Parametric Surfaces

CS 418
Intro to Computer Graphics
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Space Curves

Separate into three coordinate functions

\[ \mathbf{p}(t) = (x(t), y(t), z(t)) \]

\[ x(t) = (1-t)^3 x_0 + 3t(1-t)^2 x_1 + 3t^2(1-t) x_2 + t^3 x_3 \]

\[ y(t) = (1-t)^3 y_0 + 3t(1-t)^2 y_1 + 3t^2(1-t) y_2 + t^3 y_3 \]

\[ z(t) = (1-t)^3 z_0 + 3t(1-t)^2 z_1 + 3t^2(1-t) z_2 + t^3 z_3 \]
Space Curves

Make your own roller-coaster ride

• Camera position along space curve
• Look at point is next position along space curve (tangent)
• Binormal is cross product of vector to next position with vector to previous position
• Up direction (normal) is cross product of binormal with tangent
Extrusion

- Two 3-D copies of each 2-D curve

\[ \mathbf{p}_0(t) = (x(t), y(t), 0) \]
\[ \mathbf{p}_1(t) = (x(t), y(t), 1) \]
\[ \mathbf{p}(t) = (x(t), y(t)) \]

- Create a mesh of quads (or tri-strip)

\[ \mathbf{p}_0(t), \mathbf{p}_1(t), \mathbf{p}_0(t + \Delta t), \mathbf{p}_1(t + \Delta t) \]
Generalized Cylinder

- Construct a 2-D profile curve
  \[ \mathbf{q}(s) = (a(s), b(s)) \]
- Construct a space curve
  \[ \mathbf{p}(t) = (x(t), y(t), z(t)) \]
- Construct a Frenet frame at each point along space curve
  \[ T(t) = \mathbf{p}(t+\Delta t) - \mathbf{p}(t) \]
  \[ B(t) = T(t) \times -T(t - \Delta t) \]
  \[ N(t) = B(t) \times T(t) \]
  (all normalized)
- Plot 2-D curve in \((N,B)\) space
  \[ \mathbf{gc}(s,t) = \mathbf{p}(t) + a(s) N(t) + b(s) B(t) \]
Generalized Cylinder

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Generalized Cylinder

- Construct a Frenet frame at each point along space curve

\[ t(s) = p(s + \Delta s) - p(s) \]
\[ b(s) = t(s) \times -t(s - \Delta s) \]
\[ n(s) = b(s) \times t(s) \]

(all normalized)
Generalized Cylinder

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  \[ gc(s,t) = p(s) + a(t) \, n(s) + b(t) \, b(s) \]
Generalized Cylinder

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• Construct a space curve
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• Plot 2-D curve in \((N,B)\) space
  \[ gc(s,t) = p(s) + a(t) \mathbf{n}(s) + b(t) \mathbf{b}(s) \]
Revolution

- Construct a 2-D profile curve
  \[ \mathbf{q}(t) = (a(t), b(t)) \]

- Rotate about y axis
  \[ \mathbf{p}(s,t) = (a(t) \cos 2\pi s, b(t), a(t) \sin 2\pi s) \]
Beziers Patches

- Bezier patch
  - Tensor product of two Bezier curves
  \[
p(s,t) = \sum_{i=1}^{n} \sum_{j=1}^{n} B^n_j(s) B^n_i(t) p_{ij}
\]
Beziers Patches

- Bezier patch
  - Tensor product of two Bezier curves
    \[ p(s, t) = \sum_{j=1}^{n} \sum_{i=1}^{n} B_j^n(s) B_i^n(t) p_{ij} \]
  - Product of Bernstein polynomials
    \[ p(s, t) = \sum_{j=1}^{n} \sum_{i=1}^{n} \left( B_j^n(s) B_i^n(t) \right) p_{ij} \]
Bezier Patches

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  - Product of Bernstein polynomials
    \[ p(s, t) = \sum_{j=1}^{n} \sum_{i=1}^{n} (B_j^n(s) B_i^n(t)) p_{ij} \]
  - Bernstein interpolation of Bernstein polynomials
    \[ p(s, t) = \sum_{j=1}^{n} B_j^n(s) \left( \sum_{i=1}^{n} B_i^n(t) (p_i) \right)_j \]
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- Works same way for B-splines
Blossoming Patches

- Curves: \( p(t) \rightarrow p(t,t,t) \)
- Patches: \( p(s,t) \rightarrow p(s,s,s;t,t,t) \)
- Variables not allowed to cross the semicolon
- In patches, bilinear interpolation replaces linear interpolation in curves
Blossoming Patches

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