Rational Curves

CS 418
Interactive Computer Graphics
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2-D Quadratic Bezier Curves

- Three control points

\[ p(t) = (1-t)^2 p_0 + 2t(1-t) p_1 + t^2 p_2 \]

- Always planar because of convex hull property and any three points always lie in some plane

- Used for True-Type fonts
Cubic Arc Approximation

\[ x(t) = C - S \left( t - \frac{\pi}{4} \right) - C \frac{(t - \frac{\pi}{4})^2}{2} + S \frac{(t - \frac{\pi}{4})^3}{6}, \]

\[ y(t) = S + C \left( t - \frac{\pi}{4} \right) - S \frac{(t - \frac{\pi}{4})^2}{2} - C \frac{(t - \frac{\pi}{4})^3}{6}, \]
Cubic Arc Approximation

\[ x(t) = C - S \left( t - \frac{\pi}{4} \right) - C \left( t - \frac{\pi}{4} \right)^2 - \frac{2}{2} + S \frac{\left( t - \frac{\pi}{4} \right)^3}{6} , \]

\[ y(t) = S + C \left( t - \frac{\pi}{4} \right) - S \left( t - \frac{\pi}{4} \right)^2 - C \frac{\left( t - \frac{\pi}{4} \right)^3}{6} , \]
Rational Bezier Curves

\[ p(t) = \frac{\sum_{i=0}^{n} w_i B_i^n(t) p_i}{\sum_{i=0}^{n} w_i B_i^n(t)} , \]

\[ p_1, w_1 = 1 \quad p_2, w_2 = 1 \]

\[ p_0, w_0 = 1 \quad p_3, w_3 = 1 \]
Rational Bezier Curves

$$p(t) = \frac{\sum_{i=0}^{n} w_i B_i^n(t) p_i}{\sum_{i=0}^{n} w_i B_i^n(t)},$$
Rational Bezier Curves

\[ p(t) = \frac{\sum_{i=0}^{n} w_i B_i^n(t) p_i}{\sum_{i=0}^{n} w_i B_i^n(t)}, \]

\[ p_1, w_1 = 5 \]
\[ p_2, w_2 = 1 \]
\[ p_0, w_0 = 1 \]
\[ p_3, w_3 = 1 \]
Homogeneous Control Points

- Think of the control point $p_i = (x_i, y_i)$ with weight $w_i$ as a homogeneous control point $P_i = (w_i x_i, w_i y_i, w_i)$
- Then $P(t) = (w x, w y, w)$ and $p(t) = P(t)/w(t)$
- $P(t)$ is an ordinary 3-D B-spline
- $w(t)$ is the denominator

\[
p(t) = \frac{\sum_{i=0}^{n} w_i B_i^n(t) p_i}{\sum_{i=0}^{n} w_i B_i^n(t)}\]
Homogeneous Control Points
Rational Bezier Arc

\[ p_0, w_0 = 1 \]

\[ p_1, w_1 = \frac{\sqrt{2}}{2} \]

\[ p_2, w_2 = 1 \]