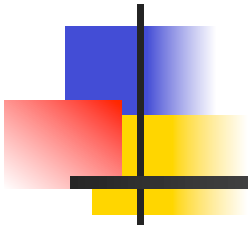


Programming Languages and Compilers (CS 421)



Elsa L Gunter

2112 SC, UIUC

<http://courses.engr.illinois.edu/cs421>

Based in part on slides by Mattox Beckman, as updated
by Vikram Adve and Gul Agha



Semantics

- Expresses the meaning of syntax
- Static semantics
 - Meaning based only on the form of the expression without executing it
 - Usually restricted to type checking / type inference



Dynamic semantics

- Method of describing meaning of executing a program
- Several different types:
 - Operational Semantics
 - Axiomatic Semantics
 - Denotational Semantics



Dynamic Semantics

- Different languages better suited to different types of semantics
- Different types of semantics serve different purposes



Operational Semantics

- Start with a simple notion of machine
- Describe how to execute (implement) programs of language on virtual machine, by describing how to execute each program statement (ie, following the *structure* of the program)
- Meaning of program is how its execution changes the state of the machine
- Useful as basis for implementations



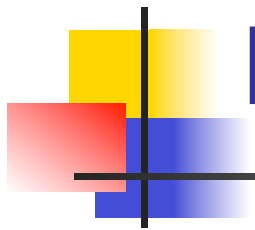
Axiomatic Semantics

- Also called Floyd-Hoare Logic
- Based on formal logic (first order predicate calculus)
- Axiomatic Semantics is a logical system built from *axioms* and *inference rules*
- Mainly suited to simple imperative programming languages



Axiomatic Semantics

- Used to formally prove a property (*post-condition*) of the *state* (the values of the program variables) after the execution of program, assuming another property (*pre-condition*) of the state before execution
- Written :
 {Precondition} Program {Postcondition}
- Source of idea of *loop invariant*



Denotational Semantics

- Construct a function \mathcal{M} assigning a mathematical meaning to each program construct
- Lambda calculus often used as the range of the meaning function
- Meaning function is compositional: meaning of construct built from meaning of parts
- Useful for proving properties of programs



Natural Semantics

- Aka Structural Operational Semantics, aka “Big Step Semantics”
- Provide value for a program by rules and derivations, similar to type derivations
- Rule conclusions look like

$$(C, m) \Downarrow m'$$

or

$$(E, m) \Downarrow v$$



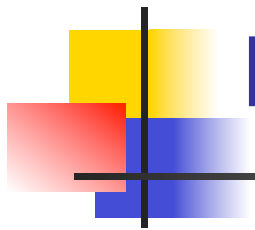
Simple Imperative Programming Language

- $I \in \textit{Identifiers}$
- $N \in \textit{Numerals}$
- $B ::= \text{true} \mid \text{false} \mid B \ \& \ B \mid B \ \text{or} \ B \mid \text{not} \ B$
 $\mid E < E \mid E = E$
- $E ::= N \mid I \mid E + E \mid E * E \mid E - E \mid - E$
- $C ::= \text{skip} \mid C; C \mid I ::= E$
 $\mid \text{if } B \text{ then } C \text{ else } C \text{ fi} \mid \text{while } B \text{ do } C \text{ od}$



Natural Semantics of Atomic Expressions

- Identifiers: $(I, m) \Downarrow m(I)$
- Numerals are values: $(N, m) \Downarrow N$
- Booleans: $(\text{true}, m) \Downarrow \text{true}$
 $(\text{false}, m) \Downarrow \text{false}$



Booleans:

$$\frac{(B, m) \Downarrow \text{false}}{(B \ \& \ B', m) \Downarrow \text{false}}$$

$$\frac{(B, m) \Downarrow \text{true} \quad (B', m) \Downarrow b}{(B \ \& \ B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(B \ \text{or} \ B', m) \Downarrow \text{true}}$$

$$\frac{(B, m) \Downarrow \text{false} \quad (B', m) \Downarrow b}{(B \ \text{or} \ B', m) \Downarrow b}$$

$$\frac{(B, m) \Downarrow \text{true}}{(\text{not } B, m) \Downarrow \text{false}}$$

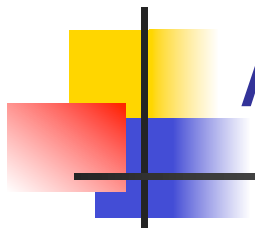
$$\frac{(B, m) \Downarrow \text{false}}{(\text{not } B, m) \Downarrow \text{true}}$$



Relations

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \sim V = b}{(E \sim E', m) \Downarrow b}$$

- By $U \sim V = b$, we mean does (the meaning of) the relation \sim hold on the meaning of U and V
- May be specified by a mathematical expression/equation or rules matching U and V



Arithmetic Expressions

$$\frac{(E, m) \Downarrow U \quad (E', m) \Downarrow V \quad U \text{ op } V = N}{(E \text{ op } E', m) \Downarrow N}$$

where N is the specified value for $U \text{ op } V$

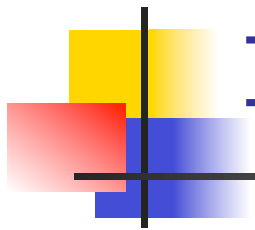


Commands

Skip: $(\text{skip}, m) \Downarrow m$

Assignment:
$$\frac{(E, m) \Downarrow V}{(I ::= E, m) \Downarrow m[I \leftarrow V]}$$

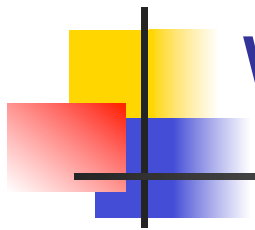
Sequencing:
$$\frac{(C, m) \Downarrow m' \quad (C', m') \Downarrow m''}{(C; C', m) \Downarrow m''}$$



If Then Else Command

$$\frac{(B, m) \Downarrow \text{true} \quad (C, m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$

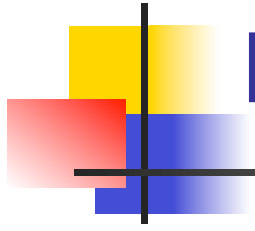
$$\frac{(B, m) \Downarrow \text{false} \quad (C', m) \Downarrow m'}{(\text{if } B \text{ then } C \text{ else } C' \text{ fi}, m) \Downarrow m'}$$



While Command

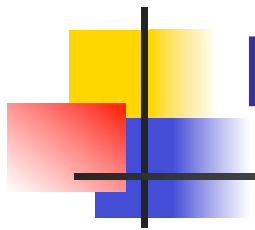
$$\frac{(B, m) \Downarrow \text{false}}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m}$$

$$\frac{(B, m) \Downarrow \text{true} \quad (C, m) \Downarrow m' \quad (\text{while } B \text{ do } C \text{ od}, m') \Downarrow m''}{(\text{while } B \text{ do } C \text{ od}, m) \Downarrow m''}$$



Example: If Then Else Rule

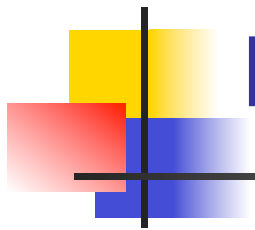
(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}) \Downarrow ?$



Example: If Then Else Rule

$$(x > 5, \{x \rightarrow 7\}) \Downarrow ?$$

$$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?$$



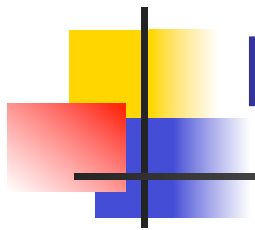
Example: Arith Relation

$? > ? = ?$

$(x, \{x \rightarrow 7\}) \Downarrow ? \quad (5, \{x \rightarrow 7\}) \Downarrow ?$

$(x > 5, \{x \rightarrow 7\}) \Downarrow ?$

$(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?$



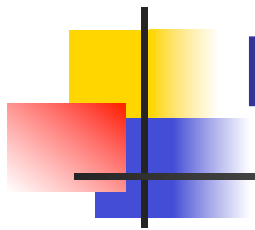
Example: Identifier(s)

$7 > 5 = \text{true}$

$(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5$

$(x > 5, \{x \rightarrow 7\}) \Downarrow ?$

(if $x > 5$ then $y := 2 + 3$ else $y := 3 + 4$ fi,
 $\{x \rightarrow 7\}) \Downarrow ?$



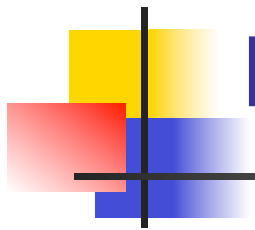
Example: Arith Relation

$$7 > 5 = \text{true}$$

$$\frac{(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5}{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}}$$

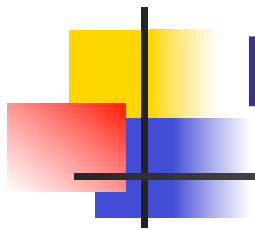
$$(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}$$

$$\frac{}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?}$$



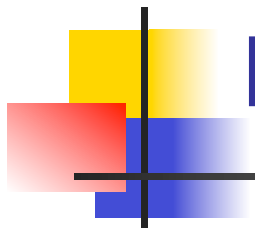
Example: If Then Else Rule

$$\begin{array}{c} 7 > 5 = \text{true} \\ \hline \frac{(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5}{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}} \quad \frac{}{(y := 2 + 3, \{x \rightarrow 7\}) \Downarrow ?} \\ \hline \text{(if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\ \quad \{x \rightarrow 7\}) \Downarrow ? \end{array}$$



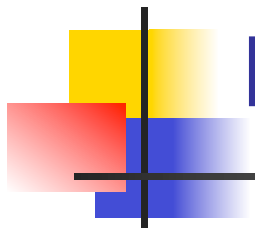
Example: Assignment

$$\frac{\frac{7 > 5 = \text{true}}{(x, \{x \rightarrow 7\}) \Downarrow 7} \quad (5, \{x \rightarrow 7\}) \Downarrow 5}{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}} \quad \frac{(2+3, \{x \rightarrow 7\}) \Downarrow ?}{(y := 2 + 3, \{x \rightarrow 7\}) \Downarrow ?}$$
$$\frac{}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?}$$



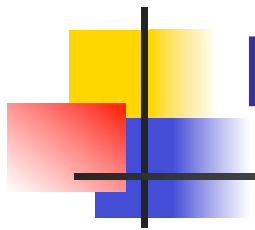
Example: Arith Op

$$\begin{array}{c}
 \text{? + ? = ?} \\
 \hline
 (2, \{x \rightarrow 7\}) \Downarrow ? \quad (3, \{x \rightarrow 7\}) \Downarrow ? \\
 \hline
 \begin{array}{cc}
 7 > 5 = \text{true} & (2+3, \{x \rightarrow 7\}) \Downarrow ? \\
 \hline
 (x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5 & (y := 2 + 3, \{x \rightarrow 7\}) \\
 \hline
 (x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} & \Downarrow ? \\
 \hline
 \text{(if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\
 \{x \rightarrow 7\}) \Downarrow ?
 \end{array}
 \end{array}$$



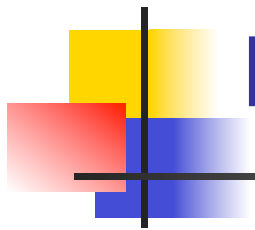
Example: Numerals

$$\begin{array}{c}
 2 + 3 = 5 \\
 \hline
 (2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3 \\
 \hline
 (2+3, \{x \rightarrow 7\}) \Downarrow ? \\
 \hline
 (y := 2 + 3, \{x \rightarrow 7\}) \\
 \Downarrow ? \\
 \hline
 \begin{array}{c}
 7 > 5 = \text{true} \\
 \hline
 (x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5 \\
 \hline
 (x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \\
 \hline
 \text{(if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\
 \{x \rightarrow 7\}) \Downarrow ?
 \end{array}
 \end{array}$$



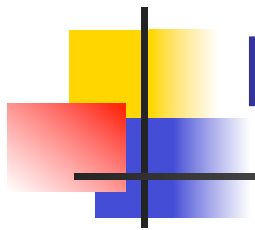
Example: Arith Op

$$\begin{array}{c} 2 + 3 = 5 \\ \hline (2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3 \\ \hline (2+3, \{x \rightarrow 7\}) \Downarrow 5 \\ \hline (y := 2 + 3, \{x \rightarrow 7\}) \Downarrow ? \\ \hline \end{array}$$
$$\begin{array}{c} 7 > 5 = \text{true} \\ \hline (x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5 \\ \hline (x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \\ \hline \end{array}$$
$$\begin{array}{c} \hline (\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi,} \\ \{x \rightarrow 7\}) \Downarrow ? \\ \hline \end{array}$$



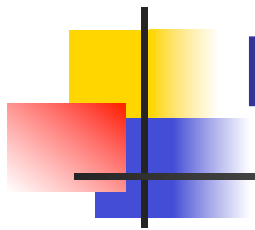
Example: Assignment

$$\begin{array}{c} \frac{\frac{7 > 5 = \text{true}}{(x, \{x \rightarrow 7\}) \Downarrow 7} \quad (5, \{x \rightarrow 7\}) \Downarrow 5}{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true}} \quad \frac{\frac{2 + 3 = 5}{(2, \{x \rightarrow 7\}) \Downarrow 2} \quad (3, \{x \rightarrow 7\}) \Downarrow 3}{(2+3, \{x \rightarrow 7\}) \Downarrow 5} \\ \frac{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \quad (y := 2 + 3, \{x \rightarrow 7\}) \Downarrow 5}{(y := 2 + 3, \{x \rightarrow 7\}) \Downarrow \{x \rightarrow 7, y \rightarrow 5\}} \\ \frac{(y := 2 + 3, \{x \rightarrow 7\}) \Downarrow \{x \rightarrow 7, y \rightarrow 5\}}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow ?} \end{array}$$



Example: If Then Else Rule

$$\begin{array}{c} \frac{\frac{7 > 5 = \text{true}}{(x, \{x \rightarrow 7\}) \Downarrow 7 \quad (5, \{x \rightarrow 7\}) \Downarrow 5} \quad \frac{\frac{2 + 3 = 5}{(2, \{x \rightarrow 7\}) \Downarrow 2 \quad (3, \{x \rightarrow 7\}) \Downarrow 3} (2+3, \{x \rightarrow 7\}) \Downarrow 5}{(y := 2 + 3, \{x \rightarrow 7\}) \Downarrow \{x \rightarrow 7, y \rightarrow 5\}} \\ \frac{(x > 5, \{x \rightarrow 7\}) \Downarrow \text{true} \quad \Downarrow \{x \rightarrow 7, y \rightarrow 5\}}{(\text{if } x > 5 \text{ then } y := 2 + 3 \text{ else } y := 3 + 4 \text{ fi, } \{x \rightarrow 7\}) \Downarrow \{x \rightarrow 7, y \rightarrow 5\}} \end{array}$$



Let in Command

$$\frac{(E, m) \Downarrow v \quad (C, m[I \leftarrow v]) \Downarrow m'}{(\text{let } I = E \text{ in } C, m) \Downarrow m''}$$

Where $m''(y) = m'(y)$ for $y \neq I$ and
 $m''(I) = m(I)$ if $m(I)$ is defined,
and $m''(I)$ is undefined otherwise



Example

$$\begin{array}{c}
 \frac{(x, \{x \rightarrow 5\}) \Downarrow 5 \quad (3, \{x \rightarrow 5\}) \Downarrow 3}{(x+3, \{x \rightarrow 5\}) \Downarrow 8} \\
 \frac{(5, \{x \rightarrow 17\}) \Downarrow 5 \quad (x := x+3, \{x \rightarrow 5\}) \Downarrow \{x \rightarrow 8\}}{(\text{let } x = 5 \text{ in } (x := x+3), \{x \rightarrow 17\}) \Downarrow ?}
 \end{array}$$



Example

$$\begin{array}{c}
 \frac{(x, \{x \rightarrow 5\}) \Downarrow 5 \quad (3, \{x \rightarrow 5\}) \Downarrow 3}{(x+3, \{x \rightarrow 5\}) \Downarrow 8} \\
 \frac{(5, \{x \rightarrow 17\}) \Downarrow 5 \quad (x := x+3, \{x \rightarrow 5\}) \Downarrow \{x \rightarrow 8\}}{(\text{let } x = 5 \text{ in } (x := x+3), \{x \rightarrow 17\}) \Downarrow \{x \rightarrow 17\}}
 \end{array}$$



Comment

- Simple Imperative Programming Language introduces variables *implicitly* through assignment
- The let-in command introduces scoped variables *explicitly*
- Clash of constructs apparent in awkward semantics



Interpretation Versus Compilation

- A **compiler** from language L1 to language L2 is a program that takes an L1 program and for each piece of code in L1 generates a piece of code in L2 of same meaning
- An **interpreter** of L1 in L2 is an L2 program that executes the meaning of a given L1 program
- Compiler would examine the body of a loop once; an interpreter would examine it every time the loop was executed



Interpreter

- An *Interpreter* represents the operational semantics of a language L1 (source language) in the language of implementation L2 (target language)
- Built incrementally
 - Start with literals
 - Variables
 - Primitive operations
 - Evaluation of expressions
 - Evaluation of commands/declarations



Interpreter

- Takes abstract syntax trees as input
 - In simple cases could be just strings
- One procedure for each syntactic category (nonterminal)
 - eg one for expressions, another for commands
- If Natural semantics used, tells how to compute final value from code
- If Transition semantics used, tells how to compute next "state"
 - To get final value, put in a loop



Natural Semantics Example

- $\text{compute_exp}(\text{Var}(v), m) = \text{look_up } v \ m$
- $\text{compute_exp}(\text{Int}(n), _) = \text{Num } (n)$
- ...
- $\text{compute_com}(\text{IfExp}(b, c1, c2), m) =$
 if $\text{compute_exp}(b, m) = \text{Bool}(\text{true})$
 then $\text{compute_com}(c1, m)$
 else $\text{compute_com}(c2, m)$



Natural Semantics Example

- $\text{compute_com}(\text{While}(b,c), m) =$
 if $\text{compute_exp}(b,m) = \text{Bool}(\text{false})$
 then m
 else compute_com
 $(\text{While}(b,c), \text{compute_com}(c,m))$
- May fail to terminate - exceed stack limits
- Returns no useful information then