Arbitrary Axis Rotations with Vector Algebra

CS418 Computer Graphics
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Vector Algebra

- Forget homogenous coordinates for the moment
- Simple vectors, e.g. \( \mathbf{a} = (a_x, a_y, a_z) \), \( \mathbf{b} = (b_x, b_y, b_z) \)
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• Simple vectors, e.g. \( \mathbf{a} = (a_x, a_y, a_z), \mathbf{b} = (b_x, b_y, b_z) \)
• Length: \( ||\mathbf{a}|| = \sqrt{a_x^2 + a_y^2 + a_z^2} \)
• Normalizing a vector \( (\mathbf{a}/||\mathbf{a}||) \) makes it unit length
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- Normalizing a vector (\( \mathbf{a}/\| \mathbf{a} \| \)) makes it unit length
- Dot product
  \[
  \mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z \\
  = \| \mathbf{a} \| \| \mathbf{b} \| \cos \theta \\
  \mathbf{a} \cdot \mathbf{a} = \| \mathbf{a} \|^2
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- Cross product
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  \mathbf{a} \times \mathbf{b} = (a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x)
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Arbitrary Axis Rotation

- Rotations about x, y and z axes
- Rotation * rotation = rotation
- Can rotate about any axis direction
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• Rotations about $x$, $y$ and $z$ axes
• Rotation $\times$ rotation $= \text{rotation}$
• Can rotate about any axis direction
• Can do simply with vector algebra
  – Ensure $||v|| = 1$
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  - Then \(p' = o + a \cos q + b \sin q\)
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- Simple solution to rotate a single point
- Difficult to generate a rotation matrix to rotate all vertices in a meshed model