Beziers Curves

Cubic Bezier Curves

A Bezier curve is a parametric polynomial curve given by:

\[ X(t) = (1 - t)^3 b_0 + 3(1 - t)^2 t b_1 + 3(1 - t)t^2 b_2 + t^3 b_3 \]

where \( b_i \) are the control points.

The tangent vector of the curve can be found by

\[ X(t) = 3(b_1 - b_0)(1 - t)^2 + 6(b_2 - b_1)(1 - t)t + 3(b_3 - b_2)t^2 \]

1. The de Casteljau Algorithm

Suppose our control points are

\[ b_0 = (-1, 0) \quad b_1 = (0, 1) \quad b_2 = (0, -1) \quad b_3 = (1, 0) \]

Use the de Casteljau algorithm to find the coordinates of \( X(1/4) \). Check that you get the same answer from using the parametric expression given above.
2. **Tangents to a Bezier Curve**
   
   **a.** What are the tangents at the controls \( b_0 \) and \( b_3 \)?
   
   Give the answer as a pair of parameterized functions.

   **b.** What is the tangent vector at \( t=0.25 \) for the curve given in question one?
3. Newtonian Physics
Suppose we have an initial particle position of (1,2,3) and velocity of < 1, -1, 2 > per second and constant acceleration < 0, 1, -1 > per second per second.

a. What is the position after 5 timesteps each with t=1, using the update equations?

b. What is the position given by integrating acceleration twice with t=5?
4. Collision Detection

The position of the sphere center C moving with velocity \( v \) is given by \( C + tv \) and the equation for a plane can be written as \( n \cdot X = d \) where \( n \) is the normal to the plane and \( d \) is a constant...meaning that all points \( X \) satisfying that equation are on the plane.

Without referring to any notes or slides, derive a formula to find the time \( t \) that the sphere will collide with plane.