CS 418: Interactive Computer Graphics Spring 2018
In-class Worksheet 5

Basic Shading and Interpolation

Phong Shading

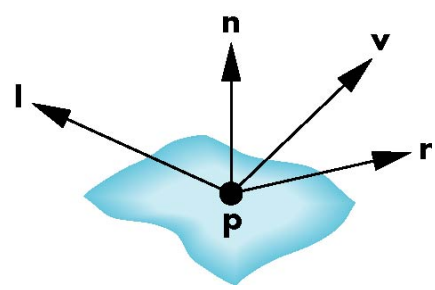
For each pixel p compute, a color for the pixel using the following reflection model for each of the Red, Green, and Blue color channels:

$$I = k_d I_d \ \mathbf{l} \cdot \mathbf{n} + k_s I_s (\mathbf{v} \cdot \mathbf{r})^a + k_a I_a$$

k: reflectance coefficient in [0,1]
l: unit vector from vertex to light
n: unit surface normal at the vertex
v: unit vector in the direction of the viewer
r: unit vector in the mirror reflectance direction

a: shininess coefficient in [0,∞]
I: Illumination indicates light intensity in [0,1]
subscripts *d*, *s*, and *a*: diffuse, specular, and ambient

1. **Shading a Vertex**
   Suppose we have the following values for a given color channel:
   n = $\langle 0,1,0 \rangle$
   v = $\langle 0, {}^{1}/_{\sqrt{2}}, {}^{1}/_{\sqrt{2}} \rangle$
   l = $\langle 0, {}^{1}/_{\sqrt{2}}, -{}^{1}/_{\sqrt{2}} \rangle$
   a = 5
   $k_d = {}^{1}/_{\sqrt{2}}$
   $I_d = 1$
   $k_s = ¼$
   $I_s = 1$
   $k_a = ¼$
   $I_a = ¼$



   Compute the vector $r = 2(l \cdot n)n - l$

$$2(0,1,0) \cdot \left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 2(0 + \frac{1}{\sqrt{2}} + 0) = 2/\sqrt{2}$$

$$\frac{2}{\sqrt{2}}(0,1,0) = (0,2/\sqrt{2},0)$$

$$r = (0,2/\sqrt{2},0) - (0,1/\sqrt{2},-1/\sqrt{2}) = (0,1/\sqrt{2},1/\sqrt{2})$$

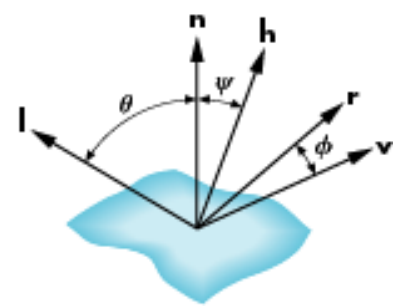
Compute the illumination in the color channel as a rational number. You may use an approximation of $^{1}/_{\sqrt{2}} \approx 7/10$ if you wish.

$$I = \frac{1}{\sqrt{2}}(1)(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) \cdot (0, 1, 0) + (1/4)1\left(\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \cdot \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\right)^5 + (1/4)1/4$$

$$I = \frac{1}{2} + \frac{1}{4}(1)^5 + \frac{1}{16}$$

$$I = \frac{1}{2} + \frac{1}{4} + \frac{1}{16}$$

$$I = \frac{13}{16}$$



2. The Halfway Vector

The Blinn-Phong shading model replaces the $r \cdot v$ computation with $n \cdot h$ where $h = \frac{l+v}{|l+v|}$

a. What is the benefit of making this substitution?
It is requires fewer arithmetic operations to compute. Also, Blinn-Phong is arguably more physically correct.

b. Will this substitution change the image being generated and if so, how?
Yes, if the shininess exponent is kept the same, the specular highlights will be larger in Blinn-Phong.

3. **Attenuation**

Suppose you wish to incorporate an attenuation term so that illumination diminishes with as the distance from a light source to the vertex increases.

a. Where in the Phong shading model will the term be present?

$$\text{attenuationTerm}*(k_d\, I_d\ \mathbf{l} \cdot \mathbf{n}\ + k_s\, I_s\, (\mathbf{v} \cdot \mathbf{r})^a)$$

b. What form does the attenuation term usually take?

$$\frac{1}{a + br + cr^2}$$

4. **Linear Interpolation of Normal Vectors**

Suppose a line segment (a,b) has a normal defined at vertex a of <1,0,0> and a normal at vertex b of <0,1,0>. What is the interpolated normal at a fragment at the midpoint of the line segment? Is this normal unit length? What does this imply for how you should implement Phong shading?

It would be <0.5, 0, 0.5>. It is not unit length. In Phong shading, interpolated normal vectors must be re-normalized inside the fragment shader.