

Bezier Blossoms

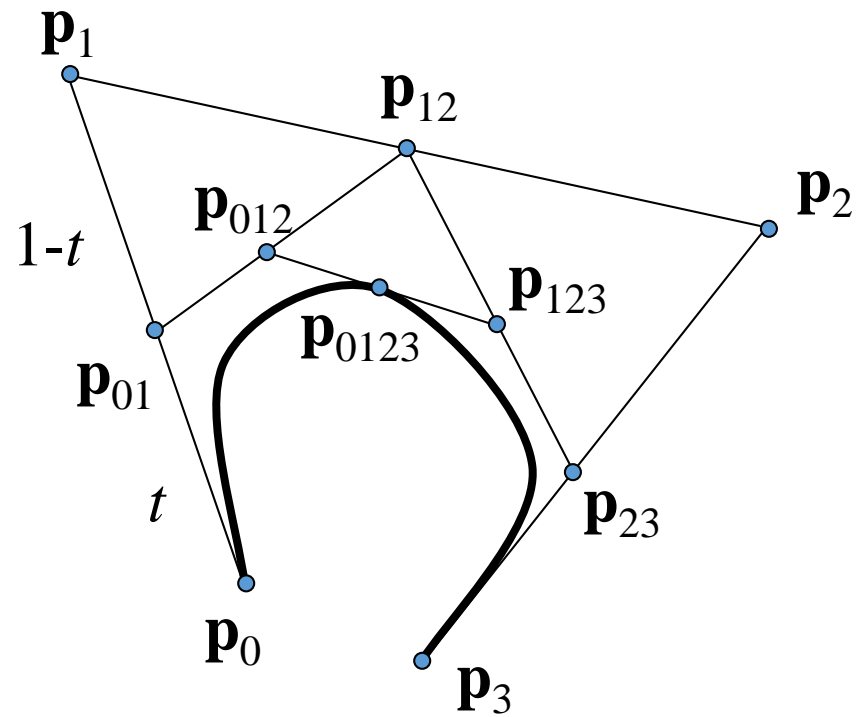
CS 418

Interactive Computer Graphics

John C. Hart

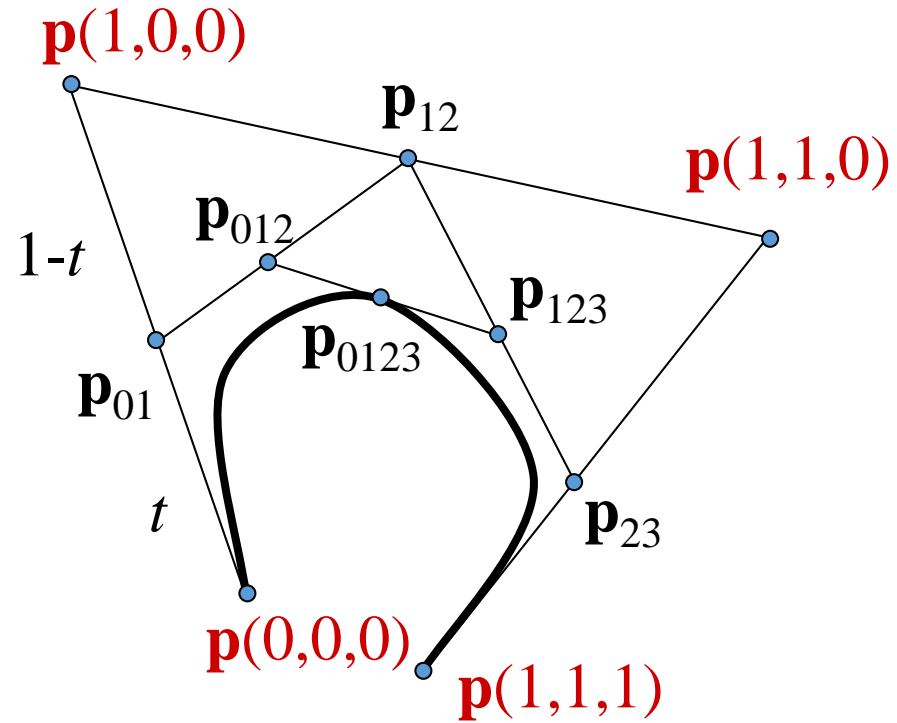
de Casteljau

- de Casteljau algorithm evaluates a point on a Bezier curve by scaffolding lerps
- Blossoming renames the control and intermediate points, like \mathbf{p}_{12} , using a polar form, like $\mathbf{p}(0,t,1)$



