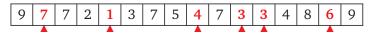
Here are several problems that are easy to solve in O(n) time, essentially by brute force. Your task is to design algorithms for these problems that are significantly faster.

- 1. Suppose we are given an array *A*[1..*n*] of *n* distinct integers, which could be positive, negative, or zero, sorted in increasing order so that *A*[1] < *A*[2] < ··· < *A*[*n*].
  - (a) Describe a fast algorithm that either computes an index *i* such that A[i] = i or correctly reports that no such index exists.
  - (b) Suppose we know in advance that A[1] > 0. Describe an even faster algorithm that either computes an index *i* such that A[*i*] = *i* or correctly reports that no such index exists. [*Hint: This is really easy.*]
- 2. Suppose we are given an array A[1..n] such that  $A[1] \ge A[2]$  and  $A[n-1] \le A[n]$ . We say that an element A[x] is a *local minimum* if both  $A[x-1] \ge A[x]$  and  $A[x] \le A[x+1]$ . For example, there are exactly six local minima in the following array:



Describe and analyze a fast algorithm that returns the index of one local minimum. For example, given the array above, your algorithm could return the integer 9, because *A*[9] is a local minimum. [*Hint: With the given boundary conditions, any array* **must** *contain at least one local minimum. Why*?]

3. Suppose you are given two sorted arrays *A*[1..*n*] and *B*[1..*n*] containing distinct integers. Describe a fast algorithm to find the median (meaning the *n*th smallest element) of the union *A*∪*B*. For example, given the input

$$A[1..8] = [0, 1, 6, 9, 12, 13, 18, 20]$$
  $B[1..8] = [2, 4, 5, 8, 17, 19, 21, 23]$ 

your algorithm should return the integer 9. [Hint: What can you learn by comparing one element of A with one element of B?]

## Harder problem to think about later:

4. Now suppose you are given two sorted arrays *A*[1..*m*] and *B*[1..*n*] and an integer *k*. Describe a fast algorithm to find the *k*th smallest element in the union *A*∪*B*. For example, given the input

$$A[1..8] = [0, 1, 6, 9, 12, 13, 18, 20]$$
  $B[1..5] = [2, 5, 7, 17, 19]$   $k = 6$ 

your algorithm should return the integer 7.