Bezier Blossoms

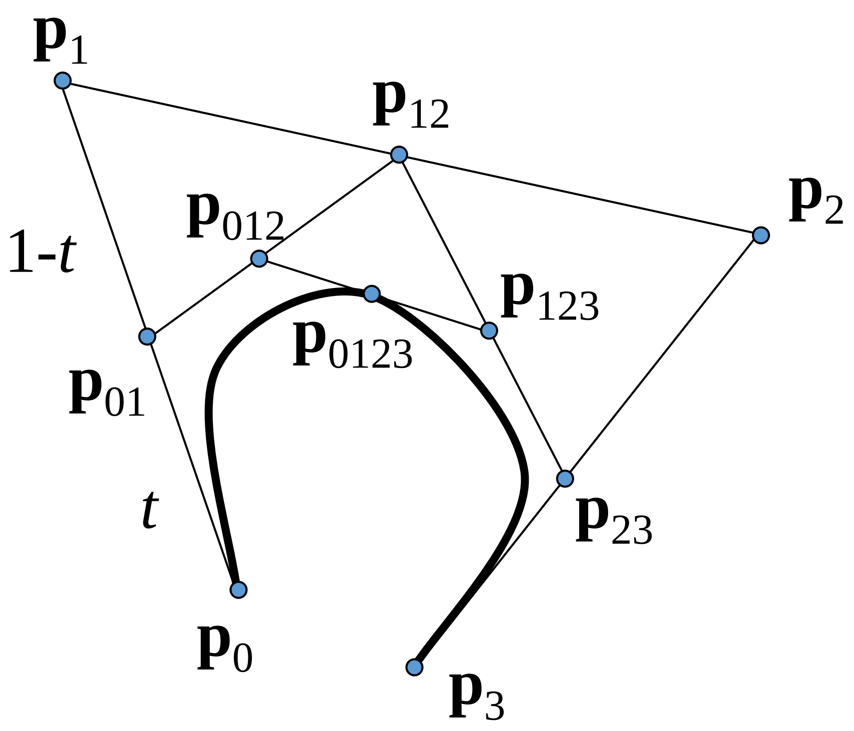
CS 418

Interactive Computer Graphics

John C. Hart

de Casteljau

- de Casteljau algorithm evaluates a point on a Bezier curve by scaffolding lerps
- Blossoming renames the control and intermediate points, like $\mathbf{p}_{12}$, using a polar form, like $\mathbf{p}(0,t,1)$



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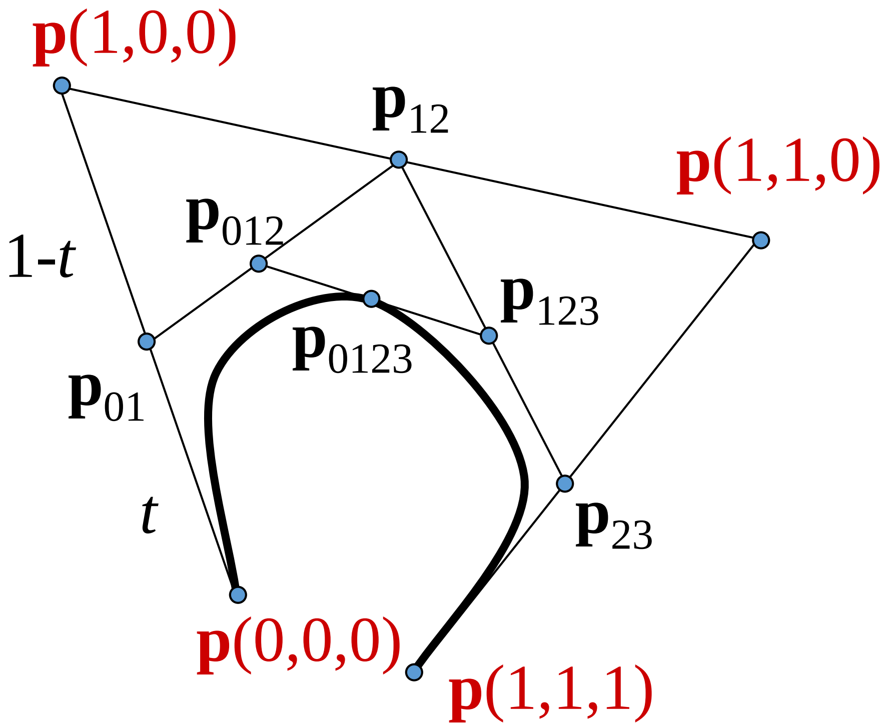

