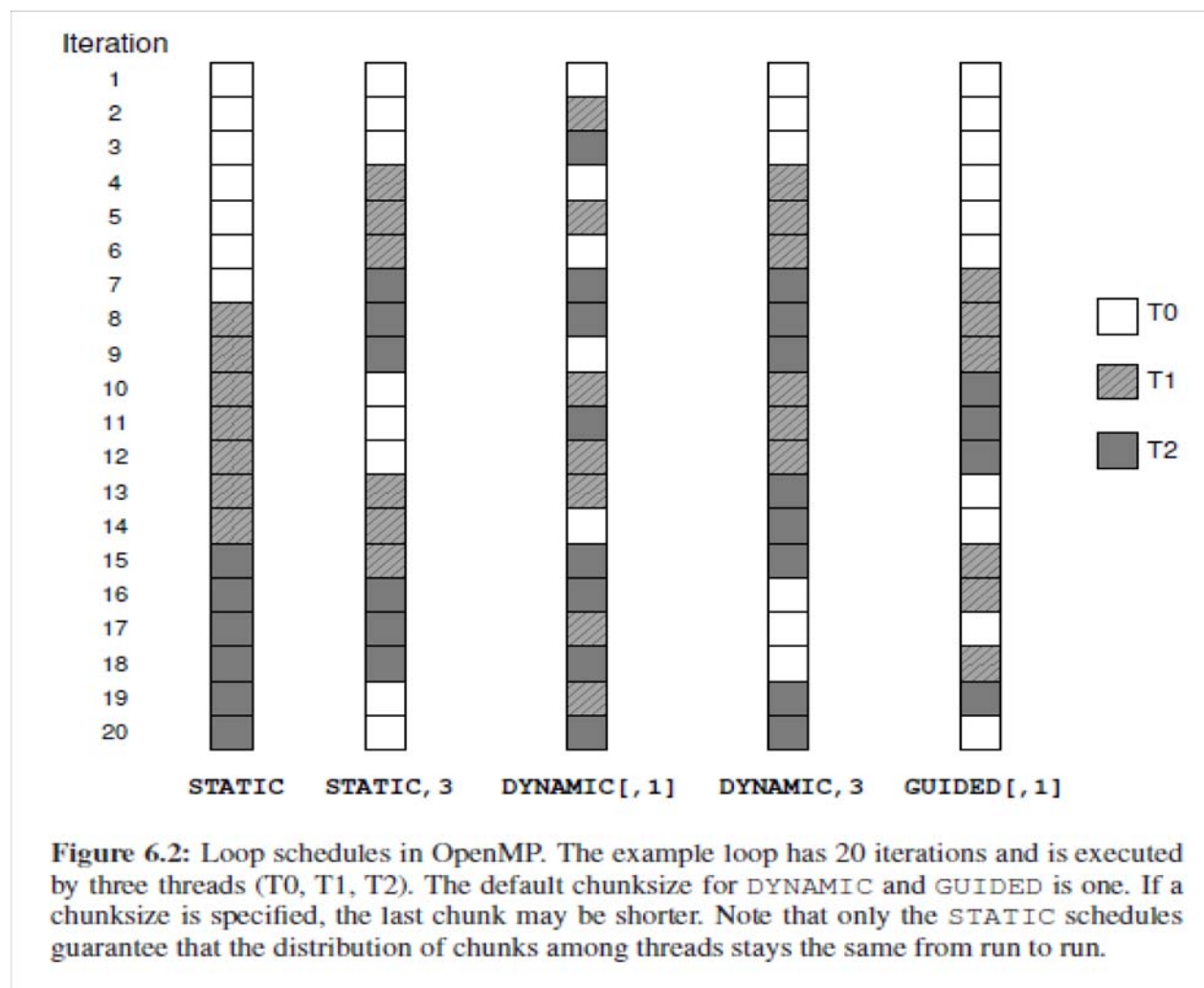


# Scheduling in OpenMP



# Tasking in OpenMP

```
1  integer i,N=1000000
2  type(object), dimension(N) :: p
3  double precision :: r
4  ...
5  !$OMP PARALLEL PRIVATE(r, i)
6  !$OMP SINGLE
7  do i=1,N
8      call RANDOM_NUMBER(r)
9      if(p(i)%weight > r) then
10 !$OMP TASK
11     ! i is automatically firstprivate
12     ! p() is shared
13     call do_work_with(p(i))
14 !$OMP END TASK
15     endif
16     enddo
17 !$OMP END SINGLE
18 !$OMP END PARALLEL
```

# The Wavefront method

First consider a simple two dimensional Fortran 77 form of this method. That is, two dimensional loop nests with a single statement inside that assigns to a two dimensional array.