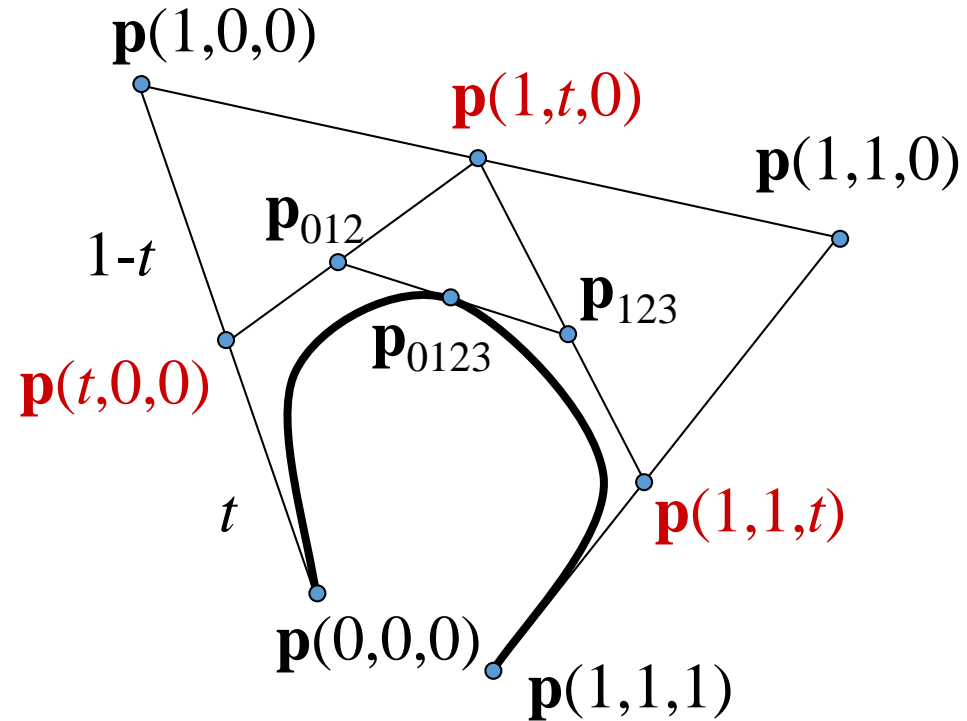
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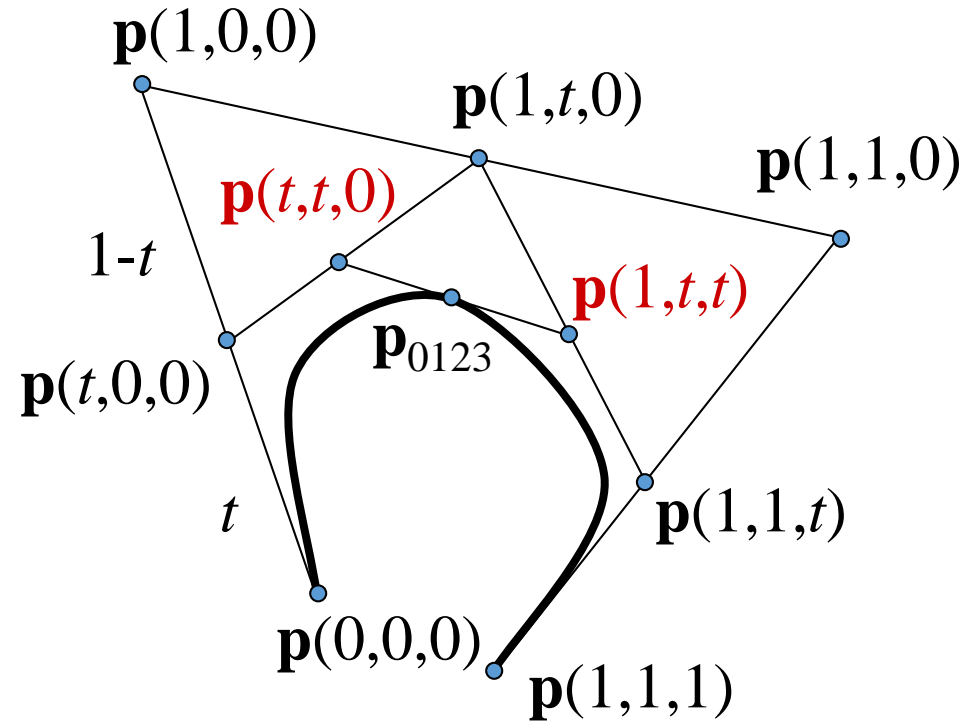
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