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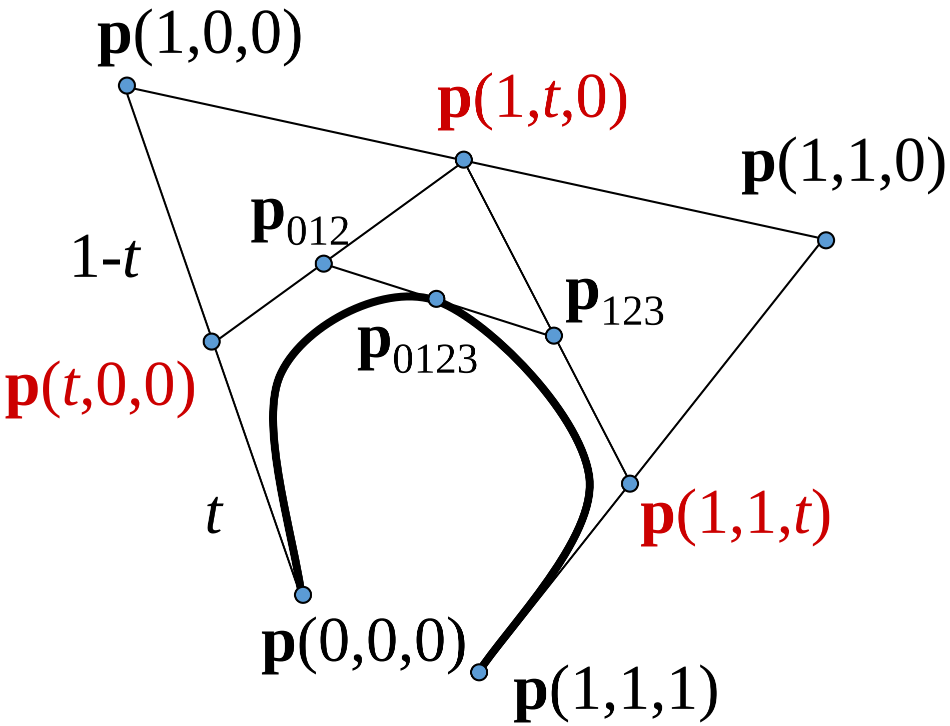

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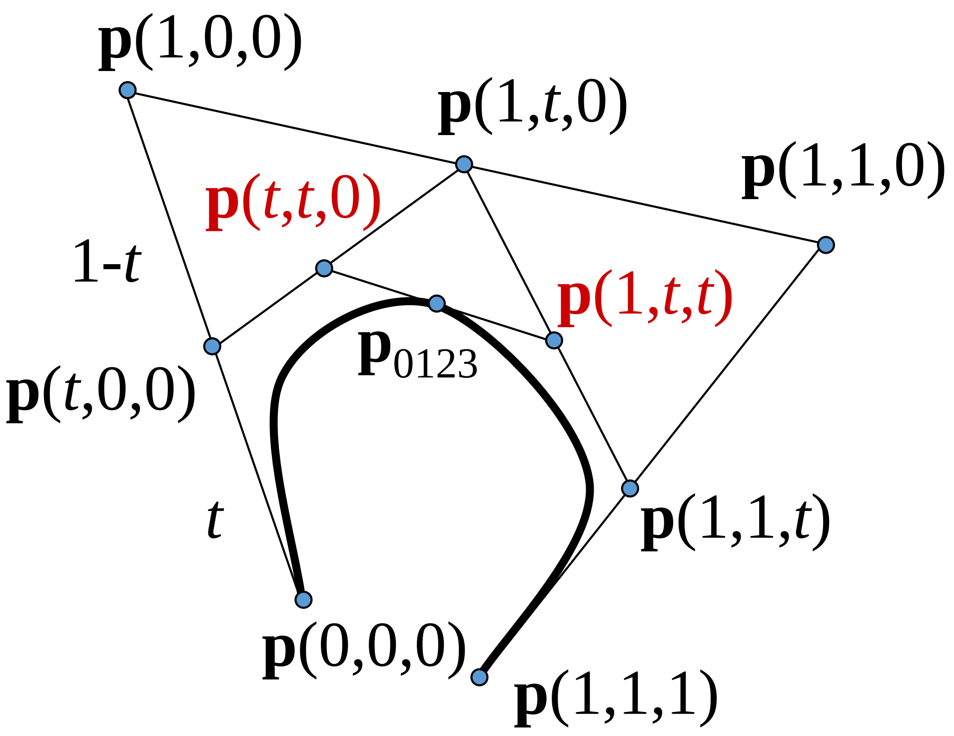

