Cubic Curves

CS 418
Interactive Computer Graphics
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Why Cubic Curves

- Polynomials (like degree 3 cubics) are well understood with beautiful mathematical formulations

\[ At^3 + Bt^2 + Ct + D \]

- Cubics provide enough flexibility to space curves, whereas quadratics are limited to planar curves

\[
\begin{align*}
A_x t^3 + B_x t^2 + C_x t + D_x & \quad & B_x t^2 + C_x t + D_x \\
A_y t^3 + B_y t^2 + C_y t + D_y & \quad & B_y t^2 + C_y t + D_y \\
A_z t^3 + B_z t^2 + C_z t + D_z & \quad & B_z t^2 + C_z t + D_z
\end{align*}
\]
Wiggle Theorem

(Bezout’s Theorem)

$y = x$
Wiggle Theorem

(Bezout’s Theorem)

\[ y = x \]

\[ y = x^2 - 1 \]
Wiggle Theorem (Bezout’s Theorem)

\[ y = x \]

\[ y = x^2 - 1 \]

\[ y = x^3 - x \]
Wiggle Theorem

(Bezout’s Theorem)

\[ y = x \]

\[ y = x^2 - 1 \]

\[ y = x^3 - x \]

\[ y = \sin x \]
Wiggle Theorem

\[ y = x \]

\[ y = x^2 - 1 \]

\[ y = x^3 - x \]

\[ y = \sin x = x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 - \ldots \]
Lagrangian Interpolation

- Cubic polynomial
  \[ y = Ax^3 + Bx^2 + Cx + D \]
- Solve for
  \[ D = 0 \]
  
  \[ 1 = A(1/3)^3 + B(1/3)^2 + C(1/3) \]
  
  \[ -1 = A(2/3)^3 + B(2/3)^2 + C(2/3) \]
  
  \[ 0 = A + B + C \]
- Result
  \[ y = 27x^3 - 40\frac{1}{2}x^2 + 13\frac{1}{2}x \]
Lagrangian Interpolation

\[
\begin{align*}
(0,0) & \quad (0,1) & \quad (1,0) & \quad (2,0) \\
(1,1) & \quad (2,1) & \\
\end{align*}
\]
Lagrangian Interpolation
Lagrangian Interpolation

\[ At^5 + Bt^4 + Ct^3 + Dt^2 + Et + F \]
Lagrangian Interpolation

\[ At^5 + Bt^4 + Ct^3 + Dt^2 + Et + F \]