Beziers Patches

Bilinear Patches

A bilinear patch is a finite piece of a parametric surface defined by four control points $b_{0,0}$, $b_{1,0}$, $b_{0,1}$, and $b_{1,1}$.

Points on the patch can be found by evaluating:

$$X(u,v) = (1-v)p^u + vq^u$$

with

$$p^u = (1-u)b_{0,0} + ub_{1,0} \quad q^u = (1-u)b_{0,1} + ub_{1,1}$$

1. **A Bilinear Patch**

Suppose our control points are

$$b_{0,0}=(0,0,0) \quad b_{1,0}=(1,0,0) \quad b_{0,1}=(0,1,0) \quad b_{1,1}=(1,1,1)$$

What point on the patch is found by evaluating $X(0.25, 0.5)$?
2. Bezier Patch

Beziers Patch

A Bezier patch is a finite piece of a parametric surface defined by a control net of points. The points on the patch are evaluated using

\[ X(u, v) = \begin{bmatrix} B_0^0(u) & \cdots & B_0^n(u) \\ \vdots & \ddots & \vdots \\ B_m^0(u) & \cdots & B_m^n(u) \end{bmatrix} \begin{bmatrix} b_0,0 & \cdots & b_0,n \\ \vdots & \ddots & \vdots \\ b_m,0 & \cdots & b_m,n \end{bmatrix} \begin{bmatrix} B_0^0(v) \\ \vdots \\ B_n^0(v) \end{bmatrix} \]

a. For quadratic Bezier curves we have

\[ B_0^0(t) = (1 - t)^2, B_1^1(t) = 2t(1 - t), B_2^2(t) = t^2 \]

Suppose we have \( B = \begin{bmatrix} (0,0,6) & (3,0,0) & (6,0,0) \\ (0,3,3) & (3,3,0) & (6,3,0) \\ (0,6,6) & (3,6,0) & (6,6,0) \end{bmatrix} \)

What point is \( X(0.5,0.5) \)?