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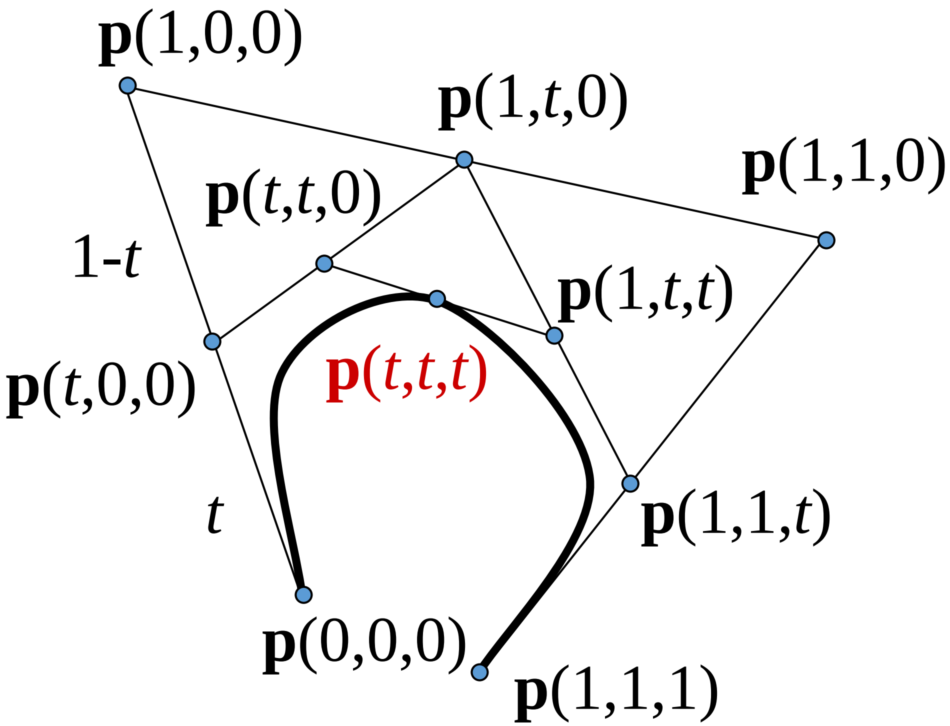
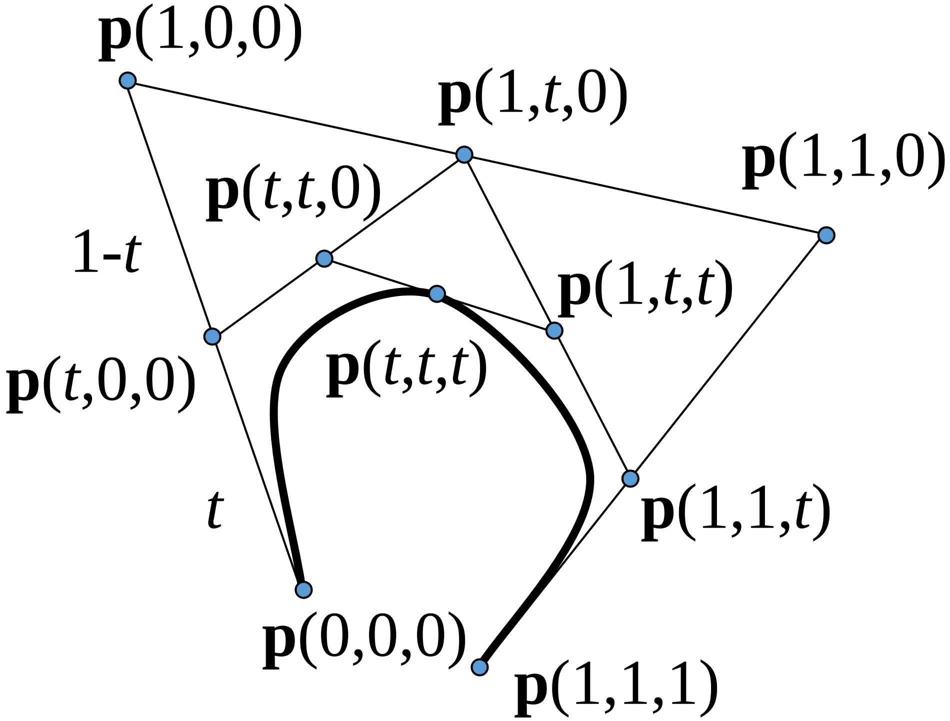


Blossoming Rules

1. # of parameters = degree

Cubic: **p**(_,_,_)



