

# Programming Languages and Compilers (CS 421)

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## LR Parsing

- Read tokens left to right (L)
- Create a rightmost derivation (R)
- How is this possible?
- Start at the bottom (left) and work your way up
- Last step has only one non-terminal to be replaced so is right-most
- Working backwards, replace mixed strings by non-terminals
- Always proceed so that there are no non-terminals to the right of the string to be replaced

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Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

$\langle \text{Sum} \rangle \Rightarrow$

=  $\bullet (0 + 1) + 0$  shift

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Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

$\langle \text{Sum} \rangle \Rightarrow$

=  $(\bullet 0 + 1) + 0$  shift  
=  $\bullet (0 + 1) + 0$  shift

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Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

$\langle \text{Sum} \rangle \Rightarrow$

$\Rightarrow (0 \bullet + 1) + 0$  reduce  
=  $(\bullet 0 + 1) + 0$  shift  
=  $\bullet (0 + 1) + 0$  shift

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Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

$\langle \text{Sum} \rangle \Rightarrow$

=  $(\langle \text{Sum} \rangle \bullet + 1) + 0$  shift  
=  $\Rightarrow (0 \bullet + 1) + 0$  reduce  
=  $(\bullet 0 + 1) + 0$  shift  
=  $\bullet (0 + 1) + 0$  shift

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Example:  $\text{<Sum>} = 0 \mid 1 \mid (\text{<Sum>})$   
 $\quad \mid \text{<Sum>} + \text{<Sum>}$

<Sum> =>

=	( <Sum> + 1 ) + 0	shift
=	( <Sum> 1 + ) + 0	shift
=>	( 0 1 + ) + 0	reduce
=	( 0 + 1 ) + 0	shift
=	0 ( 0 + 1 ) + 0	shift

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Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

<Sum> =>

=> ( <Sum> + 1 ● ) + 0	reduce
= ( <Sum> + ● 1 ) + 0	shift
= ( <Sum> ● + 1 ) + 0	shift
=> ( 0 ● + 1 ) + 0	reduce
= ( ● 0 + 1 ) + 0	shift
= ● ( 0 + 1 ) + 0	shift

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Example:  $\text{<Sum>} = 0 \mid 1 \mid (\text{<Sum>})$   
 $\quad \mid \text{<Sum>} + \text{<Sum>}$

<Sum> =>

```

=> ( <Sum> + <Sum> ● ) + 0    reduce
=> ( <Sum> + 1 ● ) + 0    reduce
= ( <Sum> + ● 1 ) + 0    shift
= ( <Sum> ● + 1 ) + 0    shift
=> ( 0 ● + 1 ) + 0    reduce
= ( ● 0 + 1 ) + 0    shift
= ● ( 0 + 1 ) + 0    shift

```

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Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

<Sum> =>

=	( <Sum> ● ) + 0	shift
=>	( <Sum> + <Sum> ● ) + 0	reduce
=>	( <Sum> + 1 ● ) + 0	reduce
=	( <Sum> + ● 1 ) + 0	shift
=	( <Sum> ● + 1 ) + 0	shift
=>	( 0 ● + 1 ) + 0	reduce
=	( ● 0 + 1 ) + 0	shift
=	( ● ( 0 + 1 ) + 0	shift

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Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

<Sum> =>

```

=> ( <Sum> ) ● + 0      reduce
= ( <Sum> ● ) + 0      shift
=> ( <Sum> + <Sum> ● ) + 0  reduce
=> ( <Sum> + 1 ● ) + 0  reduce
= ( <Sum> + ● 1 ) + 0    shift
= ( <Sum> ● + 1 ) + 0    shift
=> ( 0 ● + 1 ) + 0      reduce
= ( ● 0 + 1 ) + 0      shift
= ● ( 0 + 1 ) + 0      shift

```

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Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

<Sum> =>

```

= <Sum> ● + 0      shift
=> ( <Sum> ) ● + 0  reduce
= ( <Sum> ● ) + 0    shift
=> ( <Sum> + <Sum> ● ) + 0  reduce
=> ( <Sum> + 1 ● ) + 0  reduce
= ( <Sum> + ● 1 ) + 0  shift
= ( <Sum> ● + 1 ) + 0  shift
=> ( 0 ● + 1 ) + 0  reduce
= ( ● 0 + 1 ) + 0    shift
= ● ( 0 + 1 ) + 0    shift