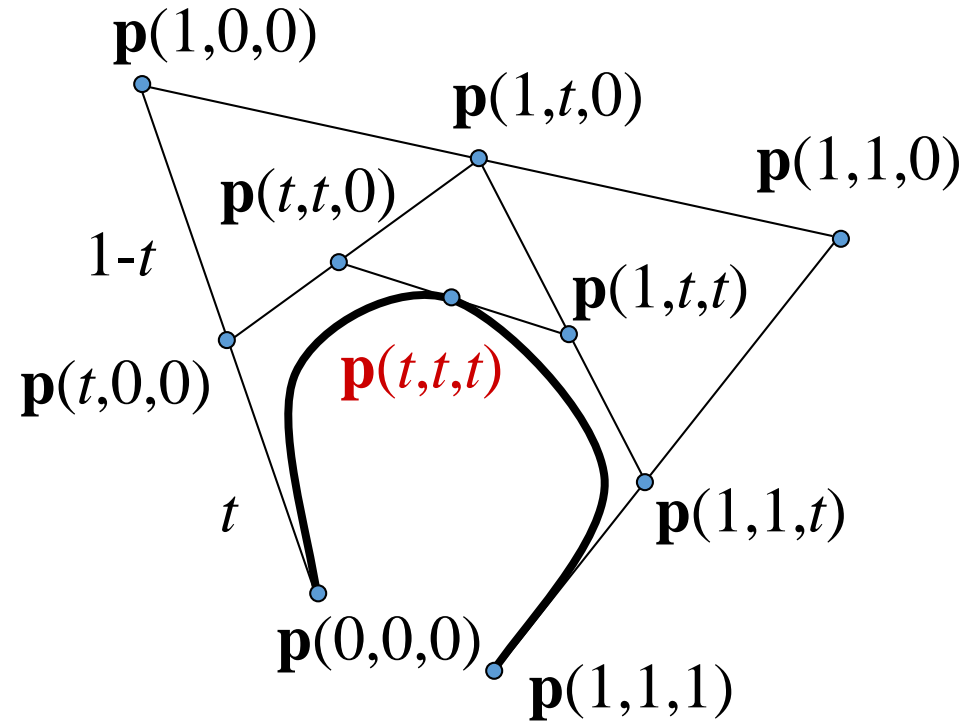
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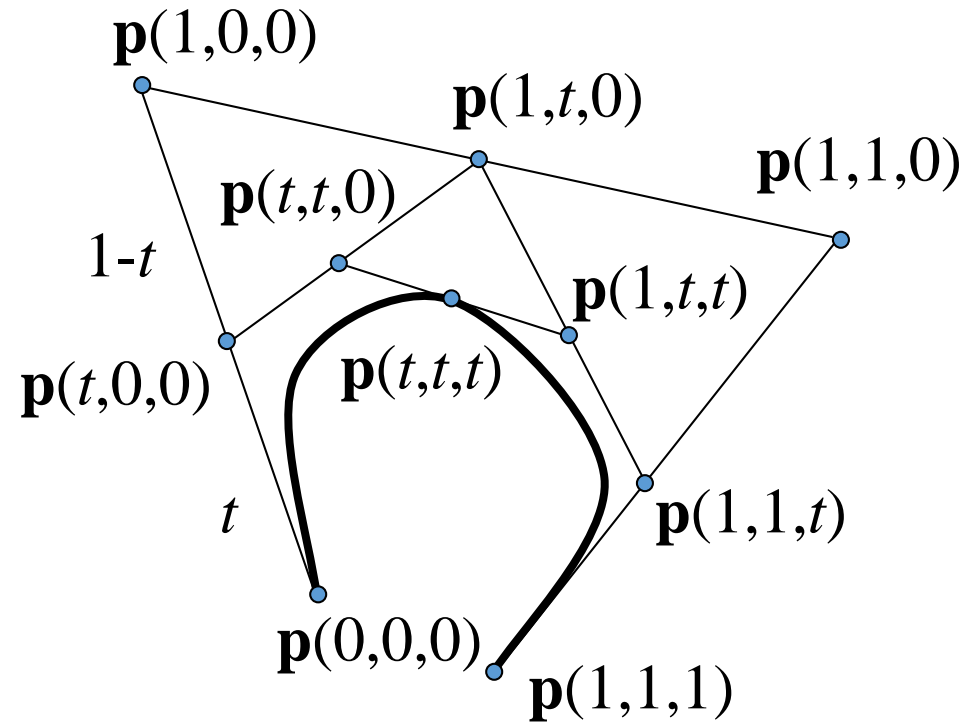
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Blossoming Rules