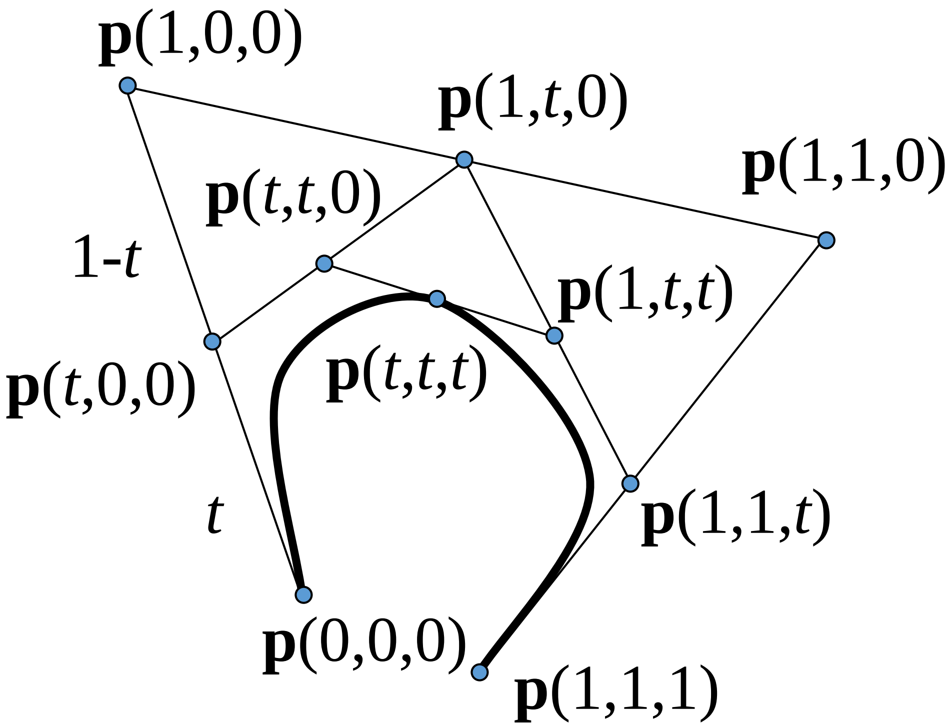
Blossoming Rules

1. # of parameters = degree

Cubic: $\mathbf{p}(\_,\_,\_)$

2. Order doesn't matter

$\mathbf{p}(a,b,c) = \mathbf{p}(b,a,c)$



Blossoming Rules

1. # of parameters = degree

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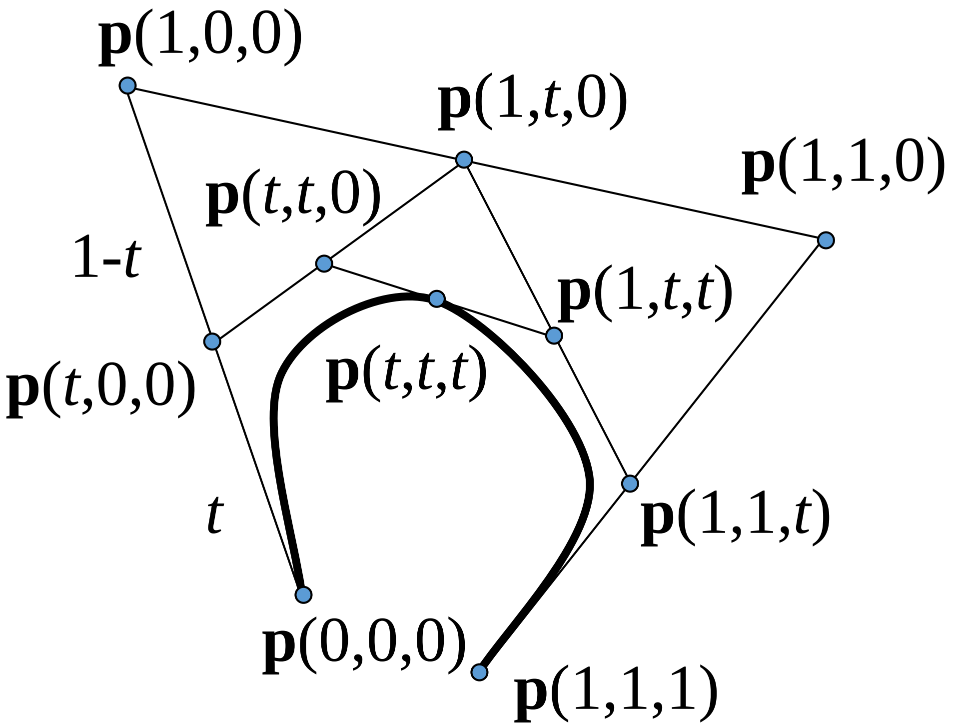
3. Setting up the board

   $\mathbf{p}(0,0,0)$, $\mathbf{p}(0,0,1)$,
   $\mathbf{p}(0,1,1)$, $\mathbf{p}(1,1,1)$



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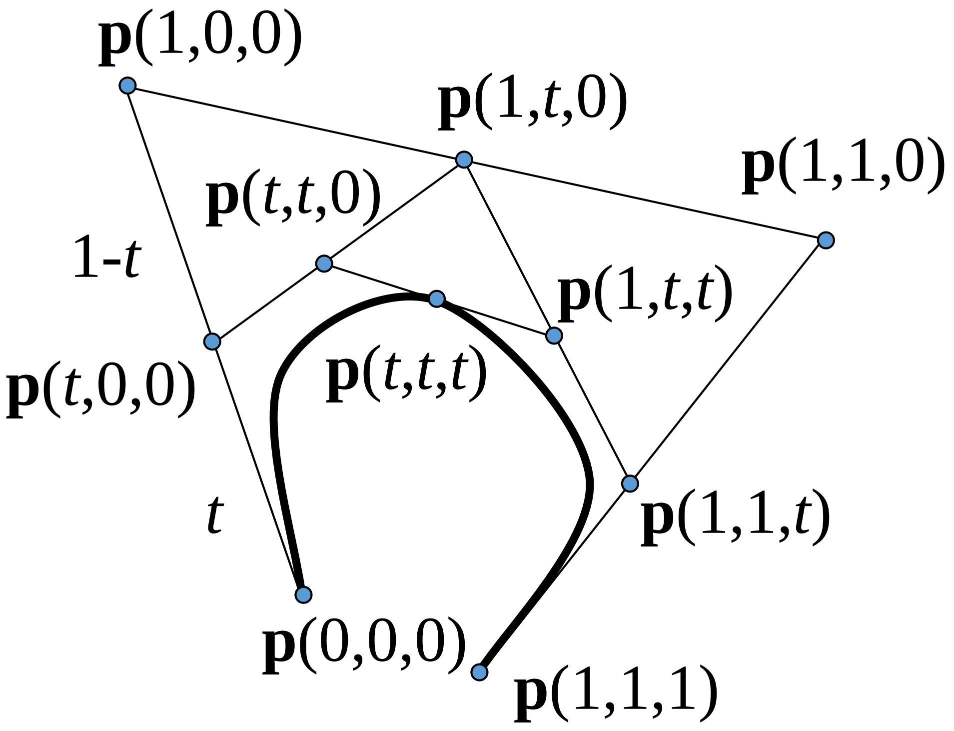
3. Setting up the board

