Subdivision Surfaces

CS 418
Intro to Computer Graphics
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Knot Insertion

[0 2 4 6 8 10]
Knot Insertion

[0 2 4 5 6 8 10]
Knot Insertion

[0 2 4 5 6 8 10]
Loop Knot Insertion
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Smoothing a Polygon
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1. Add edge midpoints
Smoothing a Polygon

1. Add edge midpoints
2. Add struts
   - Struts connect midpoints of segments from vertices to edge midpoints
   - One strut per vertex
Smoothing a Polygon

1. Add edge midpoints
2. Add struts
   - Struts connect midpoints of segments from vertices to edge midpoints
   - One strut per vertex
3. Add strut midpoints
Smoothing a Polygon

1. Add edge midpoints
2. Add struts
   - Struts connect midpoints of segments from vertices to edge midpoints
   - One strut per vertex
3. Add strut midpoints
4. Connect
Smoothing a Polygon

1. Add edge midpoints
2. Add struts
   - Struts connect midpoints of segments from vertices to edge midpoints
   - One strut per vertex
3. Add strut midpoints
4. Connect
5. Repeat
B-Spline Patches

- Tensor product of two curves
  
  \[ p(s, t) = \sum_{j=0}^{n} \sum_{i=0}^{n} N_j^n(s) N_i^n(t) p_{ij} \]

- Need to subdivide control points to create four sub-patches
- Need to generate new control points
  - vertex points (replacing control points)
  - edge points
  - face points
Face Points

- Approximate edge points as midpoint of control points
  \[ E = \frac{1}{2} p + \frac{1}{2} p \]

- Face point is midpoint of approximate edge points
  \[ F = \frac{1}{2} E + \frac{1}{2} E \]
  \[ = \frac{1}{4} p + \frac{1}{4} p + \frac{1}{4} p + \frac{1}{4} p \]
Edge Points

- Face points are midpoints between approx. edge points
- Approx. edge point is midpoint between control points
- Actual edge point is midpoint between midpoints between approx edge point and face points

\[
E = \frac{1}{2} \left( \frac{1}{2} \left( \frac{1}{2} E_0 + \frac{1}{2} E_1 \right) + \frac{1}{2} E_1 \right) + \frac{1}{2} \left( \frac{1}{2} E_1 + \frac{1}{2} \left( \frac{1}{2} E_1 + \frac{1}{2} E_2 \right) \right)
\]

\[
= \frac{1}{2} \left( \frac{1}{2} F_0 + \frac{1}{2} \left( \frac{1}{2} p_0 + \frac{1}{2} p_1 \right) \right) + \frac{1}{2} \left( \frac{1}{2} \left( \frac{1}{2} p_0 + \frac{1}{2} p_1 \right) + \frac{1}{2} F_1 \right)
\]

\[
= \frac{1}{4} (F_0 + p_0 + p_1 + F_1)
\]
Vertex Points

\[ V_0 = \frac{1}{4} E_0 + \frac{1}{2} p_0 + \frac{1}{4} E_1 \]
\[ V_2 = \frac{1}{4} E_2 + \frac{1}{2} p_2 + \frac{1}{4} E_3 \]
\[ V = \frac{1}{2} \left( \frac{1}{2} (\frac{1}{2} V_0 + \frac{1}{2} V_1) + \frac{1}{2} V_1 \right) + \frac{1}{2} \left( \frac{1}{2} V_1 + \frac{1}{2} (\frac{1}{2} V_1 + \frac{1}{2} V_2) \right) \]

\[ = \frac{1}{4} \left( \frac{1}{4} (F_0 + F_1 + p_0 + p_1) + \frac{1}{4} (F_2 + F_3 + p_1 + p_2) + 2 V_1 \right) \]

\[ = \frac{1}{4} \left( \frac{1}{4} (F_0 + F_1 + F_2 + F_3) + \frac{1}{4} (p_0 + 2 p_1 + p_2) + 2/4 (E_2 + E_3 + 2 p_1) \right) \]

\[ = \frac{1}{16} (F_0 + F_1 + F_2 + F_3 + 2E_0 + 2E_1 + 2E_2 + 2E_3 + 4p_1) \]
Catmull-Clark Subdiv
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- Add new “face” vertex at each face centroid
  centroid = average of face’s vertices
Catmull-Clark Subdiv

- Add new “face” vertex at each face centroid
centroid = average of face’s vertices

- Add new “edge” vertex at the average of each
dge’s endpoints and adjacent face centroids
Catmull-Clark Subdiv

- Add new “face” vertex at each face centroid. Centroid = average of face’s vertices.
- Add new “edge” vertex at the average of each edge’s endpoints and adjacent face centroids.
- Move each vertex to a new position that is...

\[
1 \times \text{ave. adjacent face centroids} + 2 \times \text{ave. adjacent edge midpoints} + (n-3) \times \text{current vertex position}
\]

where \( n \) is the valence of the vertex (# of neighboring edges, also # of adjacent faces)
Creases

\[ f_{i+1}^{j} = \text{Centroid of polygon} \]
\[ e_{i+1}^{j} = (v^{i} + e_{j}^{i})/2 \]

- Dart vertex (one sharp edge):
  \[ v_{i+1}^{j} = (n-2)/n \ v^{i} + 1/n^2 \ \Sigma_{j} e_{j}^{i} + 1/n^2 \ \Sigma_{j} f_{j+1}^{i} \]

- Crease vertex (two sharp edges):
  \[ v_{i+1}^{i} = (e_{j}^{i} + 6v^{i} + e_{k}^{i})/8 \]

- Corner vertex (three or more sharp edges)
  \[ v_{i+1}^{i} = v_{i}^{i} \]
Another Example
Success?