More on Affine Transformations

1. Suppose we have 2D frame with an origin at (2,2) and basis vectors 
   \( u = \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \) \( v = \left( -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \) Suppose a point \( p = (1,1) \) in \( u,v \) coordinates. What is \( p \) in \( x,y \) coordinates?

2. Construct a matrix that can convert a point in \( u,v \) coordinates to \( x,y \) coordinates for the basis described above.

3. What matrix would perform the window-to-viewport transformation specified by the function call \( \text{gl.viewPort}(0,0,960,640) \)? For this question, just ignore the z-coordinate and imagine the NDC coordinates to be transformed are in the form \((x, y,1)\). Express your answer as a 3x3 matrix and express rational numbers as fractions.

In WebGL, the window to viewport transformation is performed by the WebGL library during primitive assembly. You control it using the call \( \text{gl.viewPort}(x, y, w, h) \). It specifies the viewport by giving the lower-left coordinate of the viewport with \((x, y)\) and the width and height of the viewport with \(w\) and \(h\).
4. Suppose we have the following set of transformations that map a point in view (or camera) space to world coordinates. What transformations will map world coordinates to camera space?

\[
\begin{bmatrix}
1 & 0 & 0 & e_x \\
0 & 1 & 0 & e_y \\
0 & 0 & 1 & e_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
u_x \\
v_y \\
v_z \\
w
\end{bmatrix}
= \begin{bmatrix}
u_x \\
v_y \\
v_z \\
w
\end{bmatrix}
\]

Bonus question:
What is the inverse of matrix that performs a scale transformation?

\[
\begin{bmatrix}
s_x & 0 & 0 & 0 \\
0 & s_y & 0 & 0 \\
0 & 0 & s_z & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]