HW 2 Solution

CS 421 – Fall 2012
Revision 1.0

Assigned September 11, 2012
Due September 18, 2012, 11:59 pm
Extension 48 hours (20% penalty)

1 Change Log

1.0 Initial Release.

2 Solutions

1. (10 pts) Below is a fragment of OCaml code, with various program points indicated by numbers with comments.

   (code and solution on next page)

   For each of program points 1, 2, and 3, please describe the environment in effect after evaluation has reached that point. Finally, show step by step how the application of f_z y would be evaluated. You may assume that the evaluation begins in an empty environment, and that the environment is cumulative thereafter. The program points are supposed to indicate points at which all complete preceding declarations (including local ones) have been fully evaluated.

   Solution:

   let x = 5;;

   let plus_x = fun y -> x + y;;

   let y = 3;;

   let z = plus_x y;;

   $\rho_1 = \{x \mapsto 5, \text{plus\_x} \mapsto c_{plus}, y \mapsto 3, z \mapsto 8\}$ where $c_{plus} = <y \rightarrow x+y, \{x \mapsto 5\}>$

   let sub_z = fun x -> y - z;;

   let y = sub_z y;;

   $\rho_2 = \{\text{sub\_z} \mapsto c_{sub}, y \mapsto -5\} + \rho_1 = \{x \mapsto 5, \text{plus\_x} \mapsto c_{plus}, y \mapsto -5, z \mapsto 8, \text{sub\_z} \mapsto c_{sub}\}$
   where $c_{plus} = <y \rightarrow x+y, \{x \mapsto 5\}>$ and $c_{sub} = <x \rightarrow y\text{-}z, \rho_1>$

let f_z x = if plus_x x < z then sub_z x else plus_x z;;

$$\rho_3 \quad = \quad \{f\_z \mapsto c_f\} + \rho_2$$
$$= \quad \{f\_z \mapsto c_f,\ sub\_z \mapsto c_{sub},\ y \mapsto -5\} + \rho_1$$
$$= \quad \{x \mapsto 5,\ plus\_x \mapsto c_{plus},\ y \mapsto -5,\ z \mapsto 8,\ sub\_z \mapsto c_{sub},\ f\_z \mapsto c_f\}$$
where $c_{plus} = < y \rightarrow x + y,\ \{x \mapsto 5\} >$
and $c_{sub} = < x \rightarrow y - z, \rho_1 >$
and $c_f = < x \rightarrow$ if plus_x x $< z$ then sub_z x else plus_x z, $\rho_2 >$

f_z y;;

Eval (f_z y, $\{x \mapsto 5,\ plus\_x \mapsto c_{plus},\ y \mapsto$ -5, $z \mapsto 8,\ sub\_z \mapsto c_{sub},\ f\_z \mapsto c_f\}$)

$\Rightarrow$ Eval (f_z (-5), $\{x \mapsto 5,\ plus\_x \mapsto c_{plus},\ y \mapsto$ -5, $z \mapsto 8,\ sub\_z \mapsto c_{sub},\ f\_z \mapsto c_f\}$)

$\Rightarrow$ Eval (app $<x \rightarrow$ if plus_x x $< z$ then sub_z x else plus_x z,$\rho_2 > (-5), \{x \mapsto 5,\ plus\_x \mapsto c_{plus},\ y \mapsto$ -5, $z \mapsto 8$, sub_z $\mapsto c_{sub},\ f\_z \mapsto c_f\}$)

$\Rightarrow$ Eval (if plus_x x $< z$ then sub_z x else plus_x z,$\{x \mapsto (-5)\} + \rho_2$)

$\Rightarrow$ Eval (if plus_x x < 8 then sub_z x else plus_x z,$\{x \mapsto (-5)\} + \rho_2$)

$\Rightarrow$ Eval (if plus_x (-5) < 8 then sub_z x else plus_x z,$\{x \mapsto (-5)\} + \rho_2$)

$\Rightarrow$ Eval (if (app $<y \rightarrow$ x+y, $\{x \mapsto 5\}> (-5)) < 8$ then sub_z x else plus_x z,$\{x \mapsto (-5)\} + \rho_2$)

$\Rightarrow$ Eval (if (Eval(x+y, $\{y \mapsto (-5)\} + \{x \mapsto 5\}> (-5))) < 8$ then sub_z x else plus_x z,$\{x \mapsto (-5)\} + \rho_2$)

$\Rightarrow$ Eval (if (Eval(x+(-5), $\{y \mapsto (-5),\ x \mapsto 5\}> (-5))) < 8$ then sub_z x else plus_x z,$\{x \mapsto (-5)\} + \rho_2$)

$\Rightarrow$ Eval (if (Eval(5+(-5), $\{y \mapsto (-5),\ x \mapsto 5\}> (-5))) < 8$ then sub_z x else plus_x z,$\{x \mapsto (-5)\} + \rho_2$)

$\Rightarrow$ Eval (if $0 < 8$ then sub_z x else plus_x z,$\{x \mapsto (-5)\} + \rho_2$)

$\Rightarrow$ Eval (if true then sub_z x else plus_x z,$\{x \mapsto (-5)\} + \rho_2$)

$\Rightarrow$ Eval (sub_z x $\{x \mapsto (-5)\} + \rho_2$)

$\Rightarrow$ Eval (sub_z (-5),$\{x \mapsto$ -5, plus_x $\mapsto c_{plus},\ y \mapsto$ -5, $z \mapsto 8,\ sub\_z \mapsto c_{sub}\}$) where $c_{sub} = <x \mapsto$ y-z,$\rho_1>$

$\Rightarrow$ Eval (app $<x \rightarrow$ y-z,$\rho_1> (-5),\{x \mapsto$ -5, plus_x $\mapsto c_{plus},\ y \mapsto$ -5, $z \mapsto 8,\ sub\_z \mapsto c_{sub}\}$) where $\rho_1 = \{x \mapsto 5,$ plus_x $\mapsto c_{plus},\ y \mapsto 3,\ z \mapsto 8\}$

$\Rightarrow$ Eval (y-z,$\{x \mapsto$ -5$\} + \{x \mapsto 5,$ plus_x $\mapsto c_{plus},\ y \mapsto 3,\ z \mapsto 8\}$ where $c_{plus} = <y \mapsto$ x+y, $\{x \mapsto 5\}>$)

$\Rightarrow$ Eval (y-8,$\{x \mapsto$ -5, plus_x $\mapsto c_{plus},\ y \mapsto 3,\ z \mapsto 8\}$ where $c_{plus} = <y \mapsto$ x+y, $\{x \mapsto 5\}>$) = -5

$\Rightarrow$ Eval (3-8,$\{x \mapsto$ -5, plus_x $\mapsto c_{plus},\ y \mapsto 3,\ z \mapsto 8\}$ where $c_{plus} = <y \mapsto$ x+y, $\{x \mapsto 5\}>$) = -5