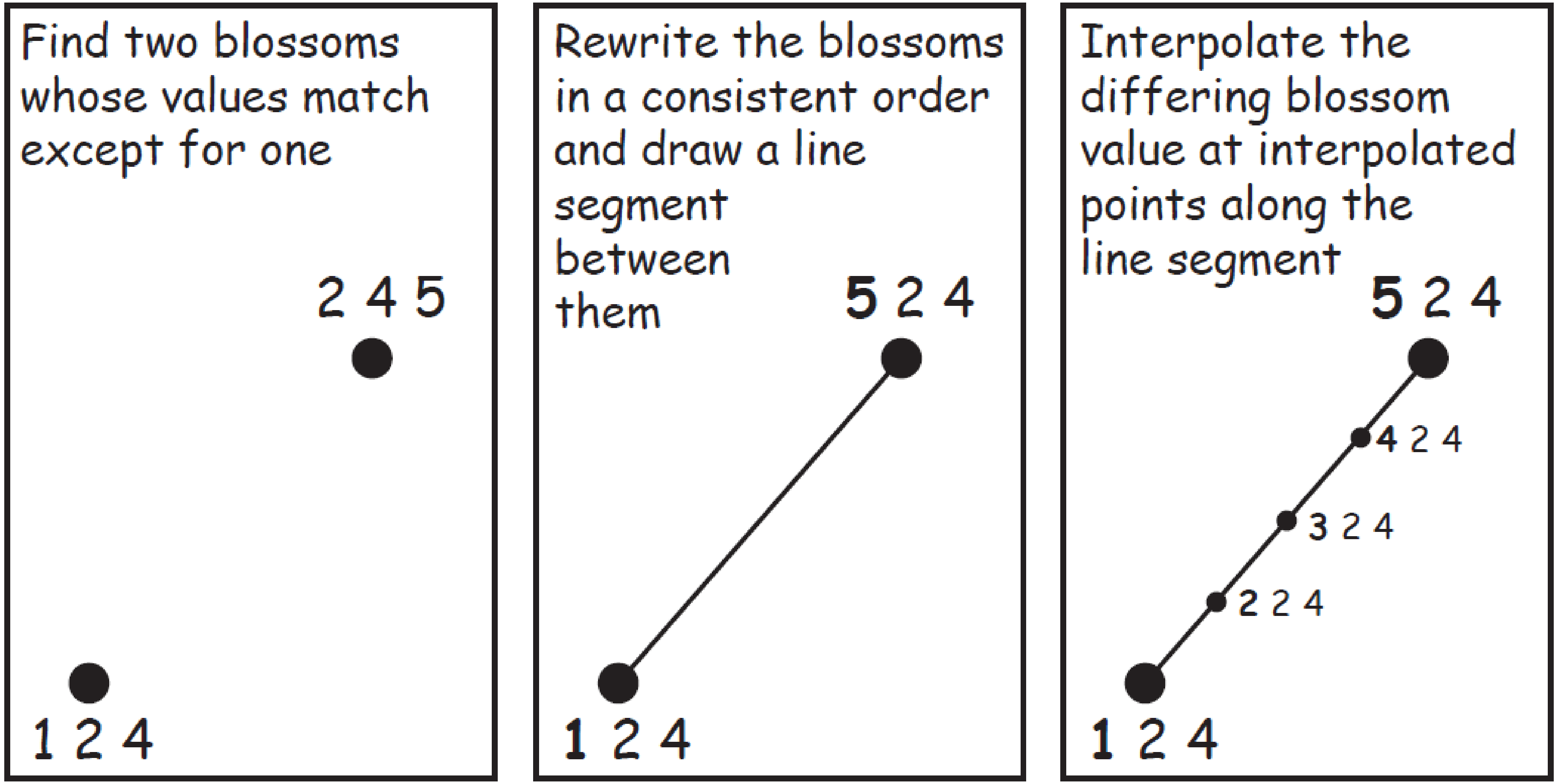
$\mathbf{p}(0,0,0)$, $\mathbf{p}(0,0,1)$,
$\mathbf{p}(0,1,1)$, $\mathbf{p}(1,1,1)$