To illustrate this method we draw a graph of the iteration space of the loop. Each iteration will be a node in the graph. The graph will take the form of a mesh with equal vertical and horizontal separation.

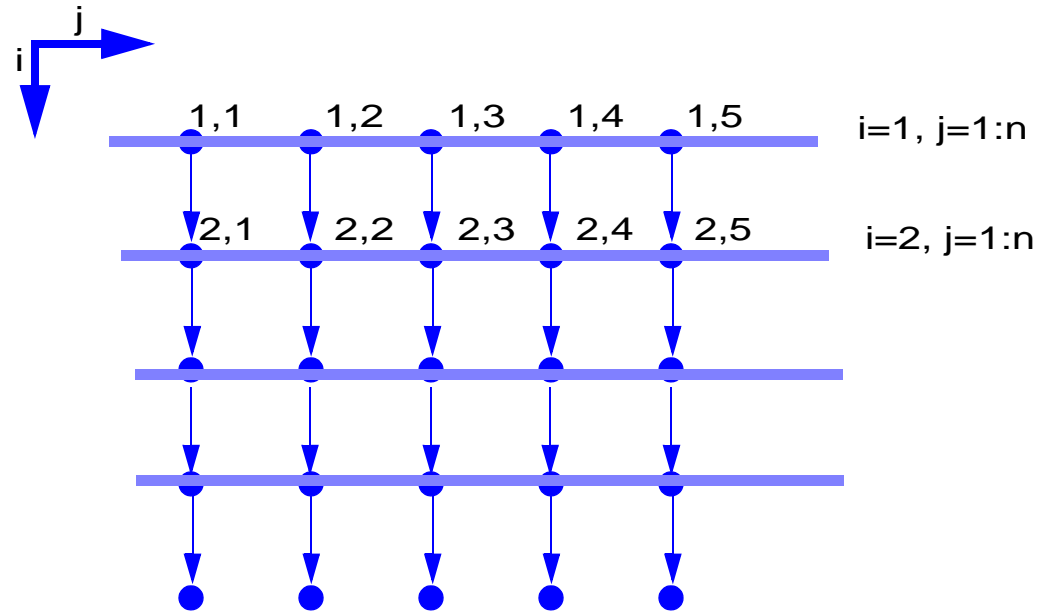
These nodes will be joined by three classes of arcs representing races (write-read, read-write, write-write). These arcs (which are called **dependences**) will always flow in the direction of execution in the original loop.

The idea is that a vector form can be obtained by finding a collection of parallel lines that are equidistant, are not parallel to any dependence arc, and pass through all the nodes in the graph.

For example, the loop

```
do i=1,n
  do j=1,n
    a(i,j)=a(i-1,j)+1
  end do
end do
```

can be represented by the following graph:

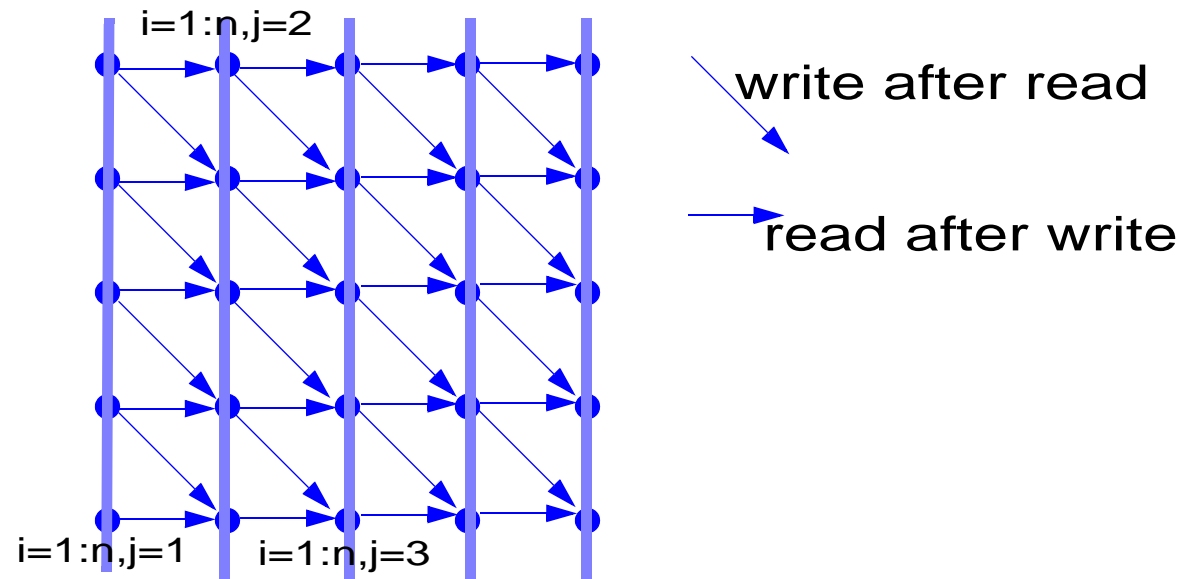


From the graph it is clear that for each  $i$  there is a vector operation in  $j$ .

```
do i=1,n
  a(i,1:n)=a(i-1,1:n)+1
end do
```

A second example:

```
do i=1,n
  do j=1,n
    a(i,j)=a(i,j-1)+a(i+1,j+1)+b(i)+c(j)
  end do
end do
```

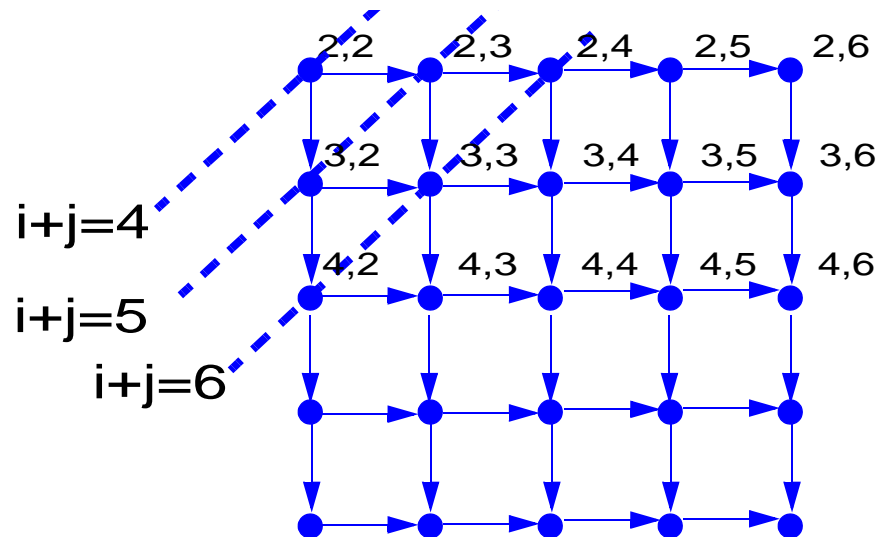


Now for each  $j$  there is a vector operation

```
do j=1,n
  a(1:n,j)=a(1:n,j-1)+a(2:n+1,j+1)+b(i)+c(1:n)
end do
```

A more complicated case:

```
do i=2,n
  do j=2,n
    a(i,j)=a(i,j-1)+a(i-1,j)
  end do
end do
```



From the equations:

$$\begin{aligned}2 &\leq i \leq n \\2 &\leq k - i \leq n \\k &= 4, 5, \dots, 2n\end{aligned}$$

We conclude that:

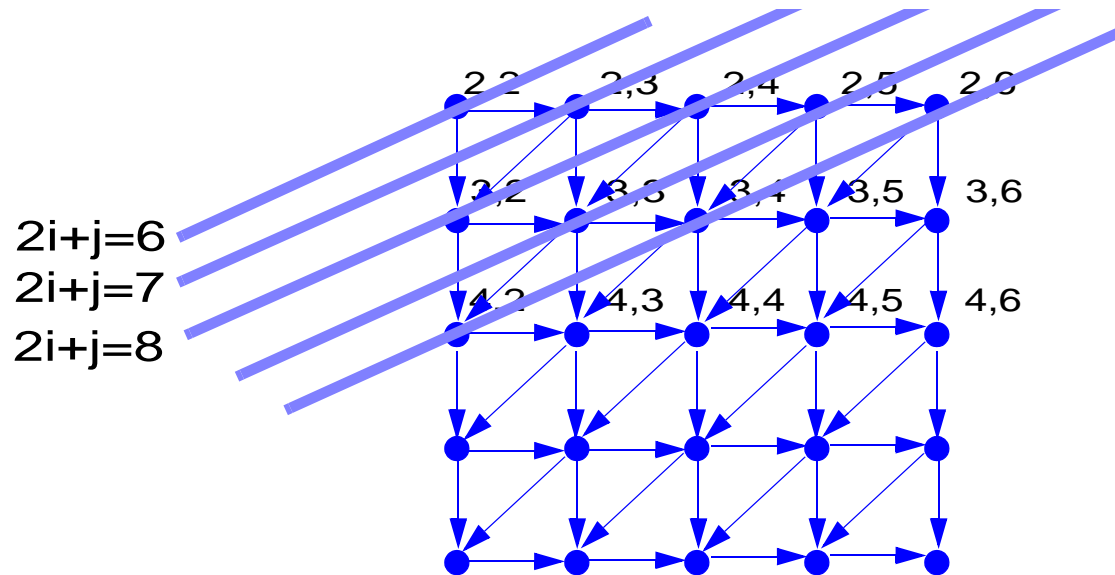
$$\max(2, k - n) \leq i \leq \min(n, k - 2)$$

From where:

```
do k=4, 2*n
  forall (i=max(2, k-n) : min(n, k-2)) a(i, k-j) = ...
end do
```

Another complex example:

```
do i=2,n
  do j=2,n
    a(i,j)=a(i+1,j-1)+a(i-1,j)+a(i,j-1)
  end do
end do
```





From the equations:

$$\begin{aligned}4 &\leq 2i \leq 2n \\ 2 &\leq k - 2i \leq n \\ k &= 6, 5, \dots, 3n\end{aligned}$$

We conclude that:

$$\max\left(2, \left\lceil \frac{k-n}{2} \right\rceil\right) \leq i \leq \min\left(n, \left\lfloor \frac{k-2}{2} \right\rfloor\right)$$

From where:

```
do k=6, 3*n
  forall (i=max(2, (k-n+1)/2) : min(n, (k-2)/2)) a(i, k-j) = ...
end do
```

**Listing 6.5:** OpenMP implementation of the 2D Jacobi algorithm on an  $N \times N$  lattice, with a convergence criterion added.

