CS 421 Midterm 1          **Name:**_____

**Rules for type derivations:**

Constants:

$$\frac{\quad\quad\quad}{\Gamma\vdash n : \text{int}} \quad (\text{assuming } n \text{ is an integer constant})$$

$$\frac{\quad\quad\quad\quad}{\Gamma\vdash \text{true} : \text{bool}} \quad\quad \frac{\quad\quad\quad\quad}{\Gamma\vdash \text{false} : \text{bool}}$$

Variables:

$$\frac{\quad\quad\quad}{\Gamma\vdash x : \sigma} \quad \text{if } \Gamma(x) = \sigma$$

Primitive operators ( $\oplus \in \{ +, -, *, \text{mod}, \ldots \}$ ):

$$\frac{\Gamma\vdash e_1 : \text{int} \quad \Gamma\vdash e_2 : \text{int}}{\Gamma\vdash e_1 \oplus e_2 : \text{int}}$$

Relations ( $\sim \in \{ <, >, =, <=, >= \}$ ):

$$\frac{\Gamma\vdash e_1 : \tau \quad \Gamma\vdash e_2 : \tau}{\Gamma\vdash e_1 \sim e_2 : \text{bool}}$$

Connectives :

$$\frac{\Gamma\vdash e_1 : \text{bool} \quad \Gamma\vdash e_2 : \text{bool}}{\Gamma\vdash e_1 \;\&\&\; e_2 : \text{bool}} \quad\quad \frac{\Gamma\vdash e_1 : \text{bool} \quad \Gamma\vdash e_2 : \text{bool}}{\Gamma\vdash e_1 \;||\; e_2 : \text{bool}}$$

If_then_else rule:

$$\frac{\Gamma\vdash e_1 : \text{bool} \quad \Gamma\vdash e_2 : \tau \quad \Gamma\vdash e_3 : \tau}{\Gamma\vdash (\text{if } e_1 \text{ then } e_2 \text{ else } e_3) : \tau}$$

Application rule:

$$\frac{\Gamma\vdash e_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma\vdash e_2 : \tau_1}{\Gamma\vdash (e_1 \; e_2) : \tau_2}$$

Function rule:

$$\frac{[x : \tau_1] + \Gamma\vdash e : \tau_2}{\Gamma\vdash \text{fun } x \rightarrow e : \tau_1 \rightarrow \tau_2}$$

Let rule:

$$\frac{\Gamma\vdash e_1 : \tau_1 \quad [x : \tau_1] + \Gamma\vdash e_2 : \tau_2}{\Gamma\vdash (\text{let } x = e_1 \text{ in } e_2) : \tau_2}$$

Let Rec rule:

$$\frac{[x : \tau_1] + \Gamma\vdash e_1 : \tau_1 \quad [x : \tau_1] + \Gamma\vdash e_2 : \tau_2}{\Gamma\vdash (\text{let rec } x = e_1 \text{ in } e_2) : \tau_2}$$