4. Winning the game

$\mathbf{p}(t,t,t)$

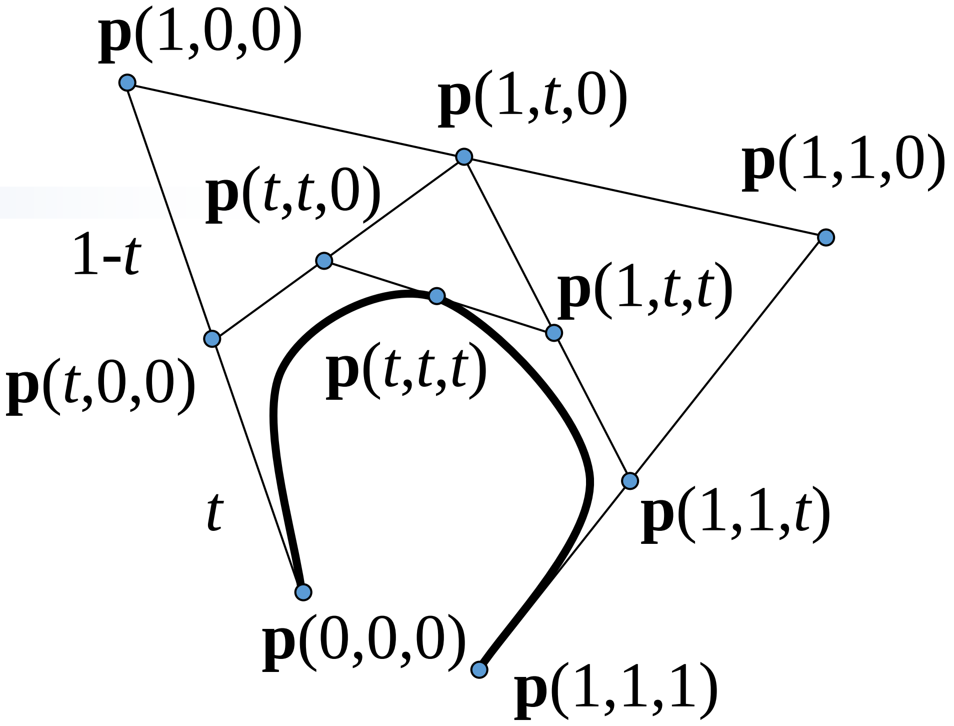


Placing Blossoms



Evaluation

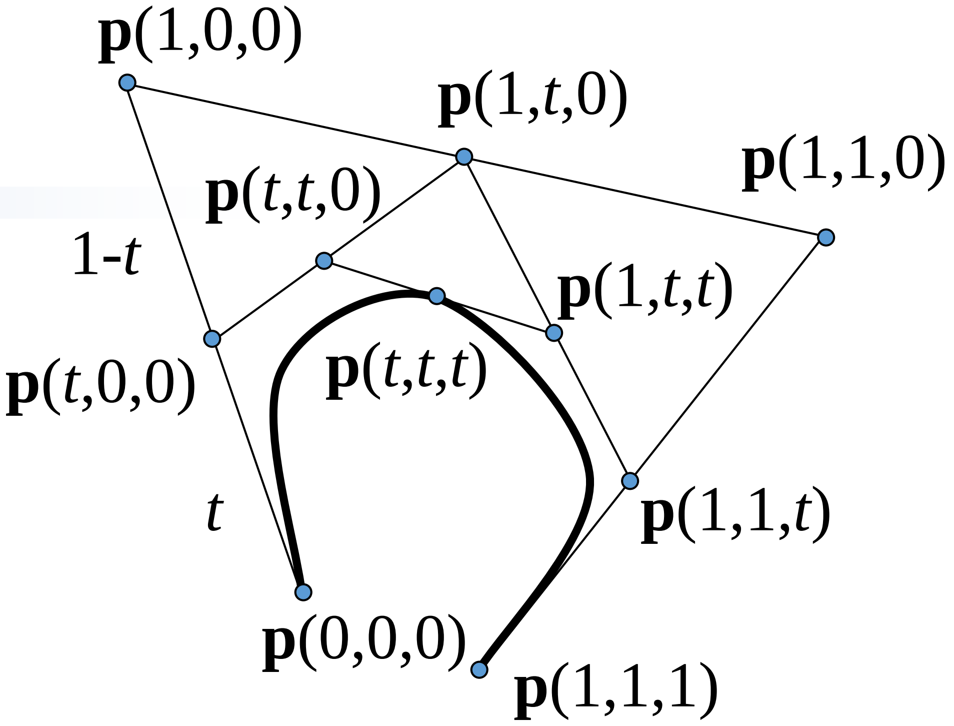
$\mathbf{p}(t) = \mathbf{p}(t,t,t)$



Evaluation

$\mathbf{p}(t) = \mathbf{p}(t,t,t)$

$= (1\text{-}t)\ \mathbf{p}(t,t,0) + t\ \mathbf{p}(t,t,1)$



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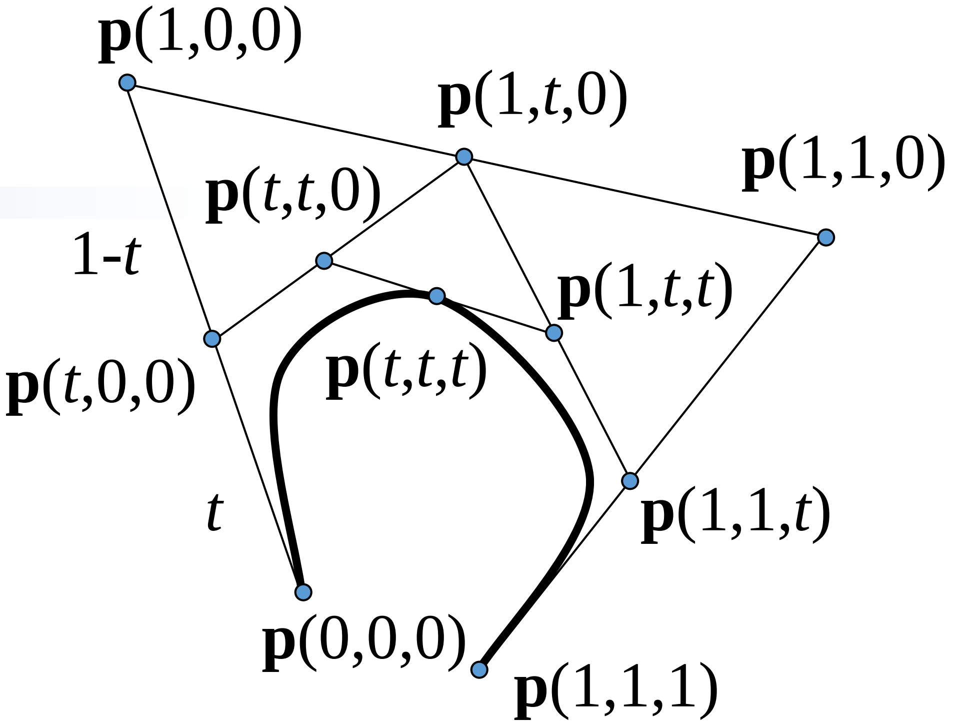
$= (1\text{-}t)\, \mathbf{p}(t,t,0) + t\, \mathbf{p}(t,t,1)$