1. # of parameters = degree

Cubic: $\mathbf{p}(_,_,_)$



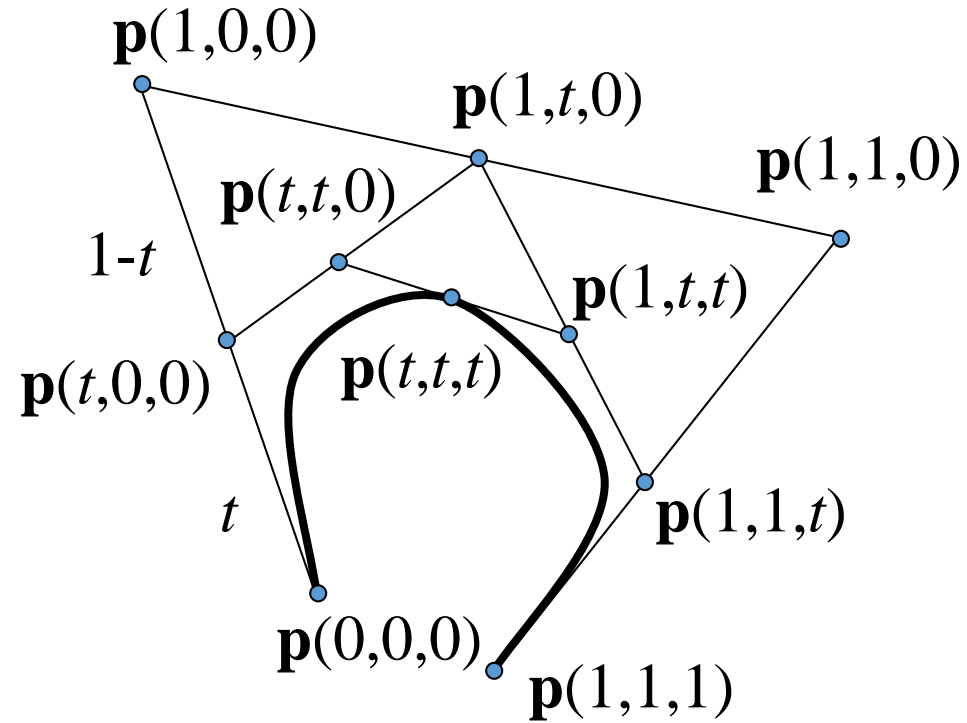
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1. # of parameters = degree

Cubic: $\mathbf{p}(_,_,_)$

2. Order doesn't matter

$\mathbf{p}(a,b,c) = \mathbf{p}(b,a,c)$



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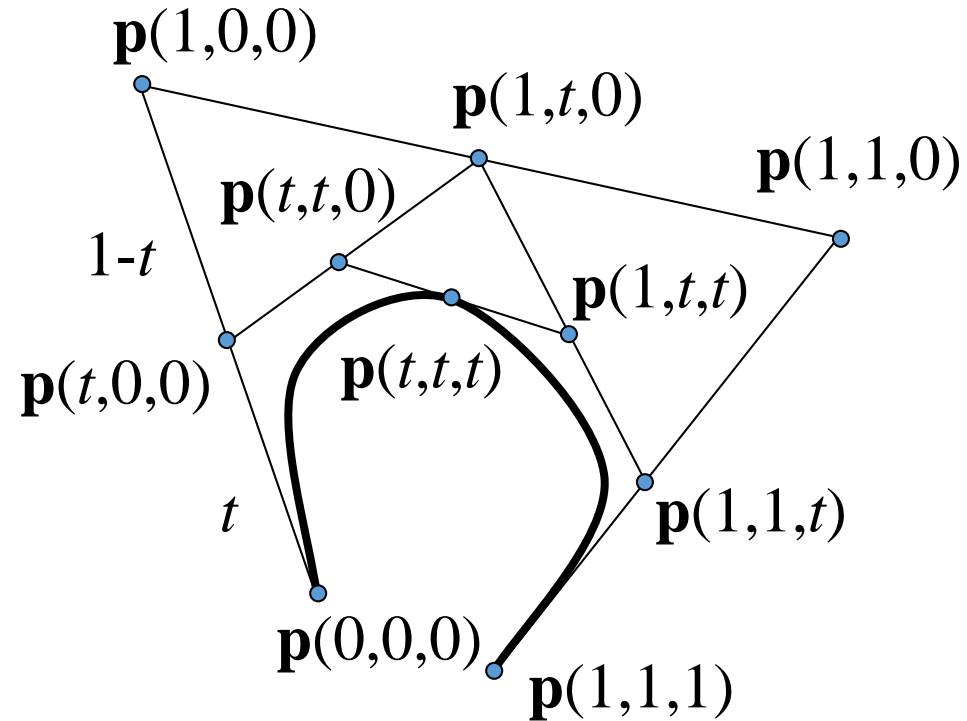
Cubic: $\mathbf{p}(_,_,_)$

