1980s - {\text{perebor - brute force exponential}}

TSP - Find shortest cycle in G, visits every vertex 0(n!) \to \text{Brute Force: Try every permutation of vertices.}

SAT - Find inputs that make a boolean formula evaluate to True \(O(2^n) \to \text{Brute Force: Try all possible values for inputs}

{\text{Reductions: IF } X \text{ is solvable quickly, then so is } Y}

\begin{align*}
\text{Independent Set} & \quad \text{Q: Largest?} \\
\text{Clique} & \quad \text{Q: Largest?} \\
\text{Vertex Cover} & \quad \text{Q: Smallest?}
\end{align*}

\text{Max Clique} \rightarrow \text{Max IndSet}

\text{Given a graph } G = (V, E) \text{, build a new graph } G' = (V', E')

\begin{align*}
V' &= V \\
E' &= \{uv \mid uv \in E \} \cup \{uv \mid (u, v) \in E \}
\end{align*}

\text{A} \subseteq V \text{ is a clique in } G \text{ iff } A \text{ is independent in } G'

\begin{align*}
\text{Clique} & \quad \rightarrow \quad \text{Max Ind Set}
\end{align*}
\[ T_{\text{clique}}(V) \leq O(V^2) + T_{\text{Ind-set}}(V) \]

**MaxIndependentSet**

- \( G \) (graph) → Complement in \( O(V^2) \) time → \( \bar{G} \) (graph)

**MaxClique**

- \( \bar{G} \) (graph) → Size of largest clique in \( \bar{G} \) → \( k \)

**MaxIndependentSet**

- \( G \) (graph) → Size of smallest vertex cover in \( G \) → \( k \)
- \( n \) (number of vertices in \( G \)) → \( n - k \)

**MaxIndSet(G):**

\[
\text{n} = \#V(G) \\
\text{return } n - \text{MinVertexCover}(G)
\]

Icosian puzzle
Hamiltonian Cycle
= cycle that visits each vertex exactly once

Given a graph $G$, does $G$ have a Ham cycle?

Directed vs. Undirected

Reduce Undir HamCycle to Dir HamCycle

Given undirected $G = (V, E)$

Construct directed $G' = (V, E')$
s.t. $G$ has Ham cycle iff $G'$ has Ham cycle.

Reduce Dir HamCycle to Undir HamCycle

Given dir graph $G$ → Build undir graph $G'$

$G$ has Ham $\iff G'$ has Ham.
Claim 1: If $G$ has $Ham$ then $G'$ has $Ham$ cycle

Given $u \rightarrow v \rightarrow \cdots \rightarrow u$ $Ham$ cycle

Claim 2: $G'$ has $Ham$ cycle $C' \implies G$ has $Ham$

$C'$ visits $v_0$

$C'$ visits $v_0 \rightarrow v_0 \rightarrow v_+$

WLOG otherwise reverse $C'$

after $v_+ \rightarrow w_+ \rightarrow w_0 \rightarrow v_+ \rightarrow x_+ \rightarrow x_0 \rightarrow x_+ \rightarrow y_-$

$C'$ traverses every edge gadget the same way by induction