below, you should expect to see questions from your MPs and HW1 on the exams. Some of these questions may be reused for the exam.

1. Given the following OCAML code:
   
   ```ocaml
   let x = 3;;
   let f y = x + y;;
   let x = 5;;
   let z = f 2;;
   let x = "hi";;
   ```

   What value will `z` have? Will the last declaration (`let x = "hi";;`) cause a type error? What is the value of `x` after this code has been executed?

   **Solution:**
   
   - `z` is bound to 5
   - `let x = "hi";;` will not cause a type error
   - `x` is bound to "hi"

2. What environment is in effect after each declaration in the code in Problem 1? What is the step-by-step formal evaluation of `f 2` in the fourth declaration, starting from the environment after the third declaration?

   **Solution:**
   
   ```ocaml
   let x = 3;;
   {x \rightarrow 3}
   
   let f y = x + y;;
   {f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle, x \rightarrow 3}
   
   let x = 5;;
   {x \rightarrow 5} + {f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle, x \rightarrow 3} =
   {x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle}
   
   let z = f 2;;
   eval(f 2, {x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle}) =
   eval(app(eval(f, {x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle}),
   eval(2, {x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle})),
   {x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle}) =
   eval(app(<y \rightarrow x+y, \{x \rightarrow 3\}\rangle, 2), {x \rightarrow 5, f \rightarrow \langle y \rightarrow x+y, \{x \rightarrow 3\}\rangle}) =
   eval(x+y, {y \rightarrow 2} + {x \rightarrow 3}) =
   eval(x, {y \rightarrow 2, x \rightarrow 3}) + eval(y, {y \rightarrow 2, x \rightarrow 3}) =
   3 + 2 = 5
   ```
{\( z \rightarrow 5, \ x \rightarrow 5, \ f \rightarrow y \rightarrow x+y, \ {x \rightarrow 3}\) }

```ocaml
let x = "hi";;
{x \rightarrow "hi"} + {\( z \rightarrow 5, \ x \rightarrow 5, \ f \rightarrow y \rightarrow x+y, \ {x \rightarrow 3}\)} =
{x \rightarrow "hi", \ z \rightarrow 5, \ f \rightarrow y \rightarrow x+y, \ {x \rightarrow 3}\} =
```

3. What is the effect of each of the following pieces of code?
   a. (fun x -> (print_string "a"; x + 2)) (print_string "b"; 4);;
      It prints ba and returns 6
   b. let f = (print_string "a"; fun x -> x + 2) in f (print_string "b"; 4);;
      It prints ab and returns 6
   c. let f = fun g -> (print_string "a"; g 2) in f (fun x -> print_string "b"; 4 + x);;
      It prints ab and returns 6

4. Consider the following two OCaml functions, `loop1` and `loop2`:
   ```ocaml
   let rec loop1 () = loop1(); ()
   let rec loop2 () = loop2();;
   val loop1 : unit -> unit = <fun>
   val loop2 : unit -> 'a = <fun>
   ```
   Suppose you were to run `loop1();` and `loop2();` in OCaml, (pressing CTRL + C after at least a minute to terminate infinite loops when necessary).
   a. For each program, what behavior would you expect to see?
   b. What is the difference between `loop1` and `loop2`?
   c. For each program state if it is:
      i. recursive,
      ii. forward recursive,
      iii. tail-recursive.

Solution:
   a. The first program generates a stack overflow, while the second program runs indefinitely.
   b. Because `loop1` is not tail-recursive, each new recursive call must push a new activation record onto the stack, hence the stack overflow, but since `loop2` is tail-recursive, each new activation record may overwrite the previous call, and thus the stack does not grow. It should also be observed that `loop2` has a more general type (`:'a`) than that of `loop1:unit -> unit`, and hence may be used in places where other return types beside `unit` are required. (Of course, it had better never actually be applied.)
   c. Both programs are recursive, and in fact forward recursive, but `loop2` is tail-recursive while `loop1` is not

5. Write an OCAML function `pair_up` that takes first a function, then an input list and returns a list of pairs of an element from input list (the second argument), paired with the result of applying the first argument to that element. What is the OCAML type of `pair_up`? What is the result of the following expressions:
   a. `pair_up (fun x -> x + 3) [6;4;1];`
   b. `pair_up ((fun x -> "Hi, "^x), ["John"; "Mary";"Dana"]);`
c. `pair_up (fun x -> x *. 2.0);`

Solution:
```
let rec pair_up f l =
  (match l with [] -> []
    | x :: xs -> (x, f x)::pair_up f xs)
alternately let pair_up f = List.map (fun x -> (x, f x))
```

`pair_up : ('a -> 'b) -> 'a list -> ('a * 'b) list`

a. `[(6, 9); (4,7); (1,4)];`

b. type error

c. A function of type float list -> (float * float) list that returns a list of pairs of an
   element from the input list paired with twice itself.

6. Write an Ocaml function `palindrome : string list -> unit` that first prints the strings in
   the list from left to right, followed by printing them right to left, recursing over the list
   only once.  (Potential extra credit problem: Do this using each of `List.fold_right` and
   `List.fold_left` but no explicit use of `let rec`.)