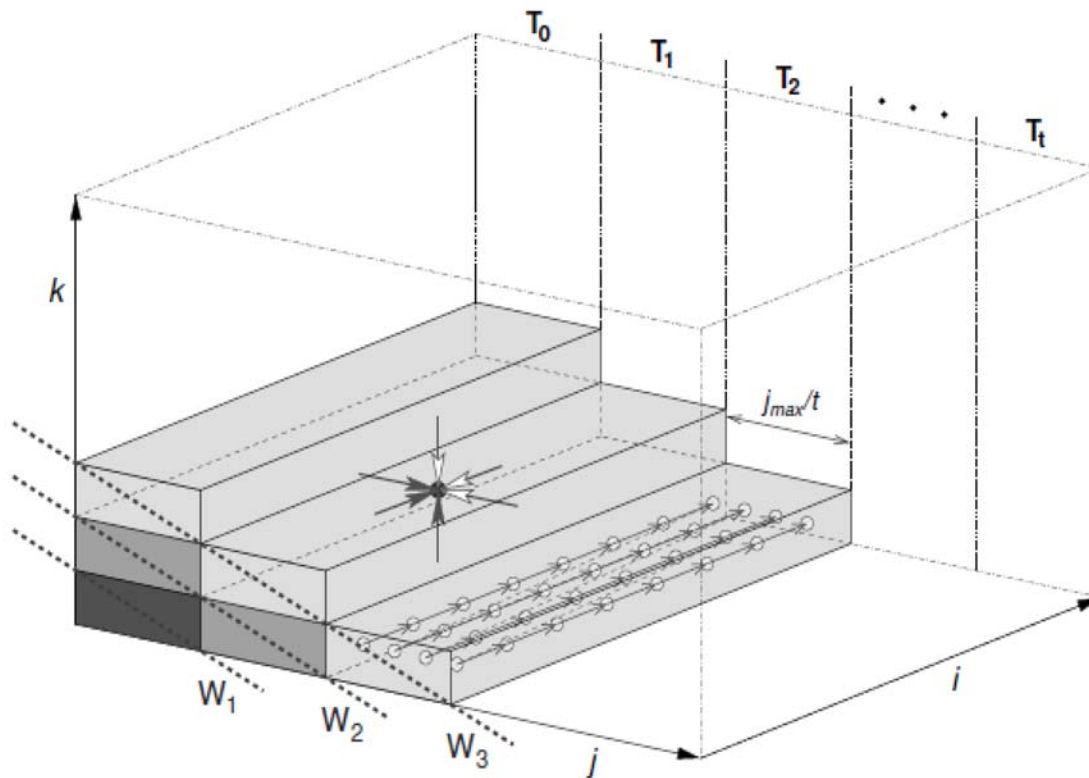
```
1  double precision, dimension(0:N+1,0:N+1,0:1) :: phi
2  double precision :: maxdelta,eps
3  integer :: t0,t1
4  eps = 1.d-14      ! convergence threshold
5  t0 = 0 ; t1 = 1
6  maxdelta = 2.d0*eps
7  do while(maxdelta.gt.eps)
8      maxdelta = 0.d0
9  !$OMP PARALLEL DO REDUCTION(max:maxdelta)
10     do k = 1,N
11         do i = 1,N
12             ! four flops, one store, four loads
13             phi(i,k,t1) = ( phi(i+1,k,t0) + phi(i-1,k,t0)
14                 + phi(i,k+1,t0) + phi(i,k-1,t0) ) * 0.25
15             maxdelta = max(maxdelta,abs(phi(i,k,t1)-phi(i,k,t0)))
16         enddo
17     enddo
18 !$OMP END PARALLEL DO
19     ! swap arrays
20     i = t0 ; t0=t1 ; t1=i
21 enddo
```

**Listing 6.6:** A straightforward implementation of the Gauss–Seidel algorithm in three dimensions. The highlighted references cause loop-carried dependencies.