2. Order doesn't matter

$\mathbf{p}(a,b,c) = \mathbf{p}(b,a,c)$

3. Setting up the board

$\mathbf{p}(0,0,0)$, $\mathbf{p}(0,0,1)$,
 $\mathbf{p}(0,1,1)$, $\mathbf{p}(1,1,1)$



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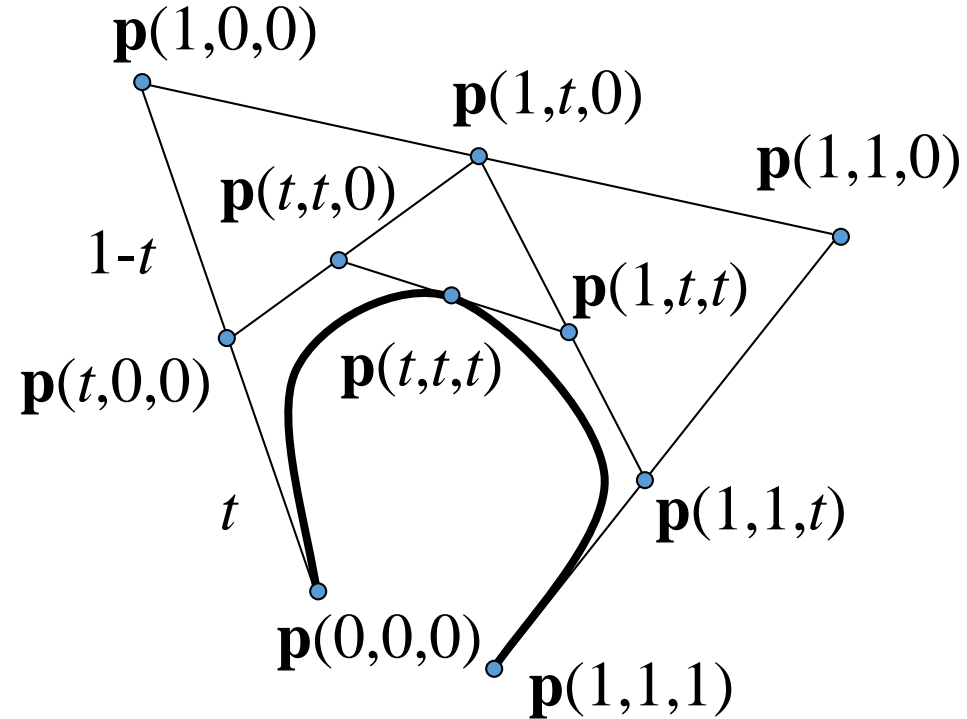
$\mathbf{p}(a,b,c) = \mathbf{p}(b,a,c)$

3. Setting up the board

$\mathbf{p}(0,0,0)$, $\mathbf{p}(0,0,1)$,
 $\mathbf{p}(0,1,1)$, $\mathbf{p}(1,1,1)$

4. Winning the game

$\mathbf{p}(t,t,t)$



Placing Blossoms

Find two blossoms whose values match except for one

●
1 2 4

●
2 4 5

Rewrite the blossoms in a consistent order and draw a line segment between them

●
1 2 4

●
5 2 4

Interpolate the differing blossom value at interpolated points along the line segment