```

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### Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

```

<Sum>   =>
          = <Sum> + ● 0      shift
          = <Sum> ● + 0      shift
          => ( <Sum> ) ● + 0  reduce
          = ( <Sum> ● ) + 0  shift
          => ( <Sum> + <Sum> ● ) + 0  reduce
          => ( <Sum> + 1 ● ) + 0  reduce
          = ( <Sum> + ● 1 ) + 0  shift
          = ( <Sum> ● + 1 ) + 0  shift
          => ( 0 ● + 1 ) + 0  reduce
          = ( ● 0 + 1 ) + 0  shift
          = ● ( 0 + 1 ) + 0  shift
  
```

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### Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

```

<Sum>   =>
          = > <Sum> + 0 ●      reduce
          = <Sum> + ● 0      shift
          = <Sum> ● + 0      shift
          => ( <Sum> ) ● + 0  reduce
          = ( <Sum> ● ) + 0  shift
          => ( <Sum> + <Sum> ● ) + 0  reduce
          => ( <Sum> + 1 ● ) + 0  reduce
          = ( <Sum> + ● 1 ) + 0  shift
          = ( <Sum> ● + 1 ) + 0  shift
          => ( 0 ● + 1 ) + 0  reduce
          = ( ● 0 + 1 ) + 0  shift
          = ● ( 0 + 1 ) + 0  shift
  
```

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### Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

```

<Sum>   => <Sum> + <Sum> ●  reduce
          => <Sum> + 0 ●  reduce
          = <Sum> + ● 0      shift
          = <Sum> ● + 0      shift
          => ( <Sum> ) ● + 0  reduce
          = ( <Sum> ● ) + 0  shift
          => ( <Sum> + <Sum> ● ) + 0  reduce
          => ( <Sum> + 1 ● ) + 0  reduce
          = ( <Sum> + ● 1 ) + 0  shift
          = ( <Sum> ● + 1 ) + 0  shift
          => ( 0 ● + 1 ) + 0  reduce
          = ( ● 0 + 1 ) + 0  shift
          = ● ( 0 + 1 ) + 0  shift
  
```

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### Example: $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

```

<Sum> ● => <Sum> + <Sum> ●  reduce
          => <Sum> + 0 ●  reduce
          = <Sum> + ● 0      shift
          = <Sum> ● + 0      shift
          => ( <Sum> ) ● + 0  reduce
          = ( <Sum> ● ) + 0  shift
          => ( <Sum> + <Sum> ● ) + 0  reduce
          => ( <Sum> + 1 ● ) + 0  reduce
          = ( <Sum> + ● 1 ) + 0  shift
          = ( <Sum> ● + 1 ) + 0  shift
          => ( 0 ● + 1 ) + 0  reduce
          = ( ● 0 + 1 ) + 0  shift
          = ● ( 0 + 1 ) + 0  shift
  
```

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### Example

$$(\quad 0 \quad + \quad 1 \quad ) \quad + \quad 0$$

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### Example

$$(\quad 0 \quad + \quad 1 \quad ) \quad + \quad 0$$

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### Example

$$(\quad 0 \quad + \quad 1 \quad ) \quad + \quad 0$$

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### Example

$$(\quad \text{<Sum>} \quad 0 \quad + \quad 1 \quad ) \quad + \quad 0$$

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$$(\quad \text{<Sum>} \quad 0 \quad + \quad 1 \quad ) \quad + \quad 0$$

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$$(\quad \text{<Sum>} \quad 0 \quad + \quad 1 \quad ) \quad + \quad 0$$

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$$(\quad \text{<Sum>} \quad 0 \quad + \quad \text{<Sum>} \quad 1 \quad ) \quad + \quad 0$$

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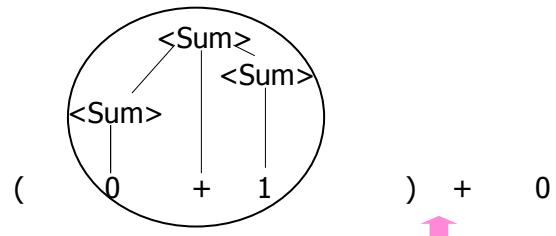
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$$(\quad \text{<Sum>} \quad 0 \quad + \quad \text{<Sum>} \quad 1 \quad ) \quad + \quad 0$$