$= (1\text{-}t)[(1\text{-t})\, \mathbf{p}(t,0,0) + t\, \mathbf{p}(t,0,1)]$
$\quad + t\, [(1\text{-}t)\, \mathbf{p}(t,0,1) + t\, \mathbf{p}(t,1,1)]$



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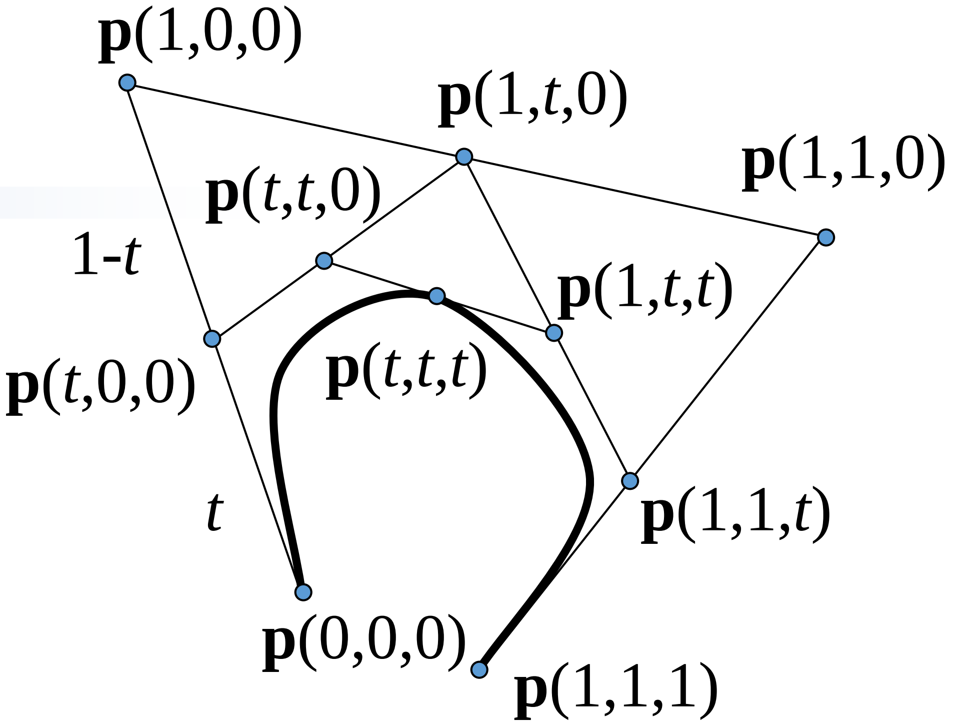
$\quad + t\ [(1\text{-}t)\ \mathbf{p}(t,0,1) + t\ \mathbf{p}(t,1,1)]$

$= (1\text{-}t)^2\ \mathbf{p}(t,0,0) + 2\ (1\text{-}t)\ t\ \mathbf{p}(t,0,1) + t^2\ \mathbf{p}(t,1,1)$