●
1 2 4

●
2 2 4

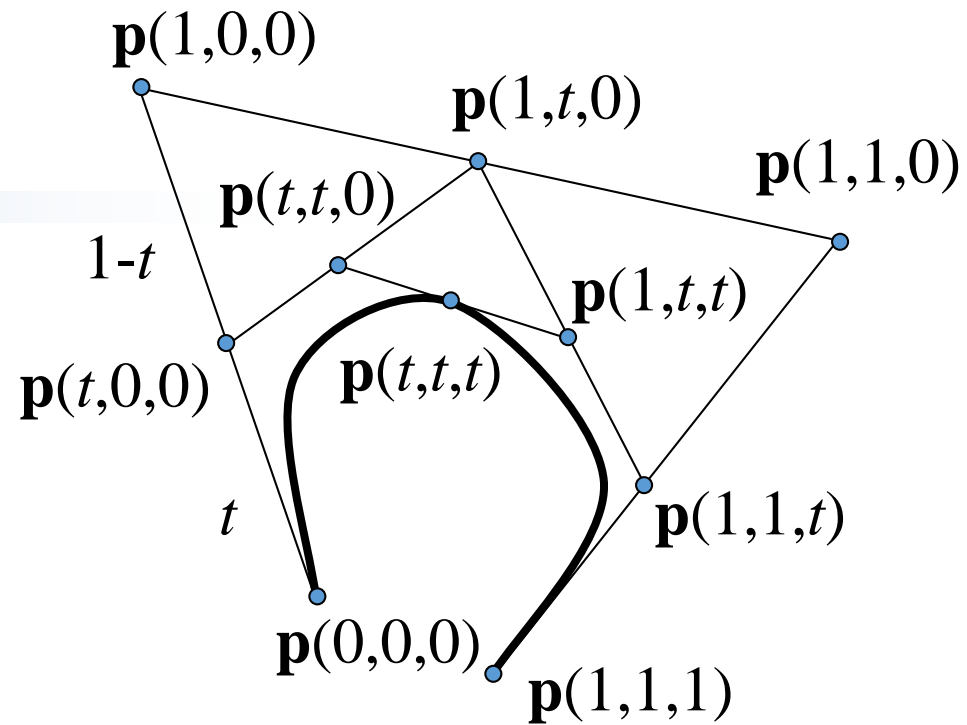
●
3 2 4

●
4 2 4

●
5 2 4

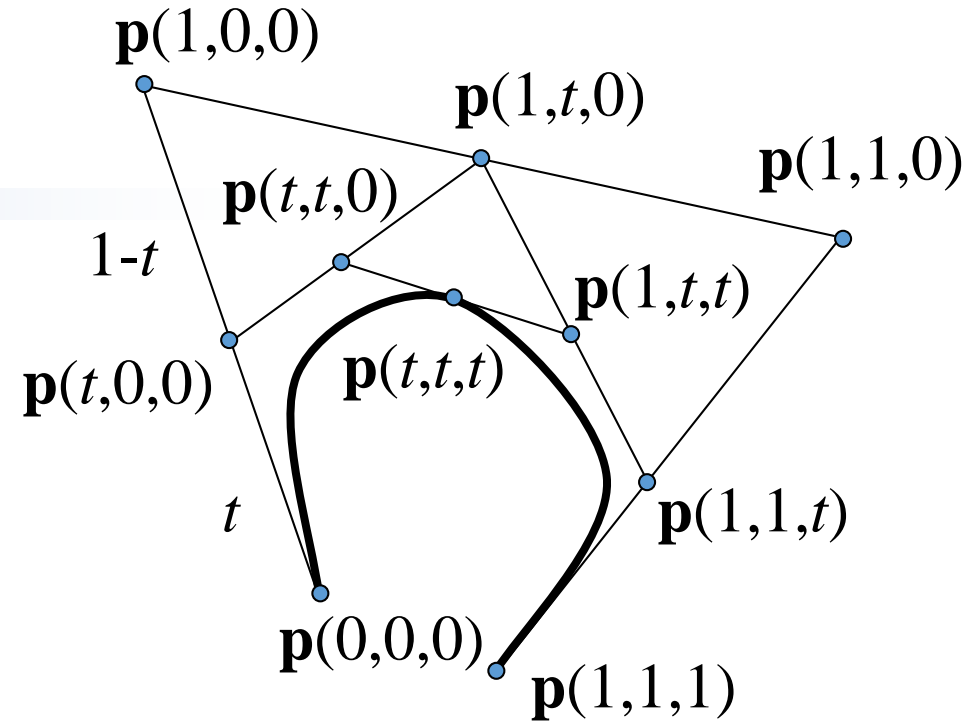
Evaluation

$$\mathbf{p}(t) = \mathbf{p}(t,t,t)$$



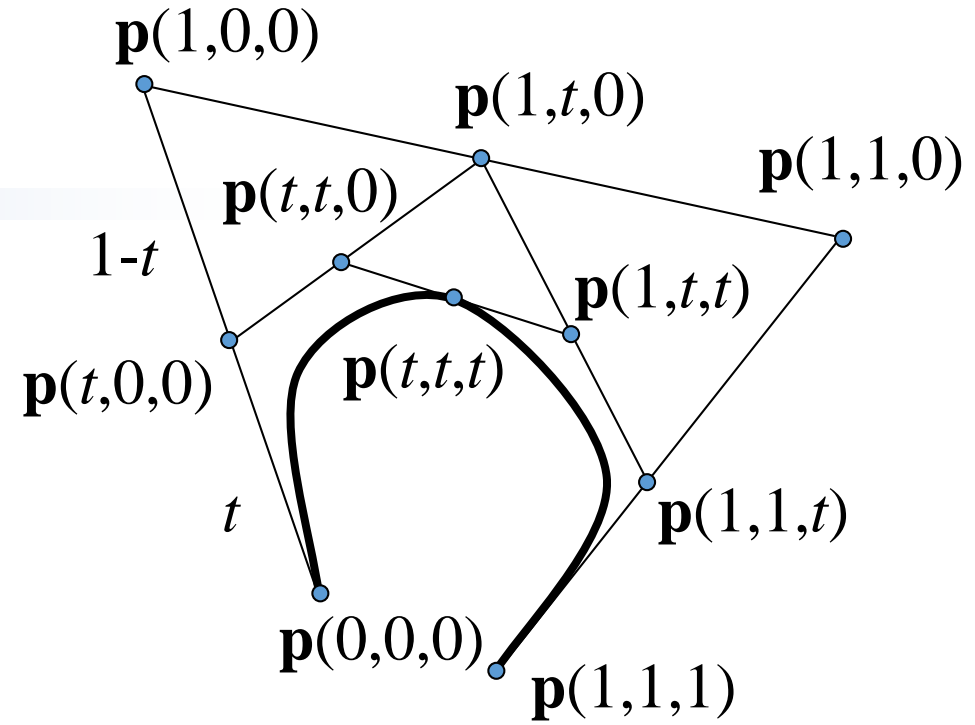
Evaluation

$$\begin{aligned}\mathbf{p}(t) &= \mathbf{p}(t,t,t) \\ &= (1-t)\mathbf{p}(t,t,0) + t\mathbf{p}(t,t,1)\end{aligned}$$



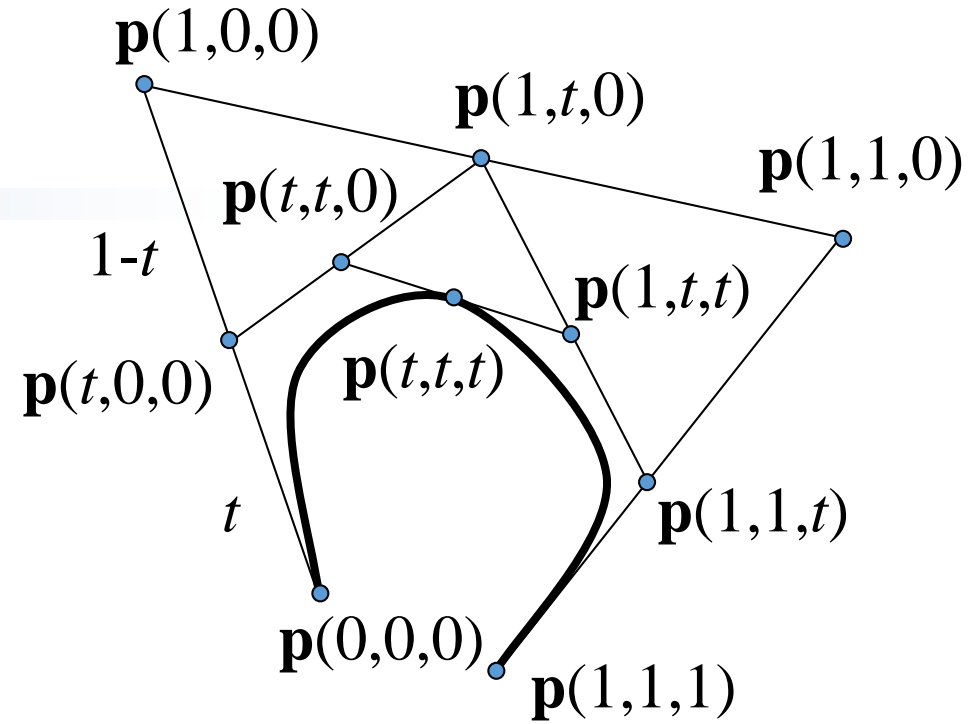
Evaluation

$$\begin{aligned}\mathbf{p}(t) &= \mathbf{p}(t,t,t) \\ &= (1-t) \mathbf{p}(t,t,0) + t \mathbf{p}(t,t,1) \\ &= (1-t)[(1-t) \mathbf{p}(t,0,0) + t \mathbf{p}(t,0,1)] \\ &\quad + t [(1-t) \mathbf{p}(t,0,1) + t \mathbf{p}(t,1,1)]\end{aligned}$$



Evaluation

$$\begin{aligned}\mathbf{p}(t) &= \mathbf{p}(t,t,t) \\ &= (1-t) \mathbf{p}(t,t,0) + t \mathbf{p}(t,t,1) \\ &= (1-t)[(1-t) \mathbf{p}(t,0,0) + t \mathbf{p}(t,0,1)] \\ &\quad + t [(1-t) \mathbf{p}(t,0,1) + t \mathbf{p}(t,1,1)] \\ &= (1-t)^2 \mathbf{p}(t,0,0) + 2 (1-t) t \mathbf{p}(t,0,1) + t^2 \mathbf{p}(t,1,1)\end{aligned}$$



