Let $L$ be an arbitrary regular language over the alphabet $\Sigma = \{0, 1\}$. Prove that the following languages are also regular.

1. **Superstrings**($L$) := $\{xyz \mid y \in L$ and $x, z \in \Sigma^*\}$. This language contains all superstrings of strings in $L$. For example:

$$\text{Superstrings}(\{10010\}) = \{10010, 01010010, 1001011, 1001001010, \ldots\}$$

*Hint: This is much easier than it looks.*

2. **Substrings**($L$) := $\{y \mid x, y, z \in \Sigma^*$ and $xyz \in L\}$. This language contains all substrings of strings in $L$. For example:

$$\text{Substrings}(\{10010\}) = \{\epsilon, 0, 1, 00, 01, 10, 001, 010, 100, 0010, 1001, 10010\}$$

3. **Cycle**($L$) := $\{xy \mid x, y \in \Sigma^*$ and $yx \in L\}$. This language contains all strings that can be obtained by splitting a string in $L$ into a prefix and a suffix and concatenating them in the wrong order. For example:

$$\text{Cycle}(\{00K!, 00KOOK\}) = \{00K!, 0K!0, K!00, !00K, 00K00, 0K00K, K00K\}$$

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**Work on these later.**

4. **FlipOdds**($L$) := $\{\text{flipOdds}(w) \mid w \in L\}$, where the function $\text{flipOdds}$ inverts every odd-indexed bit in $w$. For example:

$$\text{flipOdds}(00001110101010) = 10100101111111$$

5. **UnflipOdd1s**($L$) := $\{w \in \Sigma^* \mid \text{flipOdd1s}(w) \in L\}$, where the function $\text{flipOdd1}$ inverts every other 1 bit of its input string, starting with the first 1. For example:

$$\text{flipOdd1s}(00011100101010) = 00001010000100$$

6. **FlipOdd1s**($L$) := $\{\text{flipOdd1s}(w) \mid w \in L\}$, where the function $\text{flipOdd1s}$ is defined in the previous problem.