3D Geometry and Geometric Primitives

1. Draw a picture of a triangulated polygon that can be drawn using a single triangle fan but not a single triangle strip. You must use only the minimum number of vertices. No degenerate triangles can be used.

In a strip, no vertex can appear in more than 3 triangles.

2. Draw a picture of a triangulated polygon that can be drawn using a single triangle strip but not a single triangle fan. You must use only the minimum number of vertices. No degenerate triangles can be used.

In fan, one vertex appears in all triangles.
3. Suppose a triangle has a normal vector of $\langle 1,1,0 \rangle$ and that the vector for the view direction is $\langle 1,-2,0 \rangle$. Is the triangle front-facing or back-facing?

$$\langle 1,-2,0 \rangle \cdot \langle 1,1,0 \rangle = -1$$

Assuming the view vector is expressed as running from the eyepoint to the surface, the triangle is front-facing.

4. The following vertex buffer is suitable for drawing 3 triangles using `gl.TRIANGLES` and `gl.DRAW_ARRAYS`. Convert the buffer to one suitable for drawing the same triangles using `gl.TRIANGLE_STRIP` and `gl.DRAW_ARRAYS`. Assume we are using a CCW winding order.

<table>
<thead>
<tr>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>V3</th>
<th>V4</th>
<th>V1</th>
<th>V4</th>
<th>V3</th>
<th>V5</th>
</tr>
</thead>
</table>

We can express the mesh as a strip with this buffer:

| V5 | V4 | V3 | V1 | V2 | V5 | V4 | V3 | V1 | V2 |

5.
The Euler Characteristic

The Euler Characteristic states the following relationship for the elements of a closed and connected surface mesh:

\[ V - E + F = 2(1 - G) \]

- \( V \) is the number of vertices
- \( E \) is the number of edges
- \( F \) is the number of faces
- \( G \) is the genus of the surface (how holes/handles it has)

Show that for a triangle mesh with no holes we have \( F \approx 2V \). Hint: each face has 3 edges and each edge is shared by 2 faces.

\[
\begin{align*}
V - E + F &= 2 \\
V - \left( \frac{3}{2} \right) F + F &= 2 \\
V - \left( \frac{1}{2} \right) F &= 2 \\
V &= 2 + \left( \frac{1}{2} \right) F \\
2V &= 2 + F
\end{align*}
\]

Memory Requirements

Using the fact that \( F \approx 2V \), compare the storage requirements for an indexed face mesh and a triangle soup (in WebGL this corresponds to using \texttt{gl.drawElements} versus \texttt{gl.drawArrays}). Assume the mesh has \( V \) vertices and a number requires 4 bytes of space. Derive functions for the number of bytes the mesh will require as a function of \( V \).

- **drawArrays**: \( F \times 3 \) vertices per face \( \times 3 \) coords per vertex \( \times 4 \)B per number \( = 2V \times 36 \) = 72 bytes per vertex

- **drawElements**: \( F \times 3 \) vertex indices per face \( \times 4 \) bytes + \( V \times 3 \) coords per vertex \( \times 4 \) bytes

\( = 2V \times 12 + 12V = 36V \) = 36 bytes per vertex