Evaluation

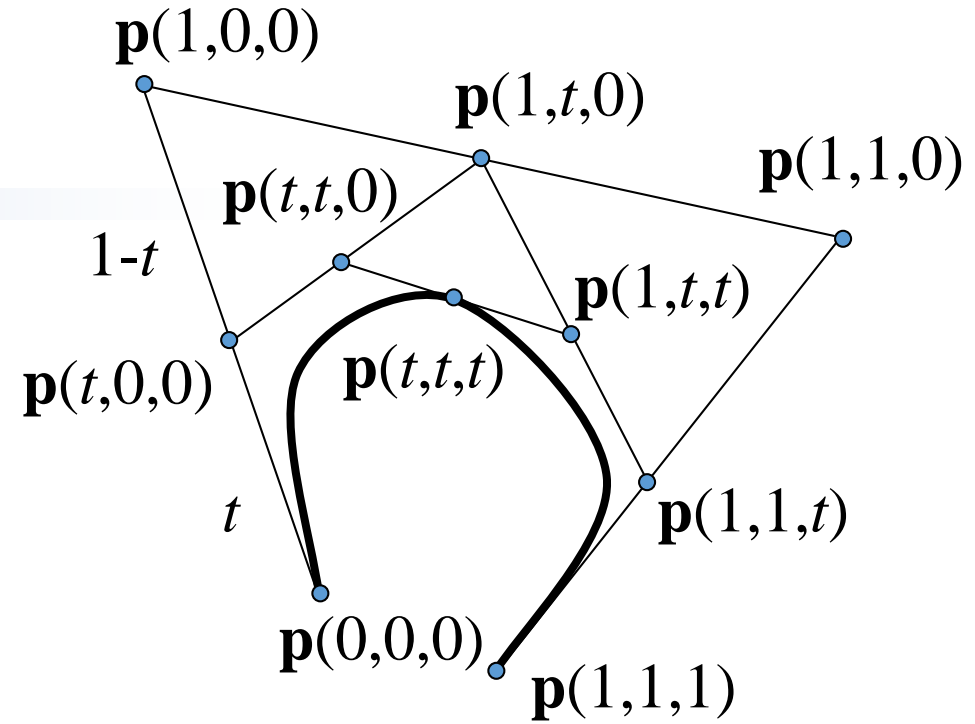
$$\mathbf{p}(t) = \mathbf{p}(t,t,t)$$

$$= (1-t) \mathbf{p}(t,t,0) + t \mathbf{p}(t,t,1)$$

$$= (1-t)[(1-t) \mathbf{p}(t,0,0) + t \mathbf{p}(t,0,1)] \\ + t [(1-t) \mathbf{p}(t,0,1) + t \mathbf{p}(t,1,1)]$$

$$= (1-t)^2 \mathbf{p}(t,0,0) + 2 (1-t) t \mathbf{p}(t,0,1) + t^2 \mathbf{p}(t,1,1)$$

$$= (1-t)^2 [(1-t)\mathbf{p}(0,0,0)+t\mathbf{p}(1,0,0)]+2(1-t)t[(1-t)\mathbf{p}(0,0,1)+t\mathbf{p}(1,0,1)]+t^2[(1-t)\mathbf{p}(0,1,1)+t\mathbf{p}(1,1,1)]$$



Evaluation

$$\begin{aligned}\mathbf{p}(t) &= \mathbf{p}(t,t,t) \\ &= (1-t) \mathbf{p}(t,t,0) + t \mathbf{p}(t,t,1) \\ &= (1-t)[(1-t) \mathbf{p}(t,0,0) + t \mathbf{p}(t,0,1)] \\ &\quad + t [(1-t) \mathbf{p}(t,0,1) + t \mathbf{p}(t,1,1)] \\ &= (1-t)^2 \mathbf{p}(t,0,0) + 2 (1-t) t \mathbf{p}(t,0,1) + t^2 \mathbf{p}(t,1,1) \\ &= (1-t)^2 [(1-t) \mathbf{p}(0,0,0) + t \mathbf{p}(1,0,0)] + 2(1-t)t [(1-t) \mathbf{p}(0,0,1) + t \mathbf{p}(1,0,1)] + t^2 [(1-t) \mathbf{p}(0,1,1) + t \mathbf{p}(1,1,1)] \\ &= (1-t)^3 \mathbf{p}(0,0,0) + 3 (1-t)^2 t \mathbf{p}(0,0,1) + 3 (1-t) t^2 \mathbf{p}(0,1,1) + t^3 \mathbf{p}(1,1,1)\end{aligned}$$