   Solution:
   ```
   let rec palindrome l =
     match l with [] -> ()
     | s::ss -> (print_string s; palindrome ss; print_string s);
   ```
   ```
   let rec palindrome l =
     List.fold_right
     (fun s -> fun print_middle -> (fun () -> (print_string s; print_middle (); print_string s)))
     l
     (fun () -> ());
   ```
   ```
   let palindrome l =
     List.fold_left
     (fun print_middle -> fun s -> (print_string s; fun () -> (print_middle (print_string s)))))
     (fun () -> ());
     l
   ```

7. Using `fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b`, but without using explicit
   recursion, write a function `concat : 'a list -> 'a list that appends all the lists in the
   input list of lists, preserving the order of elements.  You may use the append function `@`.

   Solution: let concat lst = List.fold_right (@) lst [];

8. Write an Ocaml function `list_print : string list -> unit` that prints all the strings in a list
   from left to right:
   a. using tail recursion, but no higher order functions,
   b. using `fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a but no explicit recursion.`
Solution:

a. let rec list_print lst = match lst with [ ] -> ( ) | s::ss -> (print_string s; list_print ss);;
b. let list_print lst = List.fold_left (fun ( ) -> fun s -> print_string s) () lst;;

9. Put the following function in full continuation passing style:

   let rec sum_odd n = if n <= 0 then 0 else ((2 * n) – 1) + sum_odd (n – 1);;

   Primitive operations (+, -, *, <=) do not have to be converted to CPS, but all procedure calls must be.

   Solution:

   let add_k a b k = k(a + b)
   let minus_k a b k = k(a - b)
   let times_k a b k = k(a * b)
   let leq_k a b k = k(a <= b)

   let rec sum_odd_k n k =
      leq_k n 0 (fun b -> if b then k 0
         else minus_k n 1
         (fun d -> sum_odd_k d
          (fun r -> times_k 2 n
           (fun t -> minus_k t 1
            (fun m -> add_k m r k )))))

10. Write the definition of an OCAML variant type reg_exp to express abstract syntax trees for regular expressions over a base character set of booleans. Thus, a boolean is a reg_exp, epsilon is a reg_exp, the concatenation of two reg_exp’s is a reg_exp, the “choice” of two reg_exp’s is a reg_exp, and the Kleene star of a reg_exp is a reg_exp.

   Solution:

   type reg_exp =
   Char of bool
   | Epsilon
   | Concat of (reg_exp * reg_exp)
   | Choice of (reg_exp * reg_exp)
   | Kleene_star of reg_exp
   | Paren of reg_exp (* I would accept it with this case missing *)

11. Given the following OCAML datatype:

    type int_seq = Null | Snoc of (int_seq * int)

   write a tail-recursive function in OCAML all_pos : int_seq -> bool that returns true if every integer in the input int_seq to which all_pos is applied is strictly greater than 0 and false otherwise. Thus all_pos (Snoc(Snoc(Snoc(Null,3),5),7)) should return true, but all_pos (Snoc(Null,~1)) and all_pos (Snoc(Snoc(Null,3),0)) should both return false.

   Solution:

   let rec all_pos s =
      (match s with Null -> true
12. What is **type checking**? What do the terms **static** and **dynamic** mean when referring to type checking? Given an example of a property that can be type checked statically, and an example of a property that can only be checked dynamically.

**Solution:** Type checking means checking that for every application $\text{proc}(a_1, \ldots, a_n)$, the number and types for the input arguments is the same as the number and type arguments specified.

**Static type checking** means type checking done at compile time, before object language code is produced. **Dynamic type checking** refers to perform type checking as a part of the execution of procedures at run time. That the argument to a function is a 32-bit integer can be checked **statically**. Divide by zero errors must be checked **dynamically**.

13. Using the typing rules attached, give a full type derivation for the following judgment. You should include labels stating which rules are used where.

\[
\{ \} \vdash (\text{let } x = \text{true in let } f = \text{fun } x \rightarrow x > 1 \text{ in if } x \&\& f \text{ then 17 else } 3 + 5) : \text{int}
\]
Solution:
Let C stand for the Constants rule, V stand for the Variables rule, P stand for the Primitive Operations rule, R stand for the Relations rule, K stand for the Connectives rule, I stand for the If_then_else rule, A stand for the Application rule, L stand for the Let rule, and LR stand for the Let_rec rule. The proof is

\[\text{\textbf{If_tree_proof}}\]

\[\{x: \text{int}\} \vdash \text{x: int} \quad \{x: \text{int}\} \vdash 1: \text{int} \]

\[\{x: \text{int}\} \vdash (x > 1) : \text{bool} \]

\[\{x: \text{bool}\} \vdash (\text{fun} x \rightarrow x > 1): \text{int} \rightarrow \text{bool} \]

\[\{x: \text{bool}\} \vdash \text{true: bool} \]

\[\{x: \text{bool}\} \vdash \text{let } f = \text{fun } x \rightarrow x > 1 \text{ in if } x \text{ && } f 3 \text{ then 17 else 3 + 5} : \text{int} \]

Let \(\Gamma = \{x: \text{bool}, f: \text{int} \rightarrow \text{bool}\}\). Then \textbf{If_tree_proof} is

\[\{x: \text{bool}\} \vdash \text{true: bool} \]

\[\{x: \text{bool}\} \vdash \text{let } f = \text{fun } x \rightarrow x > 1 \text{ in if } x \text{ && } f 3 \text{ then 17 else 3 + 5} : \text{int} \]

\[\{x: \text{bool}, f: \text{int} \rightarrow \text{bool}\} \vdash (\text{if } x \text{ && } f 3 \text{ then 17 else 3 + 5}) : \text{int} \]