Beziers Curves

Cubic Bezier Curves

A Bezier curve is a parametric polynomial curve given by:

\[ X(t) = (1 - t)^3 b_0 + 3(1 - t)^2 tb_1 + 3(1 - t)t^2 b_2 + t^3 b_3 \]

where \( b_i \) are the control points.

The tangent vector of the curve can be found by

\[ X(t) = 3(b_1 - b_0)(1 - t)^2 + 6(b_2 - b_1)(1 - t)t + 3(b_3 - b_2)t^2 \]

1. The de Casteljau Algorithm

Suppose our control points are

\[ b_0 = (-1, 0) \quad b_1 = (0, 1) \quad b_2 = (0, -1) \quad b_3 = (1, 0) \]

Use the de Casteljau algorithm to find the coordinates of \( X(1/4) \). Check that you get the same answer from using the parametric expression given above.
2. **Tangents to a Bezier Curve**
   a. What are the tangents at the controls \( b_0 \) and \( b_3 \)?
      Give the answer as a pair of parameterized functions.

   b. What is the tangent vector at \( t=0.25 \) for the curve given in question one?