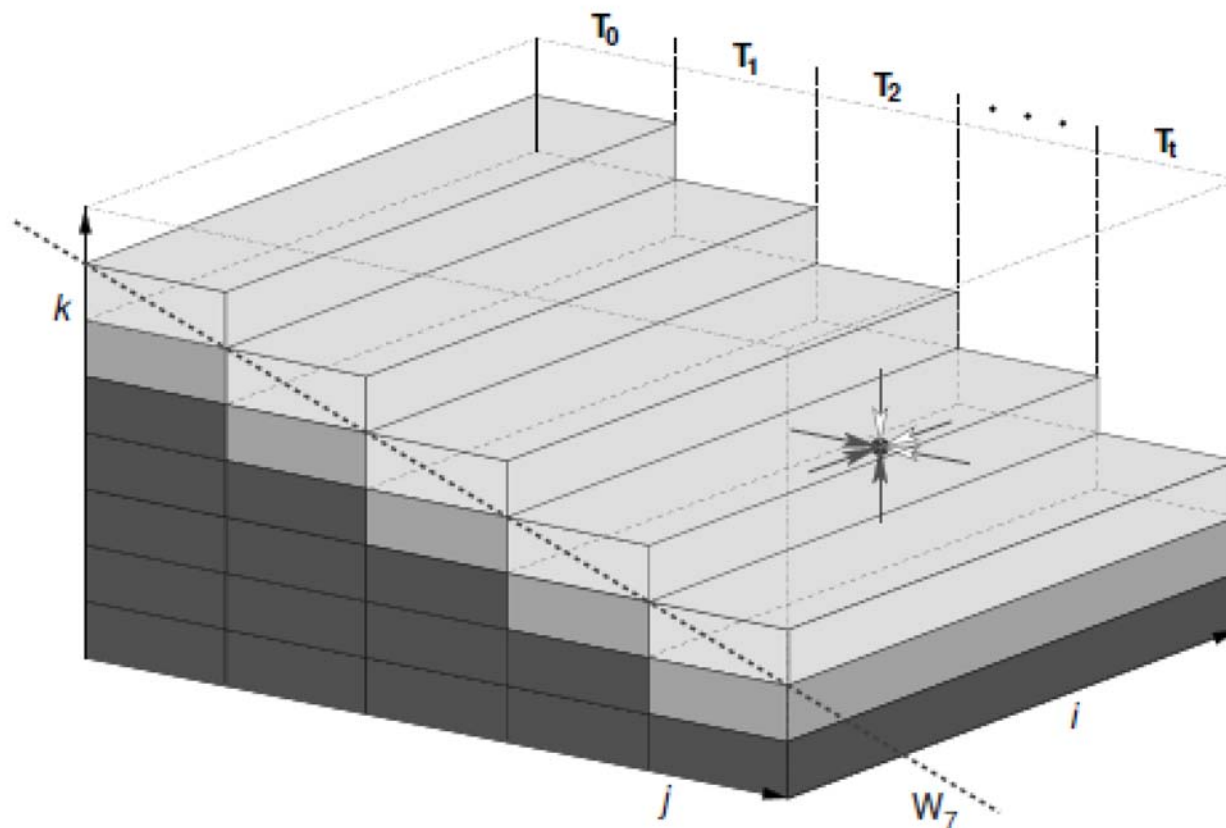
---

```
1 double precision, parameter :: osth=1/6.d0
2 do it=1,itmax ! number of iterations (sweeps)
3   ! not parallelizable right away
4   do k=1,kmax
5     do j=1,jmax
6       do i=1,imax
7         phi(i,j,k) = ( phi(i-1,j,k) + phi(i+1,j,k)
8                       + phi(i,j-1,k) + phi(i,j+1,k)
9                       + phi(i,j,k-1) + phi(i,j,k+1) ) * osth
10        enddo
11      enddo
12    enddo
13  enddo
```

---



**Figure 6.4:** Pipeline parallel processing (PPP), a.k.a. wavefront parallelization, for the Gauss-Seidel algorithm in 3D (wind-up phase). In order to fulfill the dependency constraints of each stencil update, successive wavefronts ( $W_1, W_2, \dots, W_n$ ) must be performed consecutively, but multiple threads can work in parallel on each individual wavefront. Up until the end of the wind-up phase, only a subset of all  $t$  threads can participate.



**Figure 6.5:** Wavefront parallelization for the Gauss-Seidel algorithm in 3D (full pipeline phase). All  $t$  threads participate. Wavefront  $W_7$  is shown as an example.

**Listing 6.7:** The wavefront-parallel Gauss–Seidel algorithm in three dimensions. Loop-carried dependencies are still present, but threads can work in parallel.

---

```
1  !$OMP PARALLEL PRIVATE(k, j, i, jStart, jEnd, threadID)
2    threadID=OMP_GET_THREAD_NUM()
3  !$OMP SINGLE
4    numThreads=OMP_GET_NUM_THREADS()
5  !$OMP END SINGLE
6    jStart=jmax/numThreads+threadID
7    jEnd=jStart+jmax/numThreads ! jmax is a multiple of numThreads
8    do l=1, kmax+numThreads-1
9      k=l-threadID
10     if((k.ge.1).and.(k.le.kmax)) then
11       do j=jStart, jEnd          ! this is the actual parallel loop
12         do i=1, iMax
13           phi(i, j, k) = ( phi(i-1, j, k) + phi(i+1, j, k)
14                         + phi(i, j-1, k) + phi(i, j+1, k)
15                         + phi(i, j, k-1) + phi(i, j, k+1) ) * osth
16         enddo
17       enddo
18     endif
19  !$OMP BARRIER
20  enddo
21 !$OMP END PARALLEL
```

---