

| Reminder: redundancy causes trouble |  |  |
| :---: | :---: | :---: |
| SSN | Name | Phone Number |
| 123-32-1099 123-32-1099 | Fred <br> Fred | (201) 555-1234 (206) 572-4312 |
| 909-43-4444 | Joe | (908) 464-0028 Inconsistency |
| 444 | Joe | (212) 555-4000 |
| 234-56-7890 | Jocelyn | (212) 555-4000 |
| update anomaly | = update one copy of Fred's SSN but not the other |  |
| deletion anomaly | $\begin{array}{r} =\text { dele } \\ \text { lose } \end{array}$ | all Fred's phones, his SSN as a side effect |



## Your common sense will tell you how to fix this schema

```
SSN Name
123-32-1099 Fred
909-43-4444 Joe
```

| SSN | Phone Number |
| :--- | :--- |
| $123-32-1099$ | $(201) 555-1234$ |
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| $909-43-4444$ | $(908) 464-0028$ |
| $909-43-4444$ | $(212) 555-4000$ |

No more update or delete anomalies.


Functional Dependencies

## Functional dependencies generalize the idea of a key

If two tuples agree on the attributes $A_{1}, \ldots, A_{n}$, then they must also agree on attributes $\mathrm{B}_{1}, \ldots, \mathrm{~B}_{\mathrm{n}}$.

$\mathrm{A}_{1}, \ldots, \mathrm{~A}_{\mathrm{n}}$ functionally determine $\mathrm{B}_{1}, \ldots, \mathrm{~B}_{\mathrm{n}}$.


| EmpID | Name | Phone | Office |
| :--- | :--- | :--- | :--- |
| E0045 | Alice | 9876 | SC 2119 |
| E1847 | Bob | 9876 | SC 2119A |
| E1111 | Carla | 9876 | SC 2119A |
| E9999 | David | 1234 | DCL 1320 |

EmpID $\rightarrow$ Name, Phone, Office
Office $\rightarrow$ Phone Phone $\rightarrow$ Office
Name $\rightarrow$ EmpID isn't likely to hold in all instances of this schema, though it holds in this instance
More generally, an instance can tell you many FDs that don't hold, but not all those that do.

## Use your common sense to find the FDs in the world around you

Product: name $\rightarrow$ price, manufacturer
Person: $\quad$ ssn $\rightarrow$ name, age
Company: name $\rightarrow$ stock price, president
School: student, course, semester $\rightarrow$ grade

## We can define keys in terms of FDs

Key of a relation R is a set of attributes that

1. functionally determines all attributes of $R$
2. none of its proper subsets have this property.

Superkey $=$ set of attributes that contains a key.


The closure $\mathrm{S}^{+}$of a set S of FDs is the set of all FDs logically implied by S .
$R=\{A, B, C, G, H, I\}$
$S=\{A \rightarrow B, A \rightarrow C, C G \rightarrow H, C G \rightarrow I, B \rightarrow H\}$ Does $A \rightarrow H$ hold?
You can prove whether it does!

Compute the closure $\mathrm{S}^{+}$of S using Armstrong's Axioms

1. Reflexivity
$A_{1} \ldots A_{n} \rightarrow$ every subset of $A_{1} \ldots A_{n}$
2. Augmentation

If $A_{1} \ldots A_{n} \rightarrow B_{1} \ldots B_{m}$, then $A_{1} \ldots A_{n} C_{1} \ldots C_{k} \rightarrow B_{1} \ldots B_{m} C_{1} \ldots C_{k}$
3. Transitivity

If $A_{1} \ldots A_{n} \rightarrow B_{1} \ldots B_{m}$ and $B_{1} \ldots B_{m} \rightarrow C_{1} \ldots C_{k}$, then $A_{1} \ldots A_{n} \rightarrow C_{1} \ldots C_{k}$

How to compute $\mathrm{S}^{+}$using Armstrong's Axioms
$\mathrm{S}^{+}=\mathrm{S} ;$
loop \{
For each $f$ in $S$, apply the reflexivity and augmentation rules and add the new FDs to $\mathrm{S}^{+}$.
For each pair of FDs in S,
apply the transitivity rule and add the new FDs to $\mathrm{S}^{+}$
\} until $\mathrm{S}^{+}$does not change any more.


It is easy to compute the closure of a set of attributes

Start with $X=\left\{A_{1} \ldots A_{n}\right\}$.
repeat until $X$ doesn't change do:
if $B_{1} \ldots B_{m} \rightarrow C$ is in $S$, and $B_{1} \ldots B_{m}$ are all in $X$, and $C$ is not in $X$
then add C to X .

The closure of a set of attributes contains everything they functionally determine

Given a set S of dependencies, the closure of a set of attributes $\left\{\mathrm{A}_{1} \ldots \mathrm{~A}_{n}\right\}$, written $\left\{A_{1} \ldots A_{n}\right\}^{+}$,
is $\{B$ such that any relation that satisfies $S$ also satisfies $A_{1} \ldots A_{n} \rightarrow B$ \}
$A B \rightarrow C$
$\mathrm{AD} \rightarrow \mathrm{E}$
$B \rightarrow D$
$A F \rightarrow B$
$\{\mathrm{A}, \mathrm{B}\}^{+}=\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$
$\{A, F\}^{+}=\{A, F, B, D, C, E\}$

## What is the attribute closure good for?

1. Test if $X$ is a superkey

- compute $\mathrm{X}^{+}$, and check if $\mathrm{X}^{+}$contains all attrs of $R$

2. Check if $X \rightarrow Y$ holds

- by checking if $Y$ is contained in $X^{+}$

3. Another (not so clever) way to compute closure $\mathrm{S}^{+}$of FDs

- for each subset of attributes $X$ in relation $R$, compute $\mathrm{X}^{+}$with respect to S
- for each subset of attributes $Y$ in $X^{+}$, output the FD X $\rightarrow$ Y


## Reminder: intended goals of schema refinement

- Minimize redundancy
- Avoid information loss
- Easy to check dependencies
- Ensure good query performance


## Normal Forms

First Normal Form = all attributes are atomic Second Normal Form (2NF) = obsolete

Boyce Codd Normal Form (BCNF)
Third Normal Form (3NF)
Fourth Normal Form (4NF)
Others...

## Boyce-Codd Normal Form

A relation $R$ is in BCNF if whenever there is a nontrivial FD $A_{1} \ldots A_{n} \rightarrow B$ for $R$,
$\left\{A_{1} \ldots A_{n}\right\}$ is a superkey for $R$.

An FD is trivial if all the attributes on its right-hand side are also on its left-hand side.

| SSN | Name | $\underline{\text { Phone Number }}$ |
| :--- | :--- | :--- |
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What are the nontrivial functional dependencies?
SSN $\rightarrow$ Name (plus the FDs that can be derived from that) What are the keys?

The only key is \{SSN, Phone Number\}.
How do I know? Augmentation + minimality. Is it in BCNF?

No. SSN is not a key.

## What if we are in a situation where Phone Number $\rightarrow$ SSN?

What are the nontrivial FDs?
Phone Number $\rightarrow$ SSN SSN $\rightarrow$ Name (plus FDs derived from these) What are the keys? Only \{Phone Number\}. How do I know? Augmentation, transitivity, minimality.
Is it in BCNF? No.

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A relation $R$ is in BCNF if whenever there is a nontrivial FD $A_{1} \ldots A_{n} \rightarrow B$ for $R$, $\left\{A_{1} \ldots A_{n}\right\}$ is a superkey for $R$.

What about that alternative schema we recommended earlier---are they in BCNF?

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| :--- | :--- | :--- | :--- |
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| $909-43-4444$ | Joe | $123-32-1099$ | (206) 572-4312 |
| For each relation: <br> What are its important FDs? <br> What |  |  |  |
| $909-43-4444$ | (908) 464-0028 |  |  |

What are its keys?

Is it in BCNF?

A relation $R$ is in BCNF if whenever there is a nontrivial FD A1 ... An $\rightarrow B$ for $R$, $\{A 1$... An\} is a superkey for $R$.

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SSN
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Any 2-attribute relation is in BCNF.