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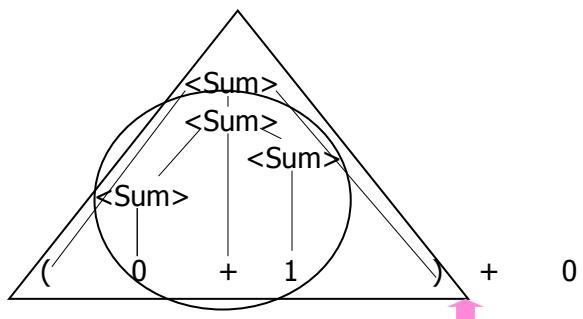
### Example



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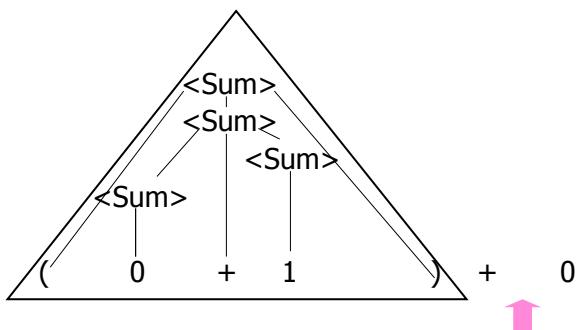
### Example



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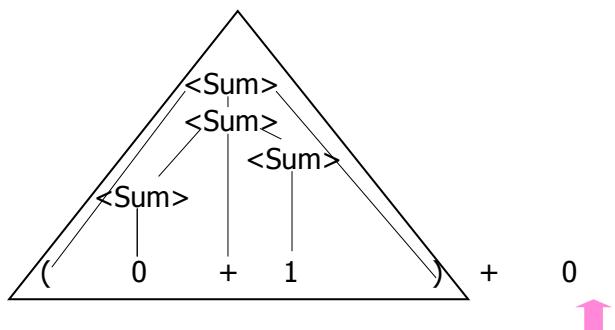
### Example



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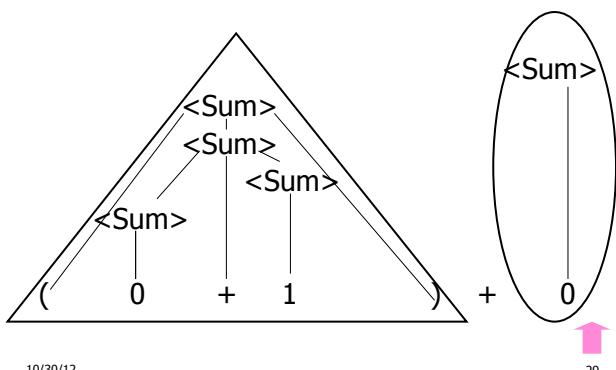
### Example



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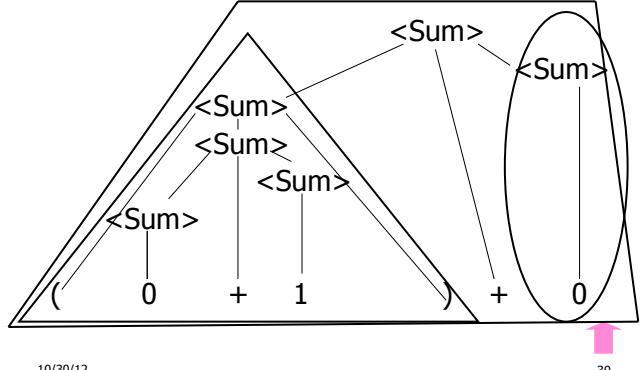
### Example



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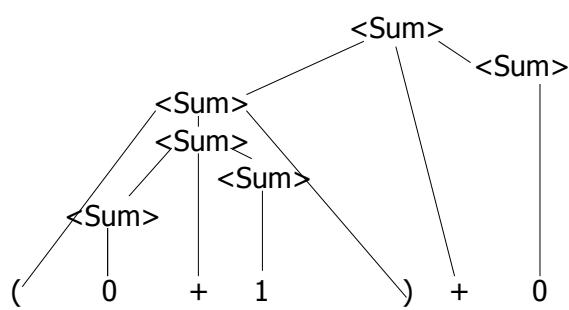
### Example