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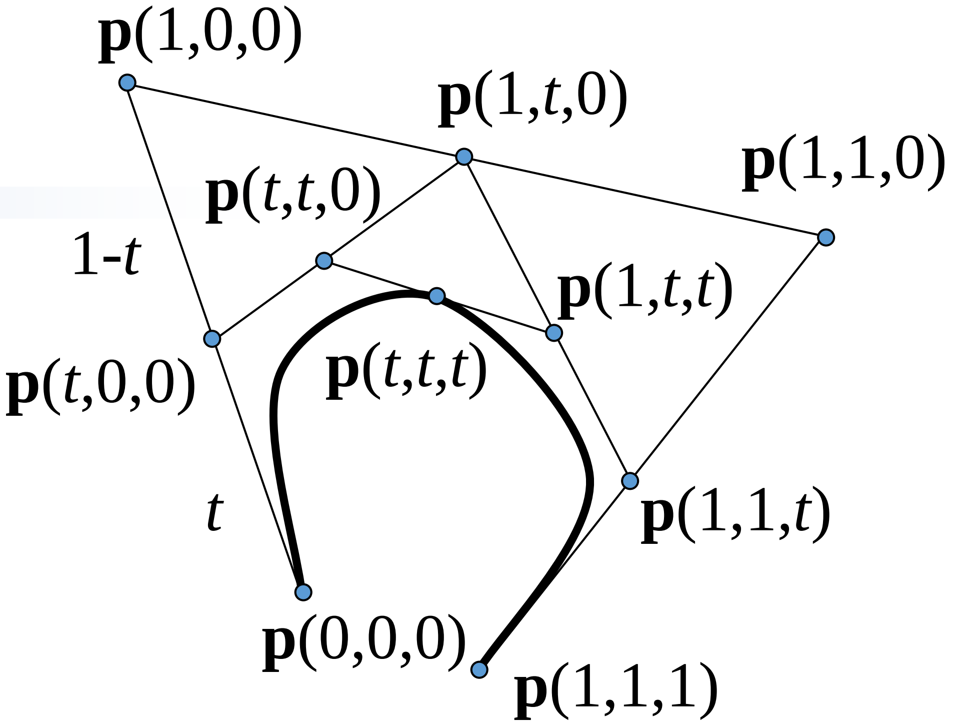
$= (1\text{-}t)^2[(1\text{-}t)\mathbf{p}(0,0,0)+t\mathbf{p}(1,0,0)]+2(1\text{-}t)t[(1\text{-}t)\mathbf{p}(0,0,1)+t\mathbf{p}(1,0,1)]+t^2[(1\text{-}t)\mathbf{p}(0,1,1)+t\mathbf{p}(1,1,1)]$



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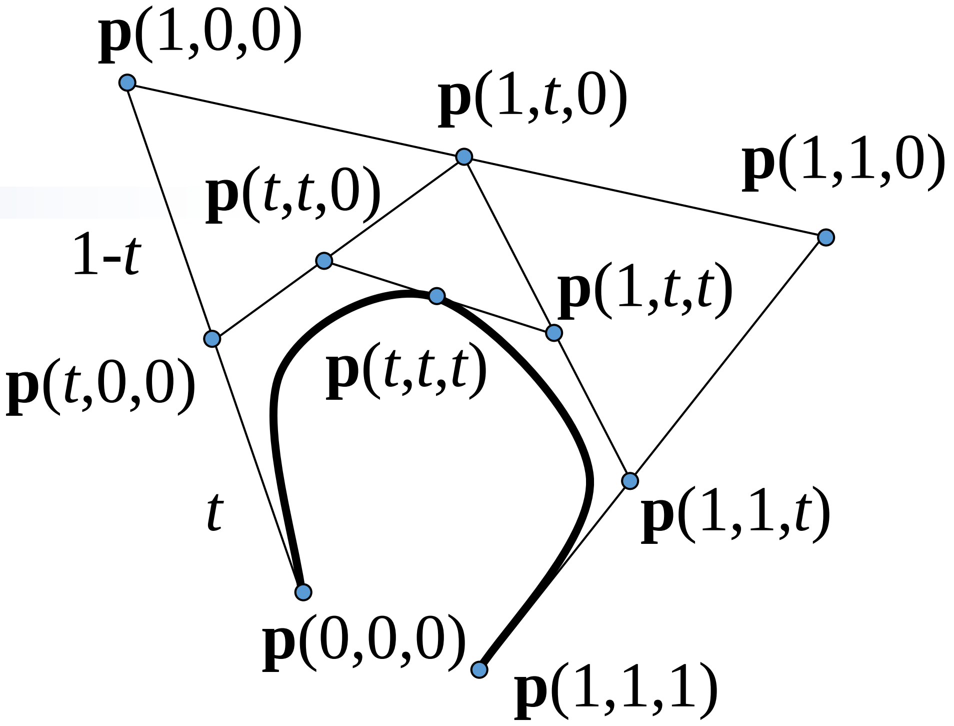

$\mathbf{p}(t) = \mathbf{p}(t,t,t)$

$= (1-t)\,\mathbf{p}(t,t,0) + t\,\mathbf{p}(t,t,1)$

$= (1-t)[(1-\mathrm{t})\,\mathbf{p}(t,0,0) + t\,\mathbf{p}(t,0,1)]$
$\quad + t\,[(1-t)\,\mathbf{p}(t,0,1) + t\,\mathbf{p}(t,1,1)]$

$= (1-t)^2\,\mathbf{p}(t,0,0) + 2\,(1-t)\,t\,\mathbf{p}(t,0,1) + t^2\,\mathbf{p}(t,1,1)$

$= (1-t)^2[(1-t)\mathbf{p}(0,0,0)+t\mathbf{p}(1,0,0)]+2(1-t)t[(1-t)\mathbf{p}(0,0,1)+t\mathbf{p}(1,0,1)]+t^2[(1-t)\mathbf{p}(0,1,1)+t\mathbf{p}(1,1,1)]$

$= (1-t)^3\,\mathbf{p}(0,0,0) + 3\,(1-t)^2\,t\,\mathbf{p}(0,0,1) + 3\,(1-t)\,t^2\,\mathbf{p}(0,1,1) + t^3\,\mathbf{p}(1,1,1)$