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## Example



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## LR Parsing Tables

- Build a pair of tables, Action and Goto, from the grammar
  - This is the hardest part, we omit here
  - Rows labeled by states
  - For Action, columns labeled by terminals and "end-of-tokens" marker
    - (more generally strings of terminals of fixed length)
  - For Goto, columns labeled by non-terminals

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## Action and Goto Tables

- Given a state and the next input, Action table says either
  - **shift** and go to state  $n$ , or
  - **reduce** by production  $k$  (explained in a bit)
  - **accept** or **error**
- Given a state and a non-terminal, Goto table says
  - go to state  $m$

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## LR(i) Parsing Algorithm

- Based on push-down automata
- Uses states and transitions (as recorded in Action and Goto tables)
- Uses a stack containing states, terminals and non-terminals

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## LR(i) Parsing Algorithm

0. Insure token stream ends in special "end-of-tokens" symbol
1. Start in state 1 with an empty stack
2. Push **state(1)** onto stack
- 3. Look at next  $i$  tokens from token stream ( $toks$ ) (don't remove yet)
4. If top symbol on stack is **state( $n$ )**, look up action in Action table at  $(n, toks)$

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## LR(i) Parsing Algorithm

5. If action = **shift**  $m$ ,
  - a) Remove the top token from token stream and push it onto the stack
  - b) Push **state( $m$ )** onto stack
  - c) Go to step 3

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## LR(i) Parsing Algorithm

6. If action = **reduce**  $k$  where production  $k$  is  $E ::= u$ 
  - a) Remove  $2 * \text{length}(u)$  symbols from stack (u and all the interleaved states)
  - b) If new top symbol on stack is **state**( $m$ ), look up new state  $p$  in  $\text{Goto}(m, E)$
  - c) Push  $E$  onto the stack, then push **state**( $p$ ) onto the stack
  - d) Go to step 3

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## LR(i) Parsing Algorithm

7. If action = **accept**
  - Stop parsing, return success
8. If action = **error**,
  - Stop parsing, return failure

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## Adding Synthesized Attributes

- Add to each **reduce** a rule for calculating the new synthesized attribute from the component attributes
- Add to each non-terminal pushed onto the stack, the attribute calculated for it
- When performing a **reduce**,
  - gather the recorded attributes from each non-terminal popped from stack
  - Compute new attribute for non-terminal pushed onto stack

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## Shift-Reduce Conflicts

- **Problem:** can't decide whether the action for a state and input character should be **shift** or **reduce**
- Caused by ambiguity in grammar
- Usually caused by lack of associativity or precedence information in grammar

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Example:  $\langle \text{Sum} \rangle = 0 \mid 1 \mid (\langle \text{Sum} \rangle \mid \langle \text{Sum} \rangle + \langle \text{Sum} \rangle)$

- $0 + 1 + 0$       shift
- >  $0 \bullet + 1 + 0$       reduce
- >  $\langle \text{Sum} \rangle \bullet + 1 + 0$       shift
- >  $\langle \text{Sum} \rangle + \bullet 1 + 0$       shift
- >  $\langle \text{Sum} \rangle + 1 \bullet + 0$       reduce
- >  $\langle \text{Sum} \rangle + \langle \text{Sum} \rangle \bullet + 0$

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## Example - cont

- **Problem:** shift or reduce?
- You can shift-shift-reduce-reduce or reduce-shift-shift-reduce
- Shift first - right associative
- Reduce first- left associative

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## Reduce - Reduce Conflicts

- **Problem:** can't decide between two different rules to reduce by
- Again caused by ambiguity in grammar
- **Symptom:** RHS of one production suffix of another
- Requires examining grammar and rewriting it
- Harder to solve than shift-reduce errors

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## Example

S ::= A | aB    A ::= abc    B ::= bc

● abc              shift  
a ● bc              shift  
ab ● c              shift  
abc ●

- Problem: reduce by B ::= bc then by S ::= aB, or by A ::= abc then S::A?

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## Using Ocamlacc

- Input attribute grammar is put in file *<grammar>.mly*
- Execute  
ocamlacc *<grammar>.mly*
- Produces code for parser in  
*<grammar>.ml*  
and interface (including type declaration for tokens) in  
*<grammar>.mli*

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## Parser Code

- *<grammar>.ml* defines one parsing function per entry point
- Parsing function takes a lexing function (lexer buffer to token) and a lexer buffer as arguments
- Returns semantic attribute of corresponding entry point

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## Ocamlacc Input

- File format:  
%{  
  *<header>*  
%}  
  *<declarations>*  
%%  
  *<rules>*  
%%  
  *<trailer>*

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## Ocamlacc *<header>*

- Contains arbitrary Ocaml code
- Typically used to give types and functions needed for the semantic actions of rules and to give specialized error recovery
- May be omitted
- *<footer>* similar. Possibly used to call parser

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## Ocamlyacc <declarations>

- **%token** *symbol ... symbol*
  - Declare given symbols as tokens
- **%token <type>** *symbol ... symbol*
  - Declare given symbols as token constructors, taking an argument of type *<type>*
- **%start** *symbol ... symbol*
  - Declare given symbols as entry points; functions of same names in *<grammar>.ml*

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## Ocamlyacc <declarations>

- **%type** *<type> symbol ... symbol*
  - Specify type of attributes for given symbols.
  - Mandatory for start symbols
- **%left** *symbol ... symbol*
  - **%right** *symbol ... symbol*
  - **%nonassoc** *symbol ... symbol*
  - Associate precedence and associativity to given symbols. Same line, same precedence; earlier line, lower precedence (broadest scope)

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## Ocamlyacc <rules>

- **nonterminal :**
  - symbol ... symbol { semantic\_action }*
  - | ...
  - | *symbol ... symbol { semantic\_action }*
- ;
- Semantic actions are arbitrary Ocaml expressions
- Must be of same type as declared (or inferred) for *nonterminal*
- Access semantic attributes (values) of symbols by position: \$1 for first symbol, \$2 to second ...

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## Example - Base types

```
(* File: expr.ml *)
type expr =
  Term_as_Expr of term
  | Plus_Expr of (term * expr)
  | Minus_Expr of (term * expr)
and term =
  Factor_as_Term of factor
  | Mult_Term of (factor * term)
  | Div_Term of (factor * term)
and factor =
  Id_as_Factor of string
  | Parenthesized_Expr_as_Factor of expr
```

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## Example - Lexer (exprlex.mll)

```
{ (*open Exprparse*) }
let numeric = ['0' - '9']
let letter =[ 'a' - 'z' 'A' - 'Z' ]
rule token = parse
  "+" {Plus_token}
  "-" {Minus_token}
  "*" {Times_token}
  "/" {Divide_token}
  "(" {Left_parenthesis}
  ")" {Right_parenthesis}
  letter (letter|numeric|"_")* as id {Id_token id}
  [' ' '\t' '\n'] {token lexbuf}
  eof {EOL}
```

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## Example - Parser (exprparse.mly)

```
%{ open Expr
%}
%token <string> Id_token
%token Left_parenthesis Right_parenthesis
%token Times_token Divide_token
%token Plus_token Minus_token
%token EOL
%start main
%type <expr> main
%%
```

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## Example - Parser (exprparse.mly)

```
expr:  
  term  
    { Term_as_Expr $1 }  
  | term Plus_token expr  
    { Plus_Expr ($1, $3) }  
  | term Minus_token expr  
    { Minus_Expr ($1, $3) }
```

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## Example - Parser (exprparse.mly)

```
term:  
  factor  
    { Factor_as_Term $1 }  
  | factor Times_token term  
    { Mult_Term ($1, $3) }  
  | factor Divide_token term  
    { Div_Term ($1, $3) }
```

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## Example - Parser (exprparse.mly)

```
factor:  
  Id_token  
    { Id_as_Factor $1 }  
  | Left_parenthesis expr Right_parenthesis  
    { Parenthesized_Expr_as_Factor $2 }  
main:  
  | expr EOL  
    { $1 }
```

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## Example - Using Parser

```
# #use "expr.ml";;  
...  
# #use "exprparse.ml";;  
...  
# #use "exprlex.ml";;  
...  
# let test s =  
let lexbuf = Lexing.from_string (s^"\n") in  
  main token lexbuf;;
```

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## Example - Using Parser

```
# test "a + b";;  
- : expr =  
Plus_Expr  
(Factor_as_Term (Id_as_Factor "a"),  
 Term_as_Expr (Factor_as_Term  
 (Id_as_Factor "